The Value of Reducing Ambiguity: Financial Advice for Smooth Ambiguity Preferences

Takao Asano[a] Katsumasa Nishide[b]

(a) Faculty of Economics, Okayama University

(b) Graduate School of Business and Finance, Waseda University

Plan of Talk

Background

- **Separation Theorem in CAPM:** All investors allocate their wealth between a risk-free asset and the efficient (market) portfolio.
- Numerous studies argue that the assumption of common beliefs about the joint distribution is overly restrictive and unrealistic.
- Estimating expected returns is particularly challenging.

Background

- Investment advice provided by financial institutions is widely prevalent in practice.
- Some securities firms employ sophisticated estimation techniques to infer the return distribution and correlation structure of individual stocks, using this information to offer stock recommendations to clients.
    - Black-Litterman approach
    - Empirical Bayes CAPM
    - Machine Learning-Enhanced CAPM
- There exists significant asymmetry in estimation capabilities between major financial institutions and individual investors.

Research Questions

- How do individual investors choose financial institutions for investment advice?
- Key determinants include:
  - precision of information
  - physical proximity
  - psychological proximity

Research questions:

1. How do risk and ambiguity preferences influence advisory fees and information acquisition by financial institutions?
2. How does market competition alter these outcomes?

Summary of Study

- This study theoretically examines competition in investment advice between two financial institutions.
- After outlining a general framework, we introduce a specific model that integrates Hara and Honda (2022) with a Hotelling (1929)-type location framework.
- Key features of the model:
  - CARA-Normal environment
  - Agents (individual investors) face both risk and ambiguity regarding expected returns
  - Financial institutions obtain superior information about the return distribution by incurring information-acquisition costs
  - Agents access this information (advice) by paying a fee
  - A Hotelling structure captures switching costs and other market frictions
- We show how ambiguity parameters and other model primitives shape the fees and information precision chosen by financial institutions.

Literature Review

- Smooth ambiguity preference: Klibanoff et al. (2005); Maccheroni et al. (2006); Hara and Honda (2022)
- Financial advice: Admati and Pfleiderer (1986, 1990); Ottaviani and Sørensen (2006); Inderst and Ottaviani (2012)

To the best of our knowledge, this study is the first to propose a theoretical model that analyzes competition in financial advice under ambiguity regarding the return distribution.

Traded Assets

- One-period economy ($t = 0, T$).
- One risky asset (market portfolio) and one risk-free asset are traded at time 0 in the market.
- The risk-free rate is set to zero for simplicity.
- R: realized return of the risky asset.
- X: amount invested in the risky asset.
- W_T^X: wealth at time T given strategy X:

$$W_T^X = W_0 + XR.$$

Utility without Advice

Smooth ambiguity preference:

- Utility of agent i is represented by:

$$U_i(X) = \int_{\pi \in \Delta} \phi_i \left(\int_{s \in S} u_i \left(W_T^X(s) \right) \mathrm{d}\pi(s) \right) \mathrm{d}m_i(\pi),$$

where:

S	sample space,
u_i	felicity function for risk,
π	return distribution regarding risk,
ϕ_i	felicity function for ambiguity,
Δ	set of probability measures on S,
m_i	subjective belief of agent i regarding ambiguity.

- $U_i^* = \max_X U_i(X)$: indirect utility of agent i without advice.

Utility with Advice

- Agent i can obtain advice from Financial Advisor k (hereafter FA k) by paying F_{ki}.
- FA k provides agent i with a more accurate measure $\hat{m}_k$.
- Ex-post utility with the new measure $\hat{m}_k$:

$$\int_{\pi \in \Delta} \phi_i \left(\int_{s \in S} u_i \left(W_T^X(s) - F_{ki} \right) \mathrm{d}\pi(s) \right) \mathrm{d}\hat{m}_k(\pi).$$

Utility with Advice

- Agent i maximizes the utility based on the advice by FA k:

$$\hat{X}_i^* = \operatorname*{argmax}_X \int_{\pi\in\Delta} \phi_i \left(\int_{s\in S} u_i \left(W_T^X(s) - F_{ki} \right) \mathrm{d}\pi(s) \right) \mathrm{d}\hat{m}_k(\pi).$$

- Ex-ante indirect utility is given by:

$$\hat{U}_i^*(F_{ki}) = \int_{\hat{m}_k\in\Delta} \int_{\pi\in\Delta} \phi_i \left(\int_{s\in S} u_i \left(W_T^{\hat{X}_i^*}(s) - F_{ki} \right) \mathrm{d}\pi(s) \right) \mathrm{d}\hat{m}_k(\pi) \mathrm{d}m_i(\hat{m}_k).$$

- Agent i prefers advice from FA k to no advice if:

$$\hat{U}_i^*(F_{ki}) > U_i^*.$$

Model Setting

- We follow Hara and Honda (2022) to construct a specific model under the CARA-Normal setting.
- Felicity functions are given by*:
$$u_i(W) = -e^{-\gamma W}, \quad \phi_i(z) = -(-z)^{\zeta}.$$
Agents are symmetric in terms of preferences.
- Belief about the return R:
$$R \sim \mathcal{N}(\mu_R, v_R).$$
- Each economic agent faces ambiguity about the expected return:
$$\mu_R \sim \mathcal{N}(\bar{\mu}, \bar{v}).$$
There is no ambiguity regarding asset volatility.

*We set $W_0 = 0$ because it does not affect the decision of each agent.

Hotelling-Type Location Model

- Agents are distributed uniformly along the real line $\boldsymbol{R}$.
- The monopolistic FA is located at point 0 (FA 0).
- Accessing advice from FA 0 incurs for agent i not only the fee F_0 but also a transportation cost of $\theta|i|$.
  - The transportation cost quantifies, in monetary terms, the physical, psychological, and other transactional frictions involved in trading with FA 0.
- Total cost F_{0i}:

$$F_{0i} = F_0 + \theta|i|.$$

Utility without Advice

- U_i is calculated as:

$$U_i(X) = \int_{-\infty}^{\infty} -\left(\int_{-\infty}^{\infty} e^{-\gamma XR} n(R;\mu_R, v_R)\,\mathrm{d}R\right)^{\zeta} n(\mu_R;\bar{\mu},\bar{v})\,\mathrm{d}\mu_R$$
$$= -\exp\left\{-\zeta\gamma\bar{\mu}X + \frac{\zeta\gamma^2(v_R+\zeta\bar{v})}{2}X^2\right\},$$

  where $n(\cdot;\mu, v)$ denotes the density function of a normal distribution with mean μ and variance v.

- Optimal strategy:

$$X^* = \frac{\bar{\mu}}{\gamma(v_R+\zeta\bar{v})}.$$

- Indirect utility without advice:

$$U_i^* = -e^{-\frac{\zeta}{2(v_R+\zeta\bar{v})}\bar{\mu}^2}.$$

Information Acquisition by FA

- FA 0 observes a noisy signal of the form:

$$s_0 = \mu_R + \varepsilon_0,$$

where the noise ε_0 follows $\mathscr{N}(0, v_{\varepsilon 0})$, independent of all other random variables.

- Projection theorem gives:

$$\mu_R \mid s_0 \sim \mathscr{N}(\hat{\mu}_0, \hat{v}(v_{\varepsilon 0})),$$

where:

$$\hat{\mu}_0 = \frac{v_{\varepsilon 0}}{\bar{v} + v_{\varepsilon 0}}\bar{\mu} + \frac{\bar{v}}{\bar{v} + v_{\varepsilon 0}}s_0, \quad \hat{v}(v_{\varepsilon 0}) = \bar{v} - \frac{\bar{v}^2}{\bar{v} + v_{\varepsilon 0}}.$$

- Acquisition of the signal incurs a cost:

$$C(v_{\varepsilon 0}) = \frac{c}{v_{\varepsilon 0}}, \quad c > 0.$$

Utility with Advice (Ex-Post)

- Agent i receives the updated distribution $\mathscr{N}(\hat{\mu}_0, \hat{v}(v_{\varepsilon 0}))$ by paying F_{0i}.
- Ex-post utility is calculated as:

$$\int_{-\infty}^{\infty} -\left(\int_{-\infty}^{\infty} e^{-\gamma(XR - F_0 - \theta|i|)} n(R; \mu_R, v_R)\,\mathrm{d}R\right)^{\zeta} n(\mu_R; \hat{\mu}_0, \hat{v}(v_{\varepsilon 0}))\,\mathrm{d}\mu_R$$
$$= -\exp\left\{\zeta\gamma(F_0 + \theta|i|) - \zeta\gamma\hat{\mu}_0 X + \frac{\zeta\gamma^2 v_R + \zeta^2\gamma^2\hat{v}(v_{\varepsilon 0})}{2} X^2\right\}.$$

- Optimal strategy:

$$\hat{X}^*(\hat{\mu}_0) = \frac{\hat{\mu}_0}{\gamma\big(v_R + \zeta\hat{v}(v_{\varepsilon 0})\big)}.$$

Utility with Advice (Ex-Ante)

- Ex-ante distribution of $\hat{\mu}_0$:

$$\hat{\mu}_0 \sim \mathcal{N}(\bar{\mu}, \bar{v} - \hat{v}(v_{\varepsilon 0})).$$

- Indirect utility with advice:

$$\hat{U}_i^*(F_{i0}) = \int_{-\infty}^{\infty} \left(-e^{\zeta\gamma(F_0+\theta|i|) - \zeta\gamma\hat{\mu}_0 \hat{X}^*(\hat{\mu}_0) + \frac{\zeta\gamma^2 v_R + \zeta^2\gamma^2 \hat{v}(v_{\varepsilon 0})}{2} \hat{X}^*(\hat{\mu}_0)^2} \right)$$
$$\times\, n(\hat{\mu}_0; \bar{\mu}, \bar{v} - \hat{v}(v_{\varepsilon 0})) \mathrm{d}\hat{\mu}_0$$
$$= -\frac{\mathrm{e}^{\zeta\gamma(F_0+\theta|i|) - \frac{\zeta}{2(v_R+\zeta\bar{v})}\bar{\mu}^2}}{\sqrt{\frac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}}} = \frac{\mathrm{e}^{\zeta\gamma(F_0+\theta|i|)}}{\sqrt{\frac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}}} U_i^*.$$

- Agent i chooses advice if:

$$\frac{\exp\{\zeta\gamma(F_0+\theta|i|)\}}{\sqrt{\frac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}}} < 1.$$

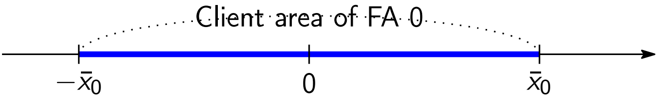
Profit of Monopolistic FA

- Let $\bar{x}_0 > 0$ be the agent who is indifferent between receiving and not receiving advice:

$$\frac{\exp\{\zeta\gamma(F_0 + \theta\bar{x}_0)\}}{\sqrt{\frac{v_R + \zeta\bar{v}}{v_R + \zeta\hat{v}(v_{\varepsilon 0})}}} = 1$$

$$\Rightarrow \bar{x}_0(F_0, v_{\varepsilon 0}) = \frac{1}{\theta}\left[\frac{1}{2\zeta\gamma}\log\left(\frac{v_R + \zeta\bar{v}}{v_R + \zeta\hat{v}(v_{\varepsilon 0})}\right) - F_0\right].$$

- FA 0 wins contracts with agents $i \in [-\bar{x}_0, \bar{x}_0]$.



- Profit of FA 0:

$$\begin{aligned}\Pi_0 =& 2F_0\bar{x}_0(F_0, v_{\varepsilon 0}) - C(v_{\varepsilon 0}) \\ =& -\frac{2}{\theta}F_0^2 + \frac{1}{\gamma\zeta\gamma}\log\left(\frac{v_R + \zeta\bar{v}}{v_R + \zeta\hat{v}(v_{\varepsilon 0})}\right)F_0 - \frac{c}{v_{\varepsilon 0}}.\end{aligned}$$

Optimal Fee and Noise Variance

Proposition 1

The optimal fee and noise variance $(F_M^*, v_{\varepsilon M}^*)$ *satisfy:*

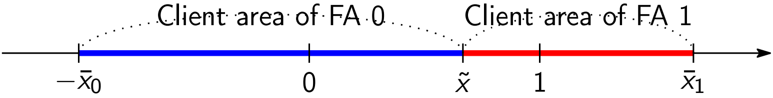
$$\begin{cases} F_M^* = \dfrac{1}{4\zeta\gamma}\log\left(\dfrac{v_R+\zeta\bar{v}}{v_R+\zeta\bar{v}\hat{\rho}(v_{\varepsilon M}^*)}\right), \\ \dfrac{1}{4\zeta\gamma^2\theta}\dfrac{\left(1-\hat{\rho}(v_{\varepsilon M}^*)\right)^2}{v_R+\zeta\bar{v}\hat{\rho}(v_{\varepsilon M}^*)}\log\left(\dfrac{v_R+\zeta\bar{v}}{v_R+\zeta\bar{v}\hat{\rho}(v_{\varepsilon M}^*)}\right)-\dfrac{c}{v_{\varepsilon M}^{*2}}=0, \end{cases}$$

where:

$$\hat{\rho}(v_{\varepsilon M}^*) = \frac{v_{\varepsilon M}^*}{\bar{v}+v_{\varepsilon M}^*}.$$

Competition between Two FAs

- Consider a setting with two financial institutions that are symmetric in their cost of acquiring information.
- FA 0 is located at point 0, while FA 1 is located at point 1.
- If $\bar{x}_M^* = \bar{x}(F_M^*, v_{\varepsilon M}^*) \leq 1/2$, the equilibrium coincides with that of the monopoly market. Therefore, we focus on the case where $\bar{x}_M^* > 1/2$.
- Let $\tilde{x} \in [0,1]$ denote the point at which the agent is indifferent between receiving advice from FA 0 and from FA 1.



(Full coverage in $[0,1]$)

Profit of FA

- We have:

$$-\frac{e^{\zeta\gamma(F_0+\theta\tilde{x})}}{\sqrt{\frac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}}}U^* = -\frac{e^{\zeta\gamma(F_1+\theta(1-\tilde{x}))}}{\sqrt{\frac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 1})}}}U^*$$

$$\Rightarrow \tilde{x}(F_0,F_1,v_{\varepsilon 0},v_{\varepsilon 1}) = \frac{1}{2} - \frac{F_0-F_1}{2\theta} + \frac{1}{4\zeta\gamma\theta}\log\left(\frac{v_R+\zeta\hat{v}(v_{\varepsilon 1})}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}\right).$$

- Profit of FA 0:

$$\Pi_{D0} = F_0\Big(\bar{x}_0(F_0,v_{\varepsilon 0}) + \tilde{x}(F_0,F_1,v_{\varepsilon 0},v_{\varepsilon 1})\Big) - C(v_{\varepsilon 0}).$$

Equilibrium in Duopoly Market

Proposition 2

(i) *If* $x_M^* \leq 1/2$, *then* $F_0 = F_1 = F_M^*$ *and* $v_{\varepsilon 0} = v_{\varepsilon 1} = v_{\varepsilon M}^*$ *in equilibrium.*

(ii) *If* $x_M^* > 1/2$, *the equilibrium fee and noise variance* $(F_D^*, v_{\varepsilon D}^*)$ *satisfy:*

$$\begin{cases} F_D^* = \dfrac{\theta}{5} + \dfrac{1}{5\zeta\gamma}\log\left(\dfrac{v_R + \zeta\bar{v}}{v_R + \zeta\bar{v}\hat{\rho}(v_{\varepsilon D}^*)}\right), \\ \dfrac{3}{4\gamma\theta}\dfrac{\left(1 - \hat{\rho}(v_{\varepsilon D}^*)\right)^2}{v_R + \zeta\bar{v}\hat{\rho}(v_{\varepsilon D}^*)}\left(\dfrac{\theta}{5} + \dfrac{1}{5\zeta\gamma}\log\left(\dfrac{v_R + \zeta\bar{v}}{v_R + \zeta\bar{v}\hat{\rho}(v_{\varepsilon D}^*)}\right)\right) - \dfrac{c}{v_{\varepsilon D}^{*2}} = 0. \end{cases}$$

Brief Sketch of Proof

The first-order conditions are:

$$\frac{\partial}{\partial F_0}\Pi_{D0}(F_0, F_1, v_{\varepsilon 0}, v_{\varepsilon 1}) = 0,$$
$$\frac{\partial}{\partial v_{\varepsilon 0}}\Pi_{D0}(F_0, F_1, v_{\varepsilon 0}, v_{\varepsilon 1}) = 0.$$

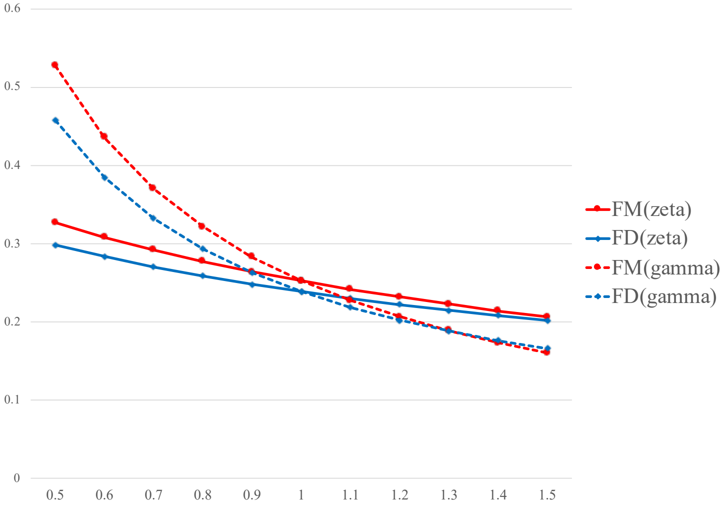
By symmetry, $F_0 = F_1$ and $v_{\varepsilon 0} = v_{\varepsilon 1}$ in equilibrium. The proposition then follows. □

Numerical Analysis

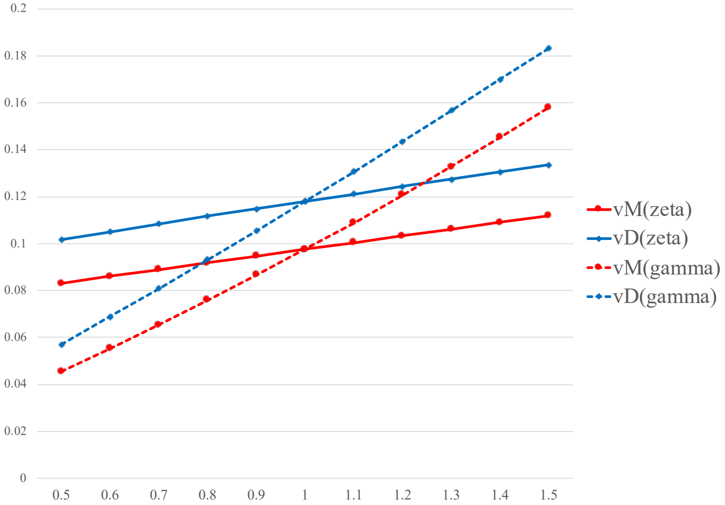
Base-case parameter values:

γ	risk aversion	1
ζ	ambiguity aversion	1
v_R	return variance	1
$\bar{v}$	return ambiguity	2
θ	transportation cost	0.2
c	signal acquisition cost	0.01

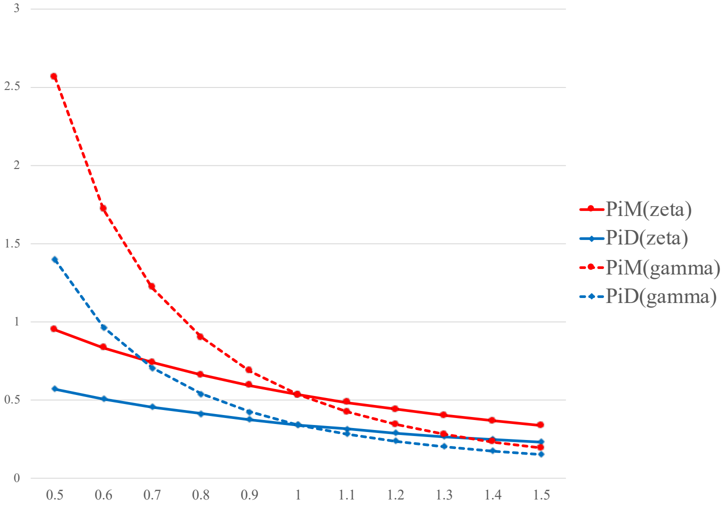
Effect of ζ and γ on Fee



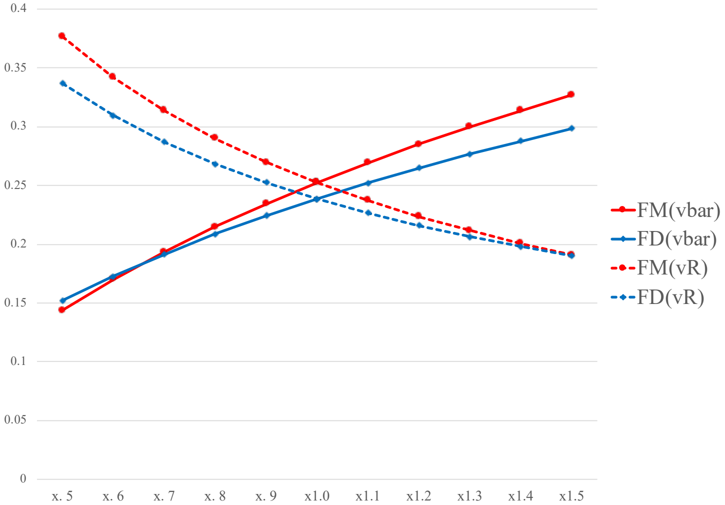
Effect of ζ and γ on Precision Choice



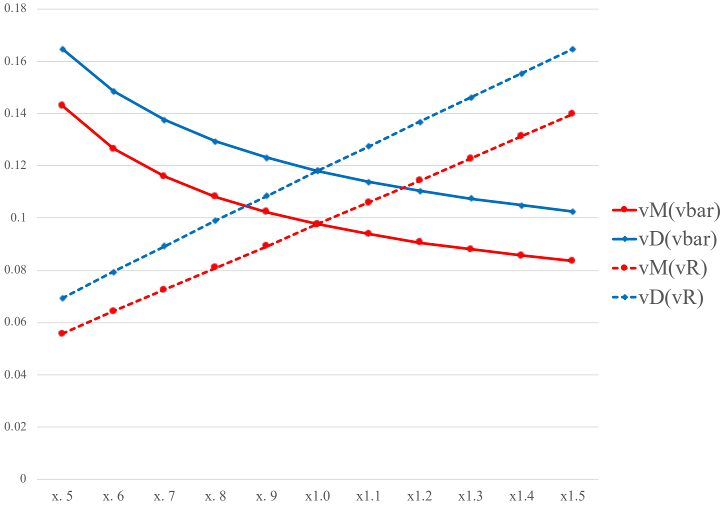
Effect of ζ and γ on FA's profits



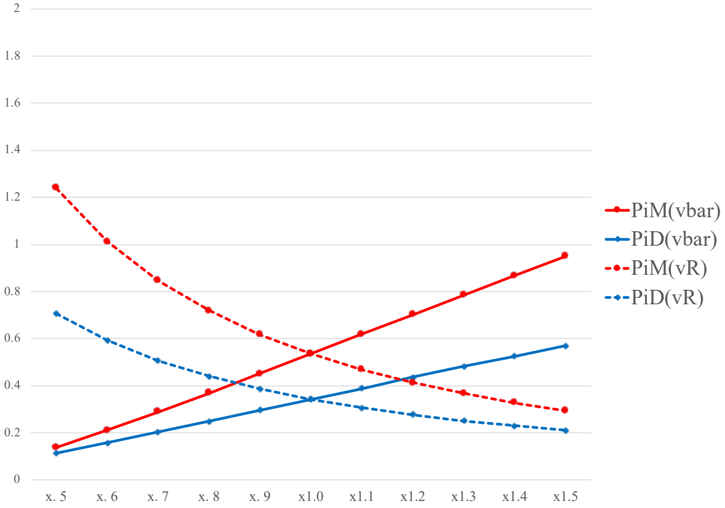
Effect of $\bar{v}$ and v_R on Fee



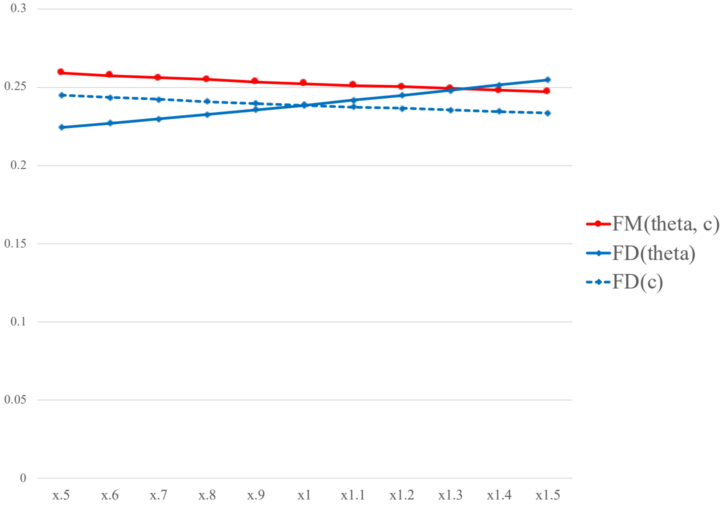
Effect of $\bar{v}$ and v_R on Precision Choice



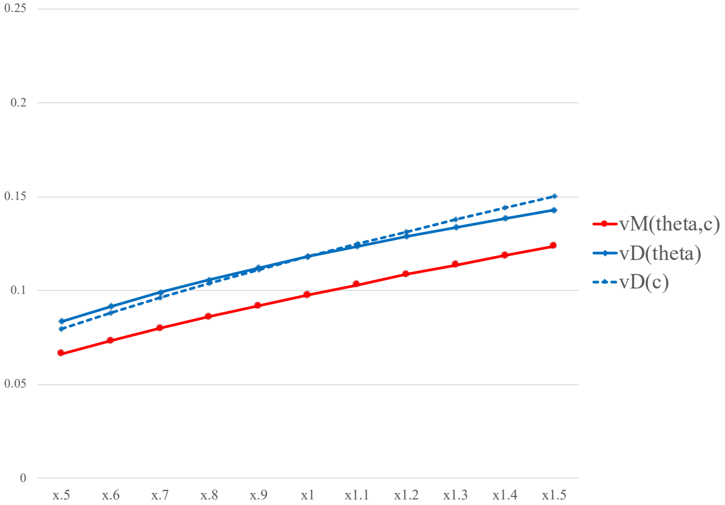
Effect of $\bar{v}$ and v_R on FA's profits



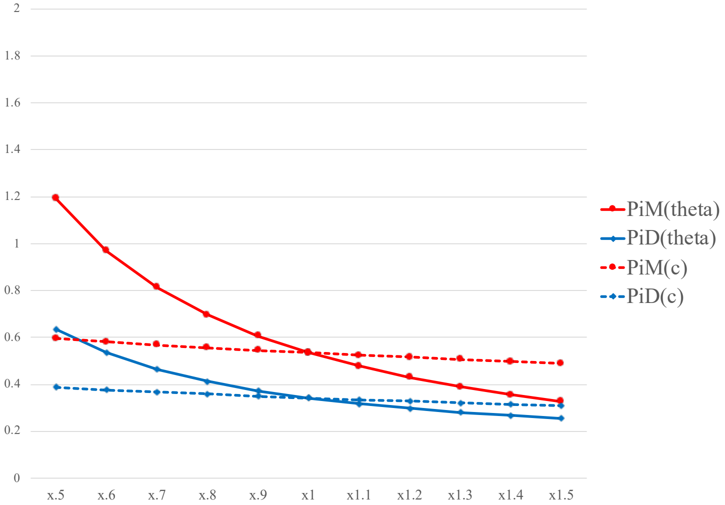
Effect of θ and c on Fee



Effect of θ and c on Precision Choice



Effect of θ and c on FA's profits



Model Implications

Summary of results:

	F_M^*	$v_{\varepsilon M}^*$	$\bar{x}_M^*$	Π_M^*	F_D^*	$v_{\varepsilon D}^*$	$\bar{x}_D^*$	Π_D^*
$\zeta\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\gamma\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\bar{v}\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$v_R\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\theta\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$c\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$

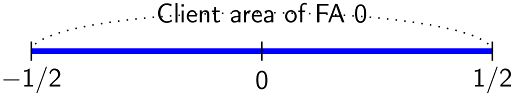
- ζ and γ have the same qualitative effect, but the magnitude is greater for γ.
- $\bar{v}$ and v_R have the opposite effect.
- θ has the opposite effect on F^*'s in the two markets.

Economic Interpretation

- **Effect of ζ and γ:** Higher ambiguity or risk aversion reduces agents' willingness to invest.
  - FAs must lower fees to attract clients.
  - A smaller of client base reduces the marginal benefit of precision, leading to higher noise variance.
- **Effect of $\bar{v}$ and v_R:** Greater ambiguity ($\bar{v}$) raises the value of advice and fees, while higher return volatility (v_R) mainly reduces investment incentives.
- **Effect of θ:** In monopoly, higher transportation cost weakens demand sensitivity $\rightarrow$ lower fee, higher variance. In duopoly, it softens competition $\rightarrow$ higher fees.

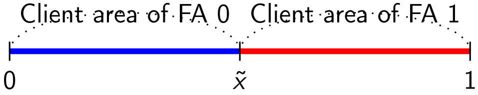
Model Setup

- Agents are uniformly distributed along a unit interval of length 1.
- Two market structures are considered:
  - Monopoly market: FA 0 is located at the midpoint of the interval.



(Full coverage)

  - Duopoly market: Two FAs are located at the endpoints of the interval.



(Full coverage)

Equilibrium in Monopoly Market

Proposition 3

(i) If $\bar{x}_0(F_M^*, v_{\varepsilon M}^*) \leq 1/2$ *in Proposition 1, the equilibrium solution coincides with that in the proposition.*
(ii) Otherwise, the fee and noise variance satisfy:

$$\begin{cases} F_M^* = \dfrac{1}{2\zeta\gamma} \log\left(\dfrac{v_R + \zeta\bar{v}}{v_R + \zeta\bar{v}\hat{\rho}(v_{\varepsilon M}^*)}\right) - \dfrac{\theta}{2}, \\ \dfrac{1}{2\gamma}\dfrac{(1-\hat{\rho}(v_{\varepsilon M}^*))^2}{v_R + \zeta\bar{v}\hat{\rho}(v_{\varepsilon M}^*)} - \dfrac{c}{v_{\varepsilon M}^{*2}} = 0. \end{cases}$$

Brief Sketch of Proof

Statement (i) is immediate. If $\bar{x}_0(F_M^*, v_{\varepsilon M}^*) > 1/2$ in Proposition 1, all agents contract with the FA 0 in equilibrium. Thus, F_M^* and $v_{\varepsilon M}^*$ maximize the following Lagrangian:

$$L = F_0 - \frac{c}{v_{\varepsilon 0}} - \lambda \left(\frac{1}{2} - \bar{x}_0(F_0, v_{\varepsilon 0}) \right).$$

The first-order conditions are:

$$\begin{aligned}
\frac{\partial L}{\partial F_0} &= 1 - \frac{\lambda}{\theta} = 0, \\
\frac{\partial L}{\partial v_{\varepsilon 0}} &= \frac{c}{v_{\varepsilon 0}^2} - \frac{\lambda}{2\zeta\gamma\theta} \frac{\zeta \hat{v}'(v_{\varepsilon 0})}{v_R + \zeta \hat{v}(v_{\varepsilon 0})} = 0, \\
\frac{\partial L}{\partial \lambda} &= \frac{1}{2} - \frac{1}{\theta} \left[\frac{1}{2\zeta\gamma} \log \left(\frac{v_R + \zeta \bar{v}}{v_R + \zeta \hat{v}(v_{\varepsilon 0})} \right) - F_0 \right] = 0.
\end{aligned}$$

The proposition then follows. □

Equilibrium in Duopoly Market

- In a duopoly market, two cases arise:
  - The two client areas are completely separated.
  - The two client areas meet at $\tilde{x}$.
- Profit of FA 0:

$$\Pi_{D0} = \begin{cases} F_0 \bar{x}_0(F_0, v_{\varepsilon 0}) - C(v_{\varepsilon 0}), & \text{if client areas are separated,} \\ F_0 \tilde{x} - C(v_{\varepsilon 0}), & \text{if client areas are connected.} \end{cases}$$

Equilibrium in Duopoly Market

Proposition 4

(i) If the solution of the following system satisfies $\bar{x}_0(F_D^*, v_{\varepsilon D}^*) < 1/2$, *then* $(F_D^*, v_{\varepsilon D}^*)$ *is the equilibrium:*

$$\begin{cases} F_D^* = \dfrac{1}{4\zeta\gamma}\log\left(\dfrac{v_R+\zeta\bar{v}}{v_R+\zeta\bar{v}\hat{\rho}(v_{\varepsilon D}^*)}\right), \\ \dfrac{1}{8\zeta\gamma^2\theta}\dfrac{(1-\hat{\rho}(v_{\varepsilon D}^*))^2}{v_R+\zeta\bar{v}\hat{\rho}(v_{\varepsilon D}^*)}\log\left(\dfrac{v_R+\zeta\bar{v}}{v_R+\zeta\bar{v}\hat{\rho}(v_{\varepsilon D}^*)}\right) - \dfrac{c}{v_{\varepsilon D}^{*2}} = 0. \end{cases}$$

(ii) Otherwise, $(F_D^*, v_{\varepsilon D}^*)$ *satisfies:*

$$\begin{cases} F_D^* = \theta, \\ \dfrac{1}{4\gamma}\dfrac{(1-\hat{\rho}(v_{\varepsilon D}^*))^2}{v_R+\zeta\bar{v}\hat{\rho}(v_{\varepsilon D}^*)} - \dfrac{c}{v_{\varepsilon D}^{*2}} = 0. \end{cases}$$

Brief Sketch of Proof

(i) If client areas are completely separated, $(F_D^*, v_{\varepsilon D}^*)$ maximizes:

$$\begin{aligned}\Pi_{D0} =& F_0\bar{x}_0 - C(v_{\varepsilon 0})\\ =& \frac{F_0}{\theta}\left[\frac{1}{2\zeta\gamma}\log\left(\frac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}\right) - F_0\right] - \frac{c}{v_{\varepsilon 0}}.\end{aligned}$$

First-order conditions:

$$\frac{\partial \Pi_{D0}}{\partial F_0} = 0, \quad \frac{\partial \Pi_{D0}}{\partial v_{\varepsilon 0}} = 0,$$

which yield the equations in (i).

Brief Sketch of Proof (cont.)

(ii) If client areas are connected, FA 0 maximizes:

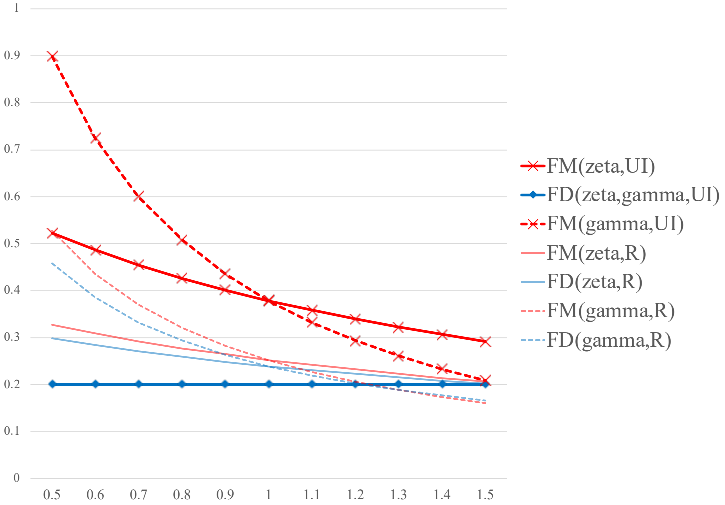
$$\begin{aligned}\Pi_{D0} =& F_0\tilde{x}(F_0, F_1, v_{\varepsilon 0}, v_{\varepsilon 1}) - C(v_{\varepsilon 0}) \\ =& \frac{F_0}{\theta}\left[\frac{1}{2} - \frac{F_0 - F_1}{2\theta} + \frac{1}{4\zeta\gamma\theta}\log\left(\frac{v_R + \zeta\hat{v}(v_{\varepsilon 1})}{v_R + \zeta\hat{v}(v_{\varepsilon 0})}\right)\right] - \frac{c}{v_{\varepsilon 0}}.\end{aligned}$$

First-order conditions:

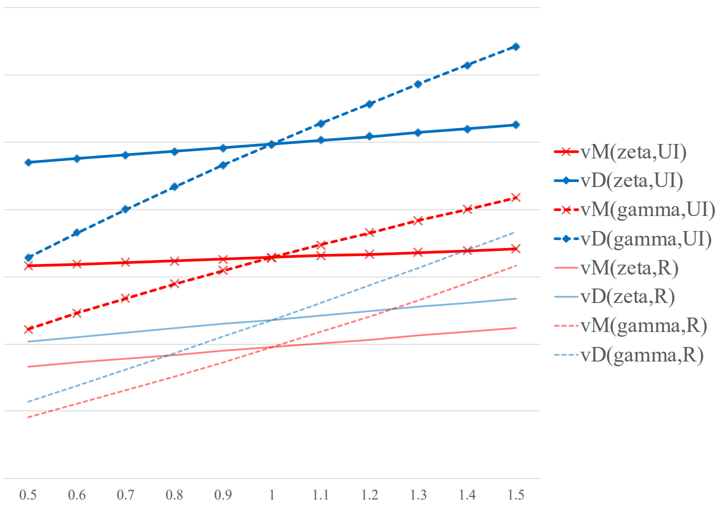
$$\frac{\partial \Pi_{D0}}{\partial F_0} = 0, \quad \frac{\partial \Pi_{D0}}{\partial v_{\varepsilon 0}} = 0.$$

Now Statement (ii) follows from noticing that $F_0 = F_1$ and $v_{\varepsilon 0} = v_{\varepsilon 1}$. □

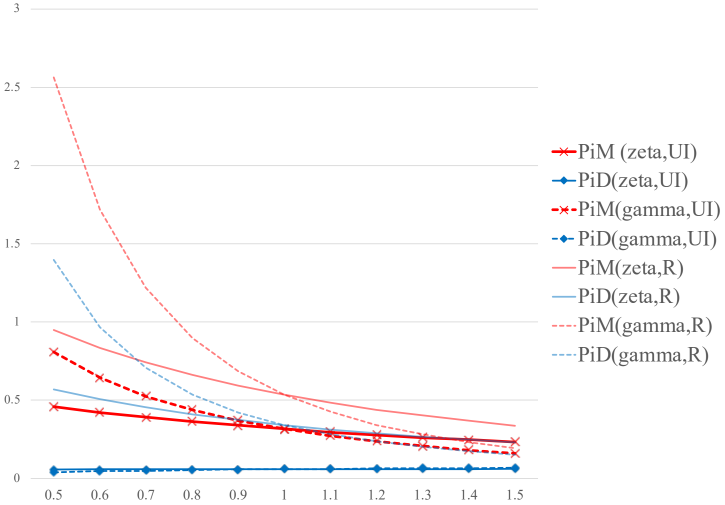
Effect of ζ and γ on Fee in Market $[0, 1]$



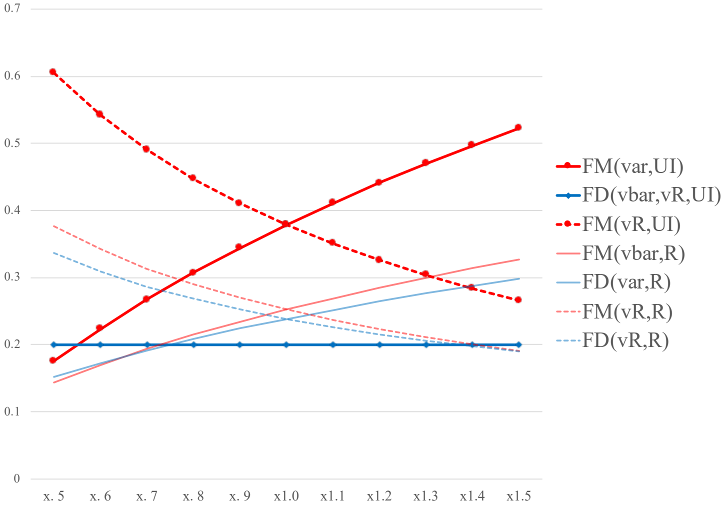
Effect of ζ and γ on Precision Choice in Market $[0, 1]$



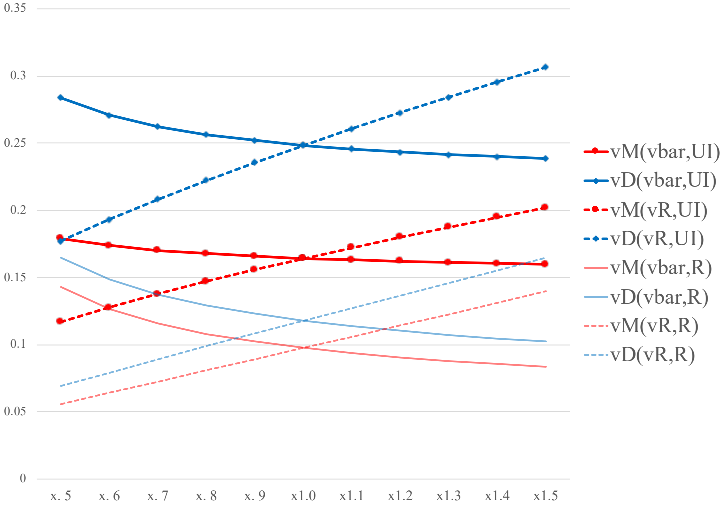
Effect of ζ and γ on FA's profits in Market $[0,1]$



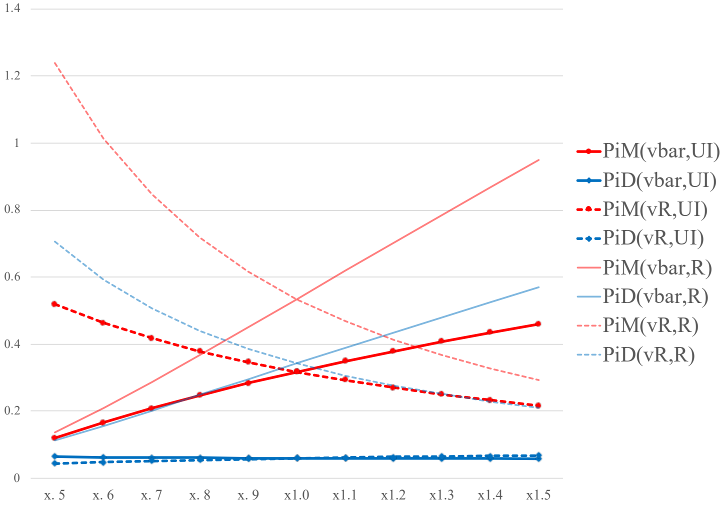
Effect of $\bar{v}$ and v_R on Fee in Market $[0,1]$



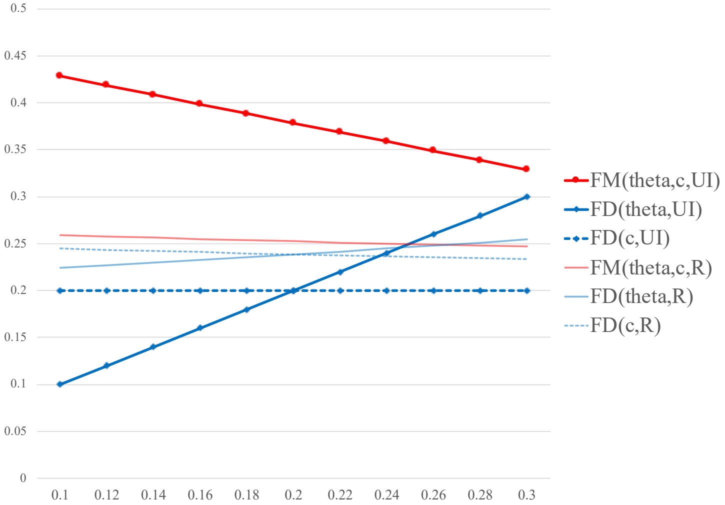
Effect of $\bar{v}$ and v_R on Precision Choice in Market $[0, 1]$



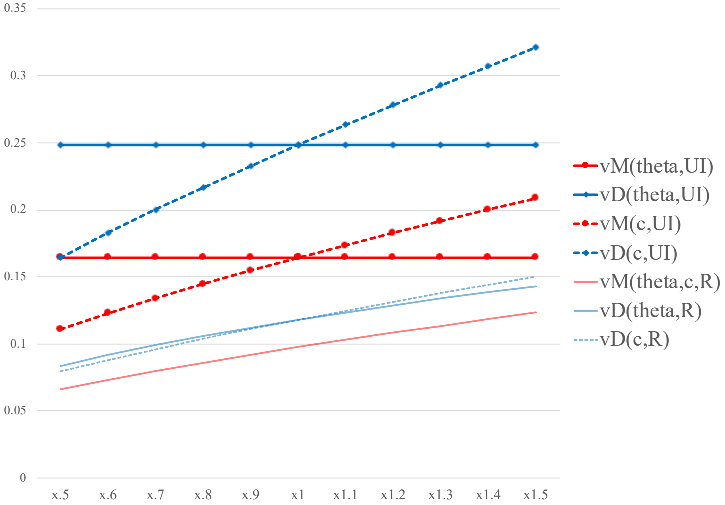
Effect of $\bar{v}$ and v_R on FA's profits in Market $[0,1]$



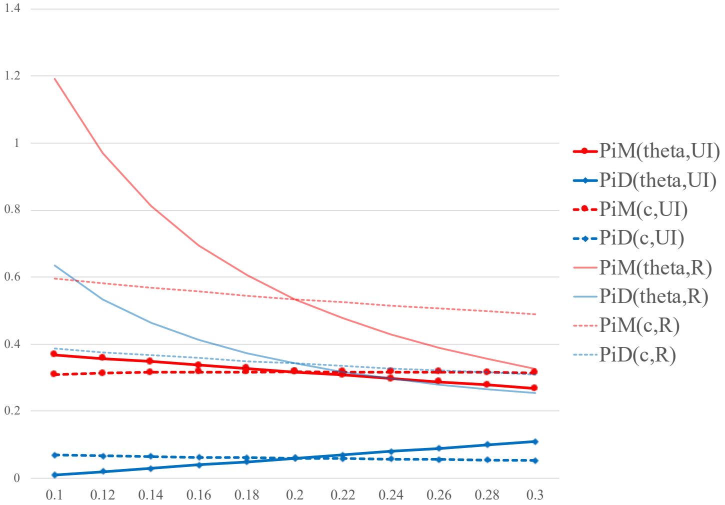
Effect of θ and c on Fee in Market $[0,1]$



Effect of θ and c on Precision Choice in Market $[0, 1]$



Effect of θ and c on FA's profits in Market $[0,1]$



Model Implications

Summary of results:

	F_M^*	$v_{\varepsilon M}^*$	Π_M^*	F_D^*	$v_{\varepsilon D}^*$	Π_D^*
$\zeta\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\rightarrow$	$\uparrow$	$\uparrow$
$\gamma\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\rightarrow$	$\uparrow$	$\uparrow$
$\bar{v}\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\rightarrow$	$\downarrow$	$\uparrow$
$v_R\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\rightarrow$	$\uparrow$	$\uparrow$
$\theta\uparrow$	$\downarrow$	$\rightarrow$	$\downarrow$	$\uparrow$	$\rightarrow$	$\uparrow$
$c\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\rightarrow$	$\uparrow$	$\downarrow$

- The effects on Π_D^* differ between the two market structures.
  - Unlike the previous case, financial institutions cannot expect to cultivate demand in monopolized regions.
- θ is a key determinant of F^*, while c plays that role for v_ε^*.

Certainty Equivalent

- In our CARA setting, certainty equivalent with respect to risk and ambiguity is given by:

$$-\frac{1}{\zeta\gamma}\log(-U_i).$$

- Certainty equivalents for indirect utilities are calculated as:

$$\begin{cases} \dfrac{\bar{\mu}^2}{2\gamma(v_R+\zeta\bar{v})} & \text{without advice,} \\ \dfrac{\bar{\mu}^2}{2\gamma(v_R+\zeta\bar{v})}+\dfrac{1}{2}\log\left(\dfrac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}\right)-F_0-\theta|i| & \text{with advice from FA 0.} \end{cases}$$

- Surplus of agent i for financial advice from FA 0 can be defined as

$$\frac{1}{2}\log\left(\frac{v_R+\zeta\bar{v}}{v_R+\zeta\hat{v}(v_{\varepsilon 0})}\right)-F_0-\theta|i|.$$

Consumer Surplus in Market $\boldsymbol{R}$

- In market $\boldsymbol{R}$, the consumer surplus from financial advice is:

$$\int_{-\bar{x}_M^*}^{\bar{x}_M^*} \left(\frac{1}{2} \log \left(\frac{v_R + \zeta \bar{v}}{v_R + \zeta \bar{v} \hat{\rho}(v_{\varepsilon M}^*)} \right) - F_0 - \theta |i| \right) \mathrm{d}i = \theta \bar{x}_M^{*2}$$

for monopoly, and

$$2 \int_{-\bar{x}_D^*}^{1/2} \left(\frac{1}{2} \log \left(\frac{v_R + \zeta \bar{v}}{v_R + \zeta \bar{v} \hat{\rho}(v_{\varepsilon D}^*)} \right) - F_0 - \theta |i| \right) \mathrm{d}i = \theta \left(\bar{x}_D^{*2} + \bar{x}_D^* - \frac{1}{4} \right).$$

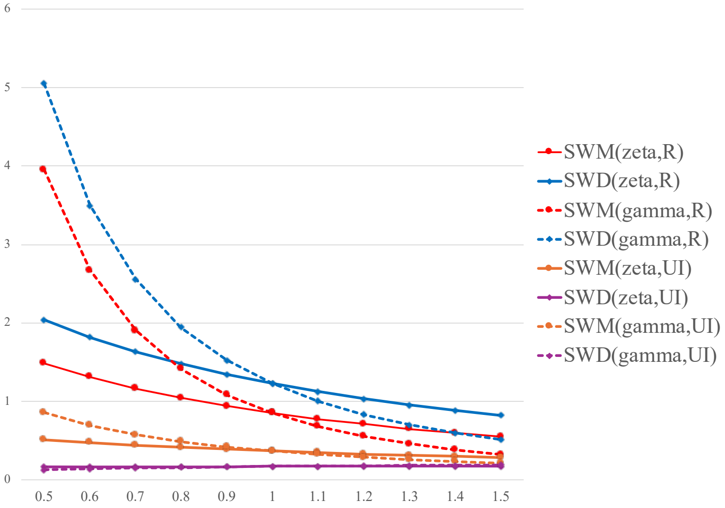
for duopoly.

- The behavior of x^* indicates how consumer surplus changes.

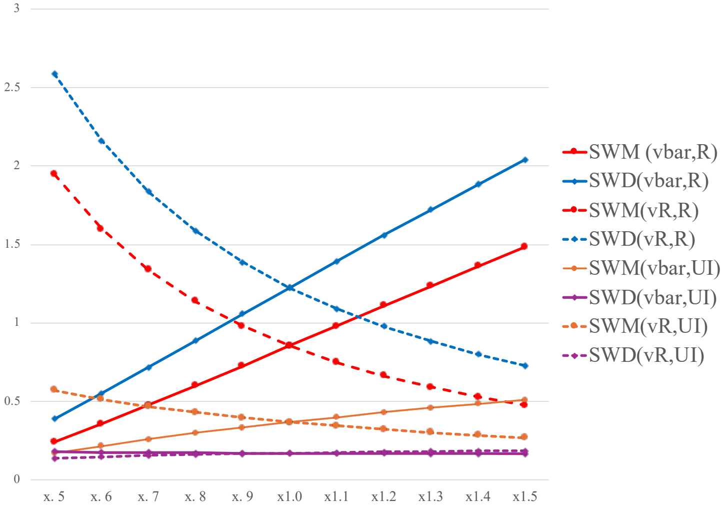
Consumer Surplus in Market $[0,1]$

- Consider the case of full coverage.
- In the unit interval market, the consumer surplus from financial advice is:
$$\int_{-1/2}^{1/2}\left(\frac{1}{2}\log\left(\frac{v_R+\zeta\bar{v}}{v_R+\zeta\bar{v}\hat{\rho}(v^*_{\varepsilon M})}\right)-F_0-\theta|i|\right)\mathrm{d}i=\frac{\theta}{4}$$
for monopoly, and
$$2\int_{0}^{1/2}\left(\frac{1}{2}\log\left(\frac{v_R+\zeta\bar{v}}{v_R+\zeta\bar{v}\hat{\rho}(v^*_{\varepsilon D})}\right)-F_0-\theta|i|\right)\mathrm{d}i=\frac{\theta}{4}.$$
for duopoly.
- θ solely determines the effect of consumer surplus.
- Competition has no impact on the consumer surplus.

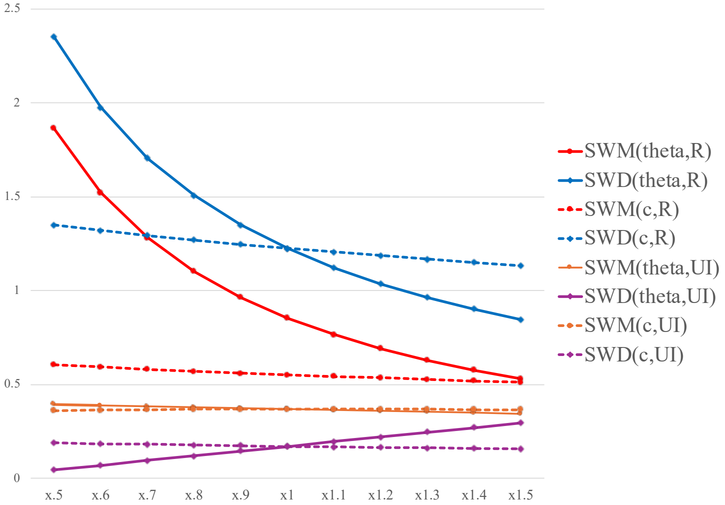
Effect of ζ and γ on Social Welfare



Effect of $\bar{v}$ and v_R on Social Welfare



Effect of θ and c on Social Welfare



Conclusion

- We examined the value of financial advice under smooth ambiguity preferences.
- Using a Hotelling spatial competition framework, we characterized equilibrium advisory fees and signal precision in monopolistic and duopolistic markets.
- Numerical analysis reveals that:
  - (i) Lower ambiguity aversion increases advice value, information precision, and market coverage.
  - (ii) Dispersion in uncertainty and ambiguity has opposite effects on fees and information precision.

References I

Admati, A. R. and P. Pfleiderer (1986), "A Monopolistic Market for Information," *Journal of Economic Theory*, **39**(2), 400–438.

Admati, A. R. and P. Pfleiderer (1990), "Direct and Indirect Sale of Information," *Econometrica*, **58**(4), 901–928.

Hara, C. and T. Honda (2022), "Implied Ambiguity: Mean-Variance Inefficiency and Pricing Errors," *Management Science*, **68**(6), 4246–4260.

Hotelling, H. (1929), "Stability in Competition," *Economic Journal*, **39**(153), 41–57.

Inderst, R. and M. Ottaviani (2012), "Competition through Commissions and Kickbacks," *American Economic Review*, **102**(2), 780–809.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005), "A Smooth Model of Decision Making under Ambiguity," *Econometrica*, **73**(6), 1849–1892.

Maccheroni, F., M. Marinacci, and A. Rustichini (2006), "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences," *Econometrica*, **74**(6), 1447–1498.

Ottaviani, M. and P. N. Sørensen (2006), "The Strategy of Professional Forecasting," *Journal of Financial Economics*, **81**(2), 441–466.

Thank you for your attention