Protest to Remove a Product from the Market: A Model of an Informal Election for a Monopoly

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Abstract

This study builds a model of a protest aiming at a firm to remove its product causing negative externalities. The firm and the consumers are uncertain about the value and the extent of feeling guilty about consumers' purchasing the product, but the consumers receive noisy signals. The firm cares about protests when it reveals the unpopularity of its product. In our model, price is a key for information aggregation. With a higher price, the utilities from the situation where the product is on sale are close between consumers receiving good and bad news about the product quality. Then, the cost of participating in the protest becomes close between these consumers, and the protest is uninformative about consumers' received signals. We show that with an endogenized price, the protest is uninformative when the consumers' signal is highly precise but is informative when the signal is less accurate. This suggests that consumers' ignorance may contribute to the protest's success.

Keywords: Protest, boycotts, information aggregation, ethical voters, pricing

JEL Classification: D42, D72, D81, D82

1. Introduction

Firms sometimes face protest campaigns to remove their products from the market due to negative externalities (Egorov and Harstad, 2017). Negative externalities include pollution, workplace conditions (e.g., the use of child labor and poor working conditions), and political statements (e.g., fostering discrimination against minorities).[1] Although protest campaigns for firms' products are common, one may doubt their effects. Firms are not obligated to discontinue product sales even when many people participate, as protest participants need not be customers. Despite this concern, protest campaigns still impact firms' decisions as they may reveal information about the product's popularity. People would be reluctant to participate in campaigns aiming for product discontinuation if the product is precious. In contrast, if the product is less valuable, the negative externality exceeds the product value, which promotes participation in the protests. Then, protest campaigns are a signal of product popularity. Once poor popularity is revealed, the firm decides to discontinue the product. This study investigates this motivation for protest campaigns and shows when protests are informative by building a game-theoretic model.

In our model, a continuum of common-interested consumers decides whether to participate in protest campaigns to remove a firm's product with a negative externality. The firm and consumers observe public signals about the protest's turnout, which is a sum of actual turnout and noise followed by a normal distribution. Observing the public signal, the firm decides whether

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[1] Many protest activities are aiming for product discontinuation, E.g., protests for Nike for labor violations (e.g., "Protesters call on NYU Bookstore to cut ties with Nike", *Washington Square News*, Oct 13, 2023), a threat against the publication of a book ("Japan firm nixes translation of U.S. book questioning trans surgery", *Japan Times*, Dec. 6, 2023), and so on.

to continue the sale. The firm's objective is to maximize its profit. The firm decides whether to discontinue the product, observing the protest's turnout. We assume the firm is uncertain about the product's value for consumers. The value includes the guilty feelings that purchasing a product causes negative externalities. Further, continuing product sales is costly. Then, if the protest is sufficiently informative and informs the firm that the product is not popular, the firm discontinues the sales. The consumers are also uncertain about the product value but receive noisy (and binary) signals about it.[2] Therefore, the protest has a function of information aggregation. Once the firm decides to continue the sale, the consumers decide whether to purchase the product. This action affects the value of protest campaigns because consumers are reluctant to participate in the protest if the purchase brings much utility. The price of the product is then a key factor in consumers' decisions to participate in the protest. We assume that the firm commits to a price before the protest campaign.

When the price is given, we show that the protest can be informative when the price is intermediate. Otherwise, the protest is no longer informative. The intuition is as follows. If the price is too low, the firm gives up continuing the product sale irrelevantly to the protest turnout. On the contrary, with a high price, the consumers gain less value from purchasing products even if they receive good signals. Then, consumers' benefits from discontinuing product sales become similar levels irrelevant to whether the signals are good or bad. Then, the protest turnout becomes less informative about the consumers' received signals.

The above finding has implications for the firm's pricing strategy. Suppose the consumers' signals are sufficiently precise. Then, the firm prefers to set a high price so that only consumers receiving good signals will purchase the product. However, this makes the protest uninformative. In summary, consumers' highly precise information makes the protest campaign uninformative. Conversely, suppose the consumers' signals are less precise. In that case, even consumers receiving good signals have less willingness to pay. So, the firm prefers to set a price so that both types (i.e., receiving good and bad signals) of consumers purchase. In this case, consumers receiving good signals obtain an information rent, while ones with bad signals do not get the rent from product purchases. This discriminates the motivations for protest participation between consumers receiving good and bad signals. Then, the protest becomes informative. To underscore the novelty of our findings, we note that the informativeness of individual signals and that of the protest may have negative correlations, a departure from previous studies. For instance, while the model in Ekmekci and Lauermann (2022) shares similarities with ours, it focuses on a protest against a government where the price is not a variable. In their case, they demonstrate that the protest becomes more informative as individual signals become more precise.

Our results seem to depend on the decision flow, in which the firm commits to a price before the consumers decide on participation. Hence, we also analyze the situation when the firm decides the price after protest turnout is observed. The difference arises in the motivation of participation in the protest. Participation in an informative protest signals the unpopularity of the product to the firm; the firm lowers the price of its product, but it also benefits the consumers. However, even in this twist, a similar result continues to hold. With higher signal precision, the protest is more likely to be uninformative. On the contrary, with intermediate signal precision, it becomes informative.

Our result suggests that consumers' ignorance contributes to the successful protest campaigns against the firm to withdraw its product.

[2] We can interpret this as a model of endogenized social norm. That is, each consumer cares about the norm if a majority of consumers care about purchasing the product causing negative externalities; otherwise, not. E.g., as a similar setting, Fischer and Huddart (2008) consider players caring average norms.

1.1. Related Literature

Many studies have investigated the consumers' motivation to participate in protests, welfare consequences, and aggregation of dispersed information. Among the studies, our study contributes to the literature on information aggregation when the threshold of the voting outcome is endogenous. Battaglini (2017) studies a model of protest where the government's decision is made after observing protest turnout. The model of Ekmekci and Lauermann (2022) is closer to ours in the sense that voters are a continuum.[3]

Our study also related to studies of boycotts. Some studies (e.g., Diermeier and Van Mieghem, 2008, Delacote, 2009) model a boycott as a discrete public good game with a fixed threshold. In this modeling, as Delacote (2009) discusses, boycott campaigns have a weak point: While the participation of consumers who have a higher willingness to pay is more significant to boycott, they have less incentive to participate. In our study, however, this point is significant in making the protest informative and signaling to the firm that their product is unpopular.

There are many other models of boycotts. For example, Egorov and Harstad (2017) built a model of a boycott as a war of attrition. Baron (2001, 2003) considers an activist launching a boycott and analyzes the effects on strategic CSR behaviors. Several studies consider a market of credence goods and model a boycott as an instrument to make firms behave to benefit consumers. Feddersen and Gilligan (2001) and Innes (2006) consider a market with moral-concerned consumers and an activist. Feddersen and Gilligan (2001) consider a boycott as a signaling by the activist, while Innes (2006) shows that boycotts can arise under symmetric information. Miyagiwa (2009) considers consumers who purchase less as they become more suspicious of firms' bad behavior. Glazer et al. (2010), Heijnen and Made (2012), and Peck (2017) consider boycotts as signaling of their moral concerns by consumers. In our study, protest campaigns can be considered to signal moral concerns by interpreting the product's value as a degree of (not) concern for morality. The main difference is that our study assumes consumers are also uncertain about their moral concerns, and a problem of information aggregation arises.

Finally, we remark that our model is related to a study of turnout in a large election. Although it is well known that almost all electorates abstain from voting in costly voting models, Feddersen and Sandroni (2006) provide a model to explain high turnout by considering a group-wise utilitarian electorate. In their model, the electorate, referred to as an *ethical voter* decides whether to abstain from voting to maximize the total utilities of people who support the same candidates. Ali and Lin (2013) show that reputation-concerned electorates behave as ethical voters. Ekmekci and Lauermann (2022) also consider ethical voters in a model of an informal election, including protests.

2. Model

A firm sells a product to a continuum of consumers with unit demand. The population is normalized to 1. The marginal cost of production is $w\kappa \geq 0$. The utility of purchasing the product is wv, where $v \in \{0, 1\}$. We refer to v as product quality[4] and w as the scale parameter. The prior probability of $v = 1$ is assumed $\frac{1}{2}$ for notational simplicity.

Selling the product has a negative externality for the consumers. Consumers gain disutility $\varsigma > 0$ when the product is sold. Consumers can conduct a protest campaign to stop the sale of the product. Participating in the campaign costs c_i for consumer i. c_i is identically and indepen-

[3] Relatedly, Correa (2022) considers a dynamic model of protest with a continuum of people. Battaglini et al. (2020) conduct an experiment of a model similar to Battaglini (2017) and show that information sharing among the group enhances information aggregation.

[4] v includes a psychological factor like feeling guilty about purchasing a product having a negative externality.

dently distributed by function F among consumers. Then, when the product is sold at price wp, the consumer's utility is as follows (χ is the indicator function).

$$u_i = w \cdot (v - p) \cdot \chi(\text{purchase}) - \varsigma - c_i \cdot \chi(\text{participate in protest}).$$

Before participating in the protest campaign, each consumer gains information about the product quality, v. The consumer receives signal $\theta \in \{\theta_H, \theta_L\}$. If $v = 1$, each consumer receives θ_H with probability $\mu \in \left(\frac{1}{2}, 1\right)$ and θ_L with probability $1 - \mu$. If $v = 0$, each receives θ_L with probability μ. Then, θ_H indicates that v is likely to be high.

After the protest campaign, the firm and consumers observe a public signal, indicating the protest campaign's scale. Let the actual participation ratio be τ. The firm and consumers observe $t = \tau + \varepsilon$, where ε is a normal noise with 0 means and σ^2 variance. Let Φ be the standard normal distribution function and φ be its density.

Observing the public signal t, the firm decides whether to discontinue the product. We assume that continuing product sales cost $wK > 0$. When the product continues to be sold, the consumers decide whether to purchase the product.

The following figure summarizes the decision flow.

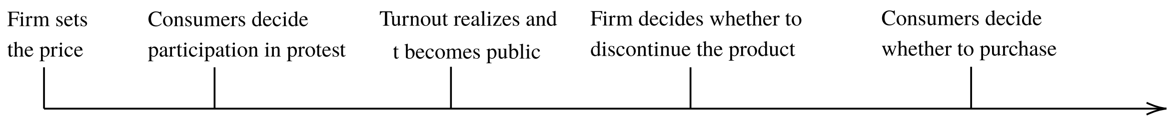

Figure 1: Timeline of the decision flow

Finally, the decision to participate in the protest follows the setting of Ekmekci and Lauermann (2022) and Feddersen and Sandroni (2006). That is, each consumer behaves to maximize the overall utility of the consumers (referred to as *ethical* consumer). In other words, each consumer participates if and only if the marginal effect of participation on expected utility exceeds the participation cost.

3. Equilibrium with a given price

3.1. After public signal observation

3.1.1. Consumer's choice

This section considers the condition of turnout when consumers purchase the product after public signal observation. Consider the consumer receiving signal θ. The consumer purchases if the expected product quality exceeds the price, that is, $\Pr(v = 1 \mid \theta, t) \geq p$. By abusing notation, we write $P_\theta(t) = \Pr(v = 1 \mid \theta, t)$. The expected value of product quality is

$$P_\theta(t) = \frac{\Pr(t \mid v = 1)\theta}{\Pr(t \mid v = 1)\theta + \Pr(t \mid v = 0)(1 - \theta)}.$$

Let λ_v be the true turnout when the realized quality is $v \in \{0, 1\}$. Then, $P_\theta(t)$ is rewritten as

$$P_\theta(t) = \frac{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta}{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta + \varphi\left(\frac{t-\lambda_0}{\sigma}\right)(1 - \theta)}.$$

By using this formula, when we assume $\lambda_0 > \lambda_1$, inequality $P_\theta(t) > p$ is rewritten as

$$t < \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\theta}{1-\theta}\right) - \ln\left(\frac{p}{1-p}\right)}{\lambda_0 - \lambda_1} =: T_\theta(p).$$

In other words, $T_\theta(p)$ is the supremum turnout with which the type-θ consumers purchase the product. The following lemma shows that the type-H consumers are likelier to purchase the product than the type-L consumers.

Lemma 1. *If $\lambda_0 > \lambda_1$, $T_L(p) < T_H(p)$ for any p.*

3.1.2. The firm's choice

Given the consumer's choice observing public signal t, this section considers the condition when the firm discontinues the product sale. We focus on a cutoff equilibrium; the firm discontinues the product sale if and only if $t > T$. Now we calculate T.

Suppose that $T_L(p) < T < T_H(p)$. Then, at $t = T$, only the H-type consumer purchases the product. In this case, the firm's profit is

$$\Pi_H := w([\Pr(v = 1 \mid T)\mu + \Pr(v = 0 \mid T)(1 - \mu)](p - \kappa) - K).$$

This is because, when $v = 1$, the population is H-type is μ while that is $1 - \mu$ when $v = 0$. As the firm's decision is independent of w, we omit the notation w for the firm's profit hereafter.

Note that $\Pr(v = 1 \mid T)\mu + \Pr(v = 0 \mid T)(1 - \mu) \in [1 - \mu, \mu]$. Therefore, if $p - \kappa \geq \frac{K}{1-\mu}$, the expected profit is no less than 0. Then, the firm always keeps the sale. In contrast, if $\frac{K}{\mu} \geq p - \kappa$, the expected profit is no more than 0. Then, the firm discontinues the sale.

Otherwise $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$. In this case, the firm discontinues the sale if and only if

$$T \leq \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)}{\lambda_0 - \lambda_1} =: T_f(p).$$

The following lemma summarizes the discussion.

Lemma 2. *Let Π_H be the least turnout signal that the firm discontinues the sale when the H-type purchases, but the L-type does not. Then, if $p - \kappa > \frac{K}{1-\mu}$, $\Pi_H > 0$. If $\frac{K}{\mu} > p - \kappa$, $\Pi_H < 0$. If $\frac{K}{1-\mu} > p - \kappa > \frac{K}{\mu}$, $\Pi_H \leq 0$ if and only if $t \leq T_f(p)$.*

Although we focus on the case only the H-type purchases when the firm discontinues the product, the remaining cases are discussed in the following sections.

3.2. Before public signal observation

Before the public signal observation, the consumer chooses whether to participate in the protest. This section examines the condition and its informativeness regarding product quality.

We focus on the cutoff strategy: each type-θ consumer participates in protest if and only if the marginal benefit of participation exceeds its cost. Following Ekmekci and Lauermann (2022), we consider *ethical* consumers: they maximize the aggregate utility of the same type of consumers written in the following formula.[5][6]

$$\int^{T - \mathrm{d}\tau} \left(w[P_\theta(t) - p]_+ - \varsigma\right)\xi_\theta(t)\,\mathrm{d}t,$$

where we define $[\cdot]_+ = \max\{\cdot, 0\}$ and $\xi_\theta(t) = \frac{1}{\sigma}\left(\theta\varphi\left(\frac{t-\lambda_1}{\sigma}\right) + (1-\theta)\varphi\left(\frac{t-\lambda_0}{\sigma}\right)\right)$. $\mathrm{d}\tau$ represents a marginal increase in the participation rate. In this formula, the firm applies a cutoff

[5] Note that assuming this utility does not affect the purchasing decision as it does not affect the other consumers' utility and the firm's decision.

[6] Our results continue to hold even when the consumer maximizes social welfare. See Section 6.

criterion: discontinuing the product if and only if $t > T$. A consumer's participation increases turnout by $\mathrm{d}\tau$, which also increases their costs by $c_i\,\mathrm{d}\tau$. Then, consumer i participates in the protest if and only if

$$\underbrace{\left(\varsigma - w[P_\theta(T) - p]_+\right)\xi_\theta(T)}_{\text{marginal benefit of participation}} \geq c_i.$$

Now let $c_\theta = \left(\varsigma - w[P_\theta(T) - p]_+\right)\xi_\theta(T)$ be the cutoff of type-θ consumer. Then, we have that

$$\lambda_1 = \mu F(c_H) + (1-\mu)F(c_L),$$
$$\lambda_0 = (1-\mu)F(c_H) + \mu F(c_L).$$

Therefore, $\lambda := \lambda_0 - \lambda_1 = (2\mu - 1)(F(c_L) - F(c_H))$. This value plays a critical role in showing the existence of an informative equilibrium.

3.3. Equilibrium characterization with given price

This section characterizes the condition for the existence of an informative equilibrium. To this end, we define the equilibrium using the optimal actions discussed in earlier sections. Let denote $\lambda = \lambda_0 - \lambda_1$. Then, as $F(c) \in [0,1]$, $|\lambda| \leq 2\mu - 1$. Now, we define the equilibrium of the game where the price is given.

Definition 1. (T, c_H, c_L) is an *equilibrium* with given price p if
(i) $E[\text{profit}\,|\,t] < 0$ if and only if $t > T$.
(ii) $c_\theta = \left(\varsigma - w[P_\theta(T) - p]_+\right)\xi_\theta(T)$.

Note that if $c_H = c_L$, which implies $\lambda_0 = \lambda_1$. Then, the public signal t is independent of v. We call this case *uninformative*.

Definition 2. An equilibrium is *informative* if $c_L > c_H$, and is *uninformative* if $c_L = c_H$.

It's important to note that the uninformative equilibrium is a constant in our model. If the firm sets $T = \infty$ or $T = -\infty$, the protest does not affect whether the product sale continues. This leads to both types of consumers having indifferent incentives. Conversely, if the protest is uninformative, the firm's decision remains unchanged regardless of the turnout. This implies that $T = \infty$ or $T = -\infty$.

Therefore, it is crucial to reiterate that our focus is on when an informative equilibrium exists. Our discussion is guided by the following proposition, which characterizes the condition concerning p for the existence of an informative equilibrium. This proposition is a key tool in our analysis (the proof is relegated to the Appendix).

Proposition 1. *Let* $\overline{P} := \mu\frac{K+(1-\mu)\kappa}{\mu(1-\mu)+(2\mu-1)K}$. *(a) If* $p \geq \max\{\mu, \overline{P}\}$ *or* $p \leq K + \kappa$, *no informative equilibrium exists. (b) Suppose that* $K + \kappa < p < \min\{\mu, \overline{P}\}$, $f(0) > 0$ *and* $F(0) > 0$. *Then, if* w *is sufficiently large, an informative equilibrium exists.*

Here, $\overline{P}$ is a threshold of p: $T_H(p) > T_f(p)$ if and only if $p < \overline{P}$. $T_H(p) > T = T_f(p)$ implies that at the threshold of the firm, on the condition that the firm gives up to continue the product sale, the H-type still has the incentive to purchase. This makes the H-type reluctant to participate in the protest, which is the source of the difference to the L-type consumer's behavior.

In contrast, if $T_f(p) \geq T_H(p)$, the H-type never purchases the product, and the incentives of participating in the protest are indifferent as both H- and L-types consumers do not gain from the product. In this case, the protest is uninformative.

4. Firm's optimal pricing

This section focuses on the most informative equilibrium for each price the firm sets and investigates when informative equilibrium exists by endogenizing the price. All proofs of this section are relegated to the Appendix.

4.1. Benchmark

As a benchmark, this section considers the model without protest. In the benchmark case, the H-type's expected product value is $E[v] = \mu$, and that of the L-type consumers is $E[v] = 1 - \mu$. If the firm keeps the product sale, the optimal price is $p = \mu$ or $p = 1 - \mu$. The expected profit when $p = \mu$ is $\frac{\mu - \kappa}{2} - K$, and that when $p = 1 - \mu$ is $1 - \mu - \kappa - K$.

4.2. Optimal price.

Now, we consider the firm's optimal pricing with the protest. First, we define the firm's profit. As the firm keeps product sale unless $t < T_f(p)$, the firm's expected profit is

$$\Pi(p) = \Pr(v=1)\left[\left(\Phi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) - \Phi\left(\frac{T_L(p)-\lambda_1}{\sigma}\right)\right)(\mu(p-\kappa)-K) + \Phi\left(\frac{T_L(p)-\lambda_1}{\sigma}\right)(p-\kappa-K)\right]$$
$$+ \Pr(v=0)\left[\left(\Phi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right) - \Phi\left(\frac{T_L(p)-\lambda_0}{\sigma}\right)\right)((1-\mu)(p-\kappa)-K) + \Phi\left(\frac{T_L(p)-\lambda_0}{\sigma}\right)(p-\kappa-K)\right].$$

The following proposition shows that the optimal price is $p = \mu$, in which case the protest is uninformative.

Proposition 2. *Suppose that* $\frac{\mu-\kappa}{2} > \max\{K, 1-\mu-\kappa\}$. *Then, for sufficiently large* $\sigma > 0$, $p = \mu$ *is the optimal price and protest is uninformative for any* λ_0, λ_1.

Note that this result is irrelevant to the size of w as w affects the size of λ_1, λ_0, but they are bounded by $[0, 1]$. Therefore, as Proposition 1 shows, an informative equilibrium exists if $p < \{\mu, \overline{P}\}$. An informative equilibrium also benefits the firm as it discontinues the product and saves fixed cost K if it observes high t, which implies $v = 0$ is likely enough.

However, the profit is smaller than that when $p = \mu$ if σ is sufficiently large. In this case, the noise of turnout is large enough, and therefore, even when the protest is informative, the benefit of gaining information is small. Then, rather than gaining information by setting a lower price, $p = \mu$ yields a greater profit.

The above observation depends on whether an uninformative equilibrium with $p = \mu$ yields a sufficient profit. If not, an informative equilibrium leads to a higher profit, as the following proposition shows.

Proposition 3. *Take sufficiently large* w *so that informative equilibrium exists. Suppose that* $1 - \mu - \kappa > K > \frac{\mu-\kappa}{2}$. *Then, at the firm's optimal price, the equilibrium is informative.*

By Propositions 2 and 3, we observe that the resulting equilibrium is uninformative if μ is high, while it is informative when μ is small. Corollary 1 summarizes the result, and Figure 2 illustrates it.

Corollary 1. *Suppose that* $\kappa < \frac{1}{2}$ *and* $K \in \left(\frac{1-2\kappa}{3}, \frac{1}{2} - \kappa\right)$. *Take sufficiently large* σ *and a sufficiently large* w *(it depends on* σ*) to guarantee the existence of an informative equilibrium.*

Then, there exists $\mu^*, \overline{\mu}, \underline{\mu}$ with $\mu^* > \overline{\mu} > \underline{\mu}$ such that the equilibrium is uninformative if $\mu \in (\overline{\mu}, \mu^*)$,[7] *and the equilibrium is informative if $\mu < \underline{\mu}$.*

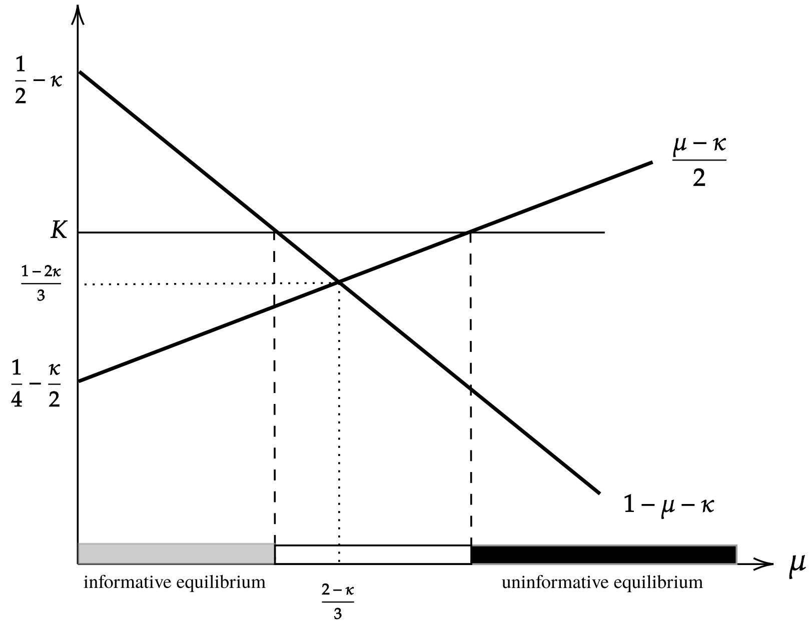

Figure 2: Classification of equilibria

This result implies that individual consumers' signal informativeness may negatively correlate with the protest's informativeness. This contrasts sharply with Ekmekci and Lauermann (2022), who shows that each person's signal informativeness positively correlates with the protest campaign's informativeness.

The main difference between their and our models is the existence of a price-setting firm. In the models of Ekmekci and Lauermann (2022) and many protest studies, citizens campaign against the policy-setting government, which affects only the policy decision. However, the firm makes two-dimensional decisions in our model: whether to discontinue the product sale and price. With a higher informativeness of each individual signal, consumers are more willing to pay, which leads to a greater profit for the firm by setting a higher price. This makes the protest uninformative by reducing the purchase incentive of H-type.

5. Uncommitted price

In the previous section, the firm is supposed to commit to a price p before the public signal is observed. This section considers the case in which the firm decides the price after the public signal observation. The timeline is depicted in Figure 3.

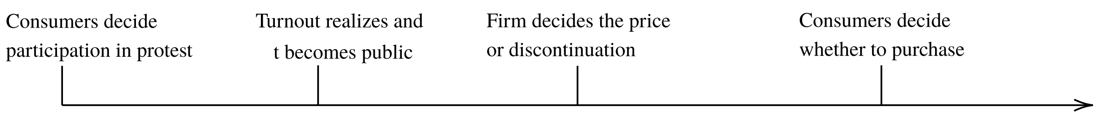

Figure 3: Timeline for the uncommitted price case

As a crucial difference to the committed price case, the turnout affects the price in the uncommitted case. With a greater turnout, if it is informative, it reduces the expected value of the product. Then, the firm sets a lower price. This makes the H-type consumers participate in the

[7] The restriction $\mu < \mu^*$ is needed because Proposition 2 is the statement when μ is fixed.

protest more than the L-type consumers, as the H-type consumers earn more information rents when purchasing. This makes the equilibrium likelier to be uninformative.

Now, we investigate how our conclusion changes. The previous section shows that with higher μ, the equilibrium is likelier to be uninformative. Even in the uncommitted price case, a similar conclusion is drawn. The following is a formal analysis. All proofs of the following section are relegated to the Appendix.

5.1. Formal analysis of the uncommitted price case

To focus on the existence of an informative equilibrium, assume that $\lambda_0 > \lambda_1$. Note that even in the uncommitted price case, the consumers' choice of product purchase is unchanged. Thus, type-θ consumer purchases the product if and only if $t < T_\theta(p)$.

Given this strategy, the firm's profit is

$$\Pi = \begin{cases} -K & \text{if } t > T_H(p) \\ [\Pr(v = 1 \mid t)\mu + \Pr(v = 0 \mid t)(1-\mu)](p-\kappa) - K & \text{if } t \in (T_L(p), T_H(p)] \\ p - \kappa - K & \text{if } t \le T_L(p). \end{cases}$$

This implies that the optimal price is either $p = P_H(t)$ or $P_L(t)$, and then, we need to compare the following equations.

$$\Pi^*_H(t) := [\Pr(v = 1 \mid t)(2\mu - 1) + 1 - \mu)](P_H(t) - \kappa) - K$$
$$\Pi^*_L(t) := P_L(t) - \kappa - K.$$

Also, the firm discontinues the product sales rather than selling it with $p = (T_L)^{-1}(t)$ if $p - \kappa < K$. In other words, $(T_L)^{-1}(t) < K + \kappa$. Equivalently, $t > T_L(K + \kappa)$.

Below, to simplify the discussion, we assume $\kappa = 0$.

Lemma 3. *Assume that $\kappa = 0$. Then, there are μ^*, μ^{**} with $\mu^* < \mu^{**}$ such that (a) If $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi^*_H(t) < \Pi^*_L(t)$ if and only if $t < \hat{t}$. (b) If $\mu < \mu^{**}$, $\Pi^*_H(t) < \Pi^*_L(t)$ for each $t < T_L(K)$. (c) $T_L(K) \to \infty$ if $\mu < \mu^*$, and $T_L(K) \to -\infty$ if $\mu > \mu^*$.*

Given this firm's behavior, we consider the consumers' incentive to participate in the protest. We first consider the case (a): there is $\hat{t} < T_L(K)$ such that $\Pi^*_L(t) > \Pi^*_H(t)$ if and only if $t < \hat{t}$. Let T^* be the solution to $\Pi^*_H(t) = 0$ with respect to t. This implies that at the optimal strategy, the firm sets price $P_L(t)$ when $t \in (-\infty, \hat{t})$, sets price $P_H(t)$ when $t \in (\hat{t}, T^*)$, and when $t > T^*$, discontinues the product sale.

Under this firm's behavior, the utility of the H-type is

$$-\varsigma \int^{T^* - \mathrm{d}\tau} \xi_H(t)\,\mathrm{d}t + w \int_{\hat{t}-d\tau}^{T^* - \mathrm{d}\tau} [P_H(t) - P_H(t + \mathrm{d}\tau)]\xi_H(t)\,\mathrm{d}t.$$
$$+ w \int^{\hat{t}-d\tau} [P_H(t) - P_L(t + \mathrm{d}\tau)]\xi_H(t)\,\mathrm{d}t.$$

Here, $P_H(t)$ is the expected value of the product while $P_L(t)$ is the price. A marginal increase of turnout $\mathrm{d}\tau$ affects the price. As the firm set price $p = P_H(t)$ if $t > \hat{t}$, type-H consumers' the informational rent $P_H(t) - P_L(t)$ vanishes. Therefore, the decision to discontinue the product only affects the externality term ς.

The differentiation concerning $\mathrm{d}\tau$ is the marginal gain by participation. Therefore, they participate in the protest if and only if $c_H \geq c_i$, where

$$c_H := \varsigma \xi_H(T^*) + w\left[\int_{\hat{t}}^{T^*} [-P'_H(t)]\xi_H(t)\,\mathrm{d}t - [P_H(\hat{t}) - P_L(\hat{t})]\xi_H(\hat{t}) + \int^{\hat{t}} [-P'_L(t)]\xi_H(t)\,\mathrm{d}t\right].$$

Note that $P_L'(t) < 0$. Then, the third term of the above equation is the gain from price manipulation by increasing turnout. In contrast, with a high turnout, the firm gives up on making all types of consumers purchase the product, and then, type-H's informational rent vanishes. This makes type H consumers reluctant to participate in the protest, captured by the second term.

Now we compare with L-type consumers' utility. The utility of the L-type is $-\varsigma \int^{T^*-\mathrm{d}\tau} \xi_L(t)\,\mathrm{d}t + w \int^{\hat{t}-d\tau} [P_L(t) - P_L(t+\mathrm{d}\tau)]\xi_L(t)\,\mathrm{d}t$ as the firm extracts type-L consumers' full surplus. Then, the type-L consumers participate in the protest if and only if

$$c_L \coloneqq \varsigma \xi_L(T^*) + w \int^{\hat{t}} [-P_L'(t)]\xi_L(t)\,\mathrm{d}t \geq c_i.$$

We compare the relation between c_H and c_L with high μ cases. As we focus on a large w case, we consider the coefficients of w in $c_L - c_H$, that is

$$D \coloneqq \int^{\hat{t}} [-P_L'(t)](\xi_L(t) - \xi_H(t))\,\mathrm{d}t - \int_{\hat{t}}^{T^*} [-P_H'(t)]\xi_H(t)\,\mathrm{d}t + [P_H(\hat{t}) - P_L(\hat{t})]\xi_H(\hat{t})$$

Suppose that $\frac{\mu}{2} > K$. Then $T^* \to \infty$ as $\sigma \to \infty$.

Proposition 4. *Suppose that $\mu > \mu^{**}$. Then, for sufficiently large σ, $D < 0$.*

Proposition 4 implies that the protest becomes uninformative with high μ. This is because this inequality $c_H > c_L$ is gained by the supposition that $\lambda_0 > \lambda_1$, which means that $c_L > c_H$. This is a contradiction. So, the equilibrium should be uninformative. This is a similar feature to the committed price case. The intuition is the following. With a high μ, the firm continues to sell even when t is sufficiently high, in which case, while the L-type gives up to purchase, the H-type purchases. Then, the H-type's incentive for price manipulation is much higher than that of the L-type. In contrast, the effect on the informational rent is small as the firm is likely to focus on selling only to the H-types when μ is high enough. This motivates the H-type to participate in the protest more than the L-type.

Next, we consider the case (b): for each $t < T_L(\kappa + K)$, $\Pi_L^*(t) > \Pi_H^*(t)$. This implies that for any $t < T_L(\kappa + K)$, the firm set price $P_L(t)$ and discontinues the product sale if $t > T_L(\kappa + K)$. To simplify the notation, we denote $T_* = T_L(\kappa + K)$.

In this case, The utility of the H-type is

$$-\varsigma \int^{T_*-\mathrm{d}\tau} \xi_H(t)\,\mathrm{d}t + w \int^{T_*-d\tau} [P_H(t) - P_L(t+\mathrm{d}\tau)]\xi_H(t)\,\mathrm{d}t.$$

The differentiation concerning $\mathrm{d}\tau$ is the marginal gain by participation. Therefore, they participate in the protest if and only if

$$c_H \coloneqq \varsigma \xi_H(T_*) - w[P_H(T_*) - P_L(T_*)]\xi_H(T_*) + w \int^{T_*} [-P_L'(t)]\xi_H(t)\,\mathrm{d}t \geq c_i.$$

Similarly, the utility of the L-type is $-\varsigma \int^{T_*-\mathrm{d}\tau} \xi_L(t)\,\mathrm{d}t + w \int^{T_*-d\tau} [P_L(t) - P_L(t+\mathrm{d}\tau)]\xi_L(t)\,\mathrm{d}t$. Then, they participate in the protest if and only if

$$c_L \coloneqq \varsigma \xi_L(T_*) + w \int^{T_*} [-P_L'(t)]\xi_L(t)\,\mathrm{d}t \geq c_i.$$

We also compare c_H with c_L and thus, focus on the coefficient of w in $c_L - c_H$,

$$\tilde{D} \coloneqq \int^{T_*} [-P_L'(t)](\xi_L(t) - \xi_H(t))\,\mathrm{d}t + [P_H(T_*) - P_L(T_*)]\xi_H(T_*)$$

Proposition 5. *Suppose that $\mu \in (\mu^*, \mu^{**})$. Then, for sufficiently large σ, $\tilde{D} > 0$.*

$c_H > c_L$ implies that the H-type is likelier to participate in the protest. Then, the equilibrium becomes informative with a large w. Intuition is the following. With lower μ, the firm gives up selling even with a small t. Then, the incentive for price manipulation becomes smaller, and the effect of losing informational rent dominates that incentive.

In conclusion, the informative equilibrium is more likely with intermediate μ cases, although not in high μ cases. Therefore, the protest's informativeness is non-monotonic to the individual informativeness, which is a similar conclusion to the committed price case.

6. Conclusion and Discussions

This study examines when the protest campaign informs about the consumers' values for the product, which determines the firm's decision. We show that the consumers' individual signal precision does not necessarily contribute to the protest's informativeness. One implication of our result is that consumers' ignorance may contribute to the success of protest campaigns.

In the remainder of this study, we discuss the assumptions in our model.

Objective of participation in protest We have assumed that consumers consider their type's aggregate utility when deciding whether to participate in the protest. However, as in Ali and Lin (2013), if this motivation comes from a reputation concern, it is more plausible to assume that consumers consider the expected social welfare. Let's consider a scenario where each consumer's decision to participate in the protest is based on maximizing the expected social welfare. The consumer's belief, represented by $\theta \in \{\theta_H, \theta_L\}$, influences their expected social welfare as follows.

$$\theta \int^{T-\mathrm{d}\tau} \big(w[P_\theta(t) - p]_+ - \varsigma\big)\xi_\theta(t)\,\mathrm{d}t + (1-\theta)\int^{T-\mathrm{d}\tau} \big(w[P_{\theta'}(t) - p]_+ - \varsigma\big)\xi_{\theta'}(t)\,\mathrm{d}t,$$

where $\theta' \in \{\theta_H, \theta_L\}$ with $\theta' \neq \theta$. In this case, the H-type participates in the protest if and only if $c_i \leq c_H^* := \mu c_H + (1-\mu)c_L$ while the L-type does if and only if $c_i \leq c_L^* := (1-\mu)c_H + \mu c_L$. Then, as long as $c_L > c_H$, the cutoff point is lower for the L-type consumers. Then, we can apply a similar discussion to show the same statements.

Concerns for negative externalities In our model, a concern for the product value plays an important role, but the concern for the negative externality itself does not. If the size of the concern regarding the negative externalities is sufficiently large, the participation rate of H- and L-type consumers becomes closer, which makes the protest uninformative. This observation continues to hold even when concerns for the externality are distinct among consumers as the firm does not care about the externality itself. In contrast, if the product quality includes guilty feelings about consuming the product causing negative externality, it would be correlated to the size of concerns for the negative externality itself. In this case, our conclusion remains true even though the concerns for negative externality are high.

Technical assumptions Our results depend on some technical assumptions: noise followed by a normal distribution whose variance is high enough, and consumers' signals are binary. Considering a sufficiently high noise is crucial for the committed price case. The firm benefits from informative equilibrium if the noise variance is small enough. The firm can save the fixed cost K when $v = 0$. In this case, if $v = 0$, the firm discontinues the sale. If $v = 1$, the optimal price can be high but less than μ for the existence of an informative equilibrium. An interesting observation is that if μ is small enough, the price becomes higher compared with the case where the protest is absent, where the price is set so that both H- and L-type consumers buy. Due to the informative protest, the firm can ignore the case that the product is unpopular. Another observation is that although the protest revealed almost complete information, the price is bounded above by individual signal precision, μ, to make the protest informative, by Proposition 1. This observation is important because it seems optimal for the firm to set a price close to 1 as the protest reveals whether $v = 1$ or $v = 0$. In fact, if doing so, the H-type's motivation becomes close to the L-type's, and the protest becomes uninformative. Then, even if the individual signal precision is high, it does not necessarily benefit the consumers, as it only increases the price.

Appendix

A. Proofs

A.1. Proof of Proposition 1

Before the proof of Proposition 1, we prepare the following lemma.

Lemma 4. *Define $\underline{P}$ be the larger solution to $\frac{1-\mu}{\mu}\frac{1-p}{p} = \frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)}$ with respect to p. Then, $T_H(p) < T_f(p)$ if and only if $p > \overline{P}$ and $T_L(p) < T_f(p)$ if and only if $p > \underline{P}$. Further, $\frac{K}{1-\mu} + \kappa > \overline{P} > \underline{P} > \frac{K}{\mu} + \kappa$.*

Proof of Lemma 4. (a) If $p \geq \frac{K}{1-\mu} + \kappa$, $T_f(p) = \infty$ by Lemma 2. If $p < \frac{K}{1-\mu} + \kappa$, $T_f(p) < T_H(p)$ if and only if $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} < \frac{\mu}{1-\mu}\frac{1-p}{p}$, which is equivalently $p > \overline{P}$. Further, a simple calculation shows $\frac{K}{1-\mu} + \kappa > \overline{P}$.

(b) If $p < \frac{K}{\mu}$, by Lemma 2, $T_f(p) = -\infty$. Consider the case where $p \geq \frac{K}{\mu}$. Note that $T_L(p) < T_f(p)$ if and only if $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} > \frac{1-\mu}{\mu}\frac{1-p}{p}$, equivalently, $g(p) > 0$, where g is a convex quadratic function (illustrated in Figure 4). Then, $T_L(p) < T_f(p)$ if and only if $p > \underline{P}$. The relation $\underline{P} > \frac{K}{\mu} + \kappa$ is shown by the fact that $\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa}-(1-\mu)} > \frac{1-\mu}{\mu}\frac{1-p}{p}$ fails if $p = \frac{K}{\mu} + \kappa$ and larger p satisfies $T_f(p) > T_H(p) > T_L(p)$.

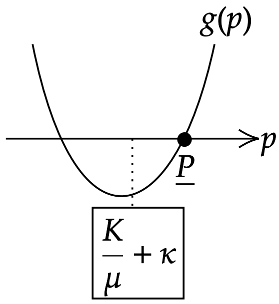

Figure 4: Illustration of g

■

Proof of Proposition 1. (1) If $p \geq \overline{P}$, as long as $t \leq T_H(p)$, $t \geq T_f(p)$. At $t > T_H(P)$, H type does not purchase the product. Then, the firm gives up to sell the product. This implies that the firm's cutoff point is $T = T_H(p)$. In this case, the equilibrium condition is

$$\begin{aligned}
\lambda_0 - \lambda_1 &= (2\mu - 1)(F(c_L) - F(c_H)) \\
c_\theta &= \varsigma \xi_\theta(T_H(p)) \\
T_H(p) &= \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu}{1-\mu} - \ln\left(\frac{p}{1-p}\right)\right)}{\lambda_0 - \lambda_1} \\
\xi_\theta(T) &= \frac{1}{\sigma}\left[\theta\varphi\left(\frac{T_H(p) - \lambda_1}{\sigma}\right) + (1-\theta)\varphi\left(\frac{T_H(p) - \lambda_0}{\sigma}\right)\right].
\end{aligned}$$

Consider the case that $p \geq \mu$. Then, $\frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu}{1-\mu} - \ln\left(\frac{p}{1-p}\right)\right)}{\lambda_0 - \lambda_1} \leq 0$ when $\lambda_0 \geq \lambda_1$. As φ is symmetric and single-peaked around 0, we show that

$$\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) \leq \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right).$$

Note also that

$$c_L - c_H = \varsigma \cdot \frac{2\mu - 1}{\sigma} \cdot \left[\varphi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right) - \varphi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)\right] \leq 0.$$

Therefore, $\lambda_0 \geq \lambda_1$ implies $c_L \leq c_H$, which shows no existence of an informative equilibrium.

(2) Consider the case $p \in \left(\underline{P}, \min\{\overline{P}, \mu\}\right)$. Then, if $\lambda_0 > \lambda_1$, $T_L(p) < T_f(p) < T_H(p)$ for any c_H, c_L

Recall that the following equations characterize the equilibriums:

$$\begin{aligned}
\lambda := \lambda_0 - \lambda_1 &= (2\mu - 1)(F(c_L) - F(c_H)), \\
c_\theta &= \left(\varsigma - w\left[P_\theta\left(T_f(p)\right) - p\right]_+\right)\xi_\theta(T_f(p)), \\
T_f(p) - \lambda_1 &= \frac{\lambda}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)}{\lambda}, \\
T_f(p) - \lambda_0 &= -\frac{\lambda}{2} + \frac{\sigma^2}{2} \frac{\ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)}{\lambda}, \\
P_\theta\left(T_f(p)\right) &= \frac{\varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right)\theta}{\varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right)\theta + \varphi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right)(1-\theta)}, \\
\xi_\theta\left(T_f(p)\right) &= \frac{1}{\sigma}\left[\theta\varphi\left(\frac{T_f(p)-\lambda_1}{\sigma}\right) + (1-\theta)\varphi\left(\frac{T_f(p)-\lambda_0}{\sigma}\right)\right].
\end{aligned}$$

This shows that c_L and c_H are depend on λ, but not on λ_0, λ_1 individually. Therefore, the equilibrium is a fixed point of a self-map $\zeta : \lambda \mapsto (2\mu - 1)(F(c_L) - F(c_H))$. We can easily verify that ζ is continuous.

By the definition of $\overline{P}$ and $\underline{P}$, $P_H\left(T_f(p)\right) > p$, and $p > P_L\left(T_f(p)\right)$. This implies that

$$\begin{aligned}
c_H &= \left(\varsigma - w\left[P_H\left(T_f(p)\right) - p\right]\right)\xi_H\left(T_f(p)\right) \\
c_L &= \varsigma\xi_L\left(T_f(p)\right) > 0.
\end{aligned}$$

Then, $c_L - c_H = \varsigma\left(\xi_L\left(T_f(p)\right) - \xi_H\left(T_f(p)\right)\right) + w\left[P_H\left(T_f(p)\right) - p\right]\xi_H\left(T_f(p)\right) \geq 0$ for large w. Note that as $\lambda \to 0$, $T_f(p) \to \infty$, which implies that $c_L \to 0$ and $c_H \to 0$. Then, take $\lambda^* > 0$ sufficiently small so that $(2\mu - 1)(F(c_L) - F(c_H)) \approx f(0) \cdot (c_L - c_H) \cdot h(\lambda^*)$ for some function h. By taking large w so that $f(0) \cdot (c_L - c_H) \cdot h(\lambda^*) > \lambda^*$. This implies that $\zeta(\lambda^*) =$

$(2\mu - 1)(F(c_L) - F(c_H)) > \lambda^*$. As $\zeta(\lambda) < \lambda$ for $\lambda > 2\mu - 1$, the intermediate value theorem implies the existence of a fixed point of ζ, and the fixed point is a positive value. This shows the existence of an informative equilibrium.

(3) Consider the case $p < \underline{P}$. This implies that the firm prefers to give up selling if $t > T_L(p)$. If $t \leq T_L(p)$, both H and L types purchase the product. This implies the firm's profit is $p - \kappa$. If $p - \kappa > K$, the give up point is $T = T_L(p)$. Then, by the definition of $T_L(p)$,

$$c_H = (\varsigma - w[P_H(T_L(p)) - p])\xi_H\big(T_f(p)\big)c_L = \varsigma\xi_L(T_L(p)) > 0.$$

A similar way to case (2) shows the existence of informative equilibrium.

If $p - \kappa \leq K$, the firm also prefers to give up at $t \leq T_L(p)$ and therefore, $T = -\infty$. Then, the equilibrium becomes uninformative. ■

A.2. Proof of Proposition 2

Before we prove Proposition 2, we provide the following lemma.

Lemma 5. *$\mu > \overline{P}$ if and only if $\frac{\mu-\kappa}{2} > K$.*

The proof immediately follows from the definition of $\overline{P}$.

Now we proceed to the proof of Proposition 2. Suppose that $\frac{\mu-\kappa}{2} > \max\{K, 1 - \mu - \kappa\}$. This implies that $\mu > \frac{2-\kappa}{3}$. Note that if $p = \mu$, as $\frac{\mu-\kappa}{2} > K$, $p > \overline{P}$ by Lemma 5. By Proposition 1, the equilibrium is uninformative and guarantees the profit $\frac{\mu-\kappa}{2} - K$.

Below, we examine the existence of an informative equilibrium that yields a profit larger than $\frac{\mu-\kappa}{2} - K$. As we consider an informative equilibrium, by Proposition 1, $p < \mu$. First, consider the case that $p < \overline{P}$. Note that the profit is at most $p - \kappa - K$. As we assume $\frac{\mu-\kappa}{2} > 1 - \mu - \kappa$, $p > 1 - \mu$. This implies that

$$\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) < 0.$$

Now we consider the limit $\sigma \to \infty$. Note that as $\lambda_0 - \lambda_1 = (2\mu - 1)(F(c_L) - F(c_H)) \in [0, 2\mu - 1] \subset [0, 1]$.[8] This shows that

$$\frac{T_L(p) - \lambda_1}{\sigma} < \frac{\sigma}{2}\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) \to -\infty$$
$$\frac{T_L(p) - \lambda_0}{\sigma} < \frac{1}{2\sigma} + \frac{\sigma}{2}\ln\left(\frac{1-p}{p}\frac{1-\mu}{\mu}\right) \to -\infty$$

Then, the firm's profit is at most $\left(\frac{1}{2}\right)(p - \kappa) - K$ at the limit. This is smaller than $\frac{\mu-\kappa}{2} - K$ as $p < \overline{P} < \mu$.

Next, we consider the case that $p > \overline{P}$.

Claim 1. *Suppose that $\mu > p > \max\left\{\frac{1}{2}, \overline{P}\right\}$. Then, $\Pi(p) < \left[\left(\frac{1}{2}\right)(p - \kappa) - K\right]$ for sufficiently large σ.*

Proof of Claim 1. Suppose that $p > \overline{P}$. Let $D(\sigma) = \Pi(p) - \left[\left(\frac{1}{2}\right)(p - \kappa) - K\right]$, which is calculated as

[8]Note that the value of σ can be taken independent of w as the value of w affects only λ. Here, λ is bounded by values that are independent of w.

$$\Pr(v=1)\left[\left(\Phi\left(\frac{T_H(p)-\lambda_1}{\sigma}\right)-1\right)(\mu(p-\kappa)-K)+\Phi\left(\frac{T_L(p)-\lambda_1}{\sigma}\right)(1-\mu)(p-\kappa)\right]$$
$$+\Pr(v=0)\left[\left(\Phi\left(\frac{T_H(p)-\lambda_0}{\sigma}\right)-1\right)((1-\mu)(p-\kappa)-K)+\Phi\left(\frac{T_L(p)-\lambda_0}{\sigma}\right)\mu(p-\kappa)\right].$$

Note that by $p\in(1-\mu,\mu)$,

$$T_H(p)-\lambda_1=\frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]>\frac{\sigma^2}{2}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]$$
$$T_H(p)-\lambda_0=-\frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]>\frac{\sigma^2}{2}\left[\ln\left(\tfrac{\mu}{1-\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]-\frac{1}{2}$$
$$T_L(p)-\lambda_1=\frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]<\frac{\sigma^2}{2}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]+\frac{1}{2}$$
$$T_L(p)-\lambda_0=-\frac{\lambda}{2}+\frac{\sigma^2}{2\lambda}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right]<\frac{\sigma^2}{2}\left[\ln\left(\tfrac{1-\mu}{\mu}\right)-\ln\left(\tfrac{p}{1-p}\right)\right].$$

By these equations, we can verify that as $\sigma\to\infty$, $T_H(p)-\lambda_v\to\infty$ and $T_L(p)-\lambda_v\to-\infty$. Then, $\Phi\left(\frac{T_H(p)-\lambda_v}{\sigma}\right)-1\to 0$ and $\Phi\left(\frac{T_L(p)-\lambda_v}{\sigma}\right)\to 0$.[9] Then, $\lim_{\sigma\to\infty}D(\sigma)=0$. Now we verify the sign of D for sufficiently large σ. To this end, we calculate $D'(\sigma)$, which is

$$D'(\sigma)=\varphi(A_{H1})\left(\frac{1}{\lambda}B_H-\frac{\lambda}{2\sigma^2}\right)(\mu(p-\kappa)-K)+\varphi(A_{L1})\left(\frac{1}{\lambda}B_L-\frac{\lambda}{2\sigma^2}\right)(1-\mu)(p-\kappa)$$
$$+\varphi(A_{H0})\left(\frac{1}{\lambda}B_H+\frac{\lambda}{2\sigma^2}\right)((1-\mu)(p-\kappa)-K)+\varphi(A_{L0})\left(\frac{1}{\lambda}B_L+\frac{\lambda}{2\sigma^2}\right)\mu(p-\kappa),$$

where

$$A_{\theta v}=\frac{T_\theta(p)-\lambda_v}{\sigma},\quad v\in\{0,1\},$$
$$B_\theta=\ln\left(\frac{\theta}{1-\theta}\right)-\ln\left(\frac{p}{1-p}\right),$$

and $B_H>0>B_L$ as $\mu>p>\frac{1}{2}>1-\mu$.

Note that

$$\frac{\varphi(A_{H1})}{\varphi(A_{L0})}=\exp\left(\left[\tfrac{\lambda}{\sigma}-\tfrac{\sigma}{\lambda}\ln\left(\tfrac{1-\mu}{\mu}\right)\right]\sigma\ln\left(\tfrac{p}{1-p}\right)\right)>\exp\left(-\sigma^2\ln\left(\tfrac{1-\mu}{\mu}\right)\ln\left(\tfrac{p}{1-p}\right)\right),$$
$$\frac{\varphi(A_{H0})}{\varphi(A_{L0})}=\exp\left(-\tfrac{\sigma}{\lambda}\ln\left(\tfrac{1-\mu}{\mu}\right)\left[\tfrac{\lambda}{\sigma}+\sigma\ln\left(\tfrac{p}{1-p}\right)\right]\right)>\exp\left(-\sigma^2\ln\left(\tfrac{1-\mu}{\mu}\right)\ln\left(\tfrac{p}{1-p}\right)\right),$$
$$\frac{\varphi(A_{L1})}{\varphi(A_{L0})}=\exp\left(\ln\left(\tfrac{p}{1-p}\right)-\ln\left(\tfrac{1-\mu}{\mu}\right)\right),$$

Then, as $\mu>\frac{1}{2}$ and $p>\frac{1}{2}$, $\frac{\varphi(A_{H1})}{\varphi(A_{L0})}\to\infty$, and $\frac{\varphi(A_{H0})}{\varphi(A_{L0})}\to\infty$ as $\sigma\to\infty$. Note also that $(\mu(p-\kappa)-K)+((1-\mu)(p-\kappa)-K)=p-\kappa-2K>0$. Using these facts, dividing $D'(\sigma)$ by $\frac{\varphi(A_{L0})}{\lambda}$ is positive for sufficiently large σ. As $\lim_{\sigma\to\infty}D(\sigma)=0$ and $D'(\sigma)>0$, $D(\sigma)<0$ for sufficiently large σ. Therefore, $\Pi(p)<\left[\frac{1}{2}(p-\kappa)-K\right]$ for sufficiently large σ. ■

Then, we conclude that for each $p<\mu$, the profit is at most $\frac{1}{2}(p-\kappa)-K$ when σ is sufficiently large. Therefore, $p=\mu$ is the optimal price, and the equilibrium is uninformative. ■

[9] Note that $\mu>\frac{1}{2}$ and $p>\frac{1}{2}$ implies that $\left|\ln\left(\frac{1-\mu}{\mu}\right)-\ln\left(\frac{p}{1-p}\right)\right|>\left|\ln\left(\frac{\mu}{1-\mu}\right)-\ln\left(\frac{p}{1-p}\right)\right|$. This also implies that $\Phi\left(\frac{T_H(p)-\lambda_v}{\sigma}\right)$ converges more slowly than $\Phi\left(\frac{T_L(p)-\lambda_v}{\sigma}\right)$. This intuition works in the following analysis.

A.3. Proof of Proposition 3

The proof consists of the following two claims, which hold under the assumption of Proposition 3.

Claim 2. *If informative equilibrium does not exist, the expected profit is at most* 0.

Claim 3. *By setting* $p = 1 - \mu$*, the equilibrium is informative. The expected profit is no less than* $(1 - \mu - \kappa - K)\frac{1}{2}$ *in this case.*

Proof of Claim 2. By Proposition 1, the situation is either $p > \min\{\mu, \overline{P}\}$ or $p < \kappa + K$. If $p > \mu$, as $P_H(T) = \mu < p$ at the uninformative equilibrium, the profit is 0 as the H type never purchases. If there is an uninformative equilibrium with $p \in (\overline{P}, \mu)$, as $P_H(T) = \mu > p > 1 - \mu$, the profit is $\frac{\mu-\kappa}{2} - K < 0$ as only the H type purchases.

At the uninformative equilibrium with $p < \kappa + K$, the profit is also 0 as the firm gives up selling. ■

Proof of Claim 3. First, consider the case where $1 - \mu \le \underline{P}$. As $p = 1 - \mu > K + \kappa$, $1 - \mu < \mu$ and $1 - \mu < \underline{P} < \overline{P}$, an equilibrium exists and it is necessarily informative by Proposition 1.

Note that $p = 1 - \mu \le \underline{P}$ implies $T = T_L(p)$. In this case, all consumer buys the product if $t < T$, and otherwise, never. Further, if $p = 1 - \mu$, $T_L(p) = \frac{\lambda_1+\lambda_0}{\sigma}$. Then, the expected profit is

$$\left[\frac{1}{2}\Phi(x) + \frac{1}{2}\Phi(-x)\right](1 - \mu - \kappa - K) = \frac{1 - \mu - \kappa - K}{2},$$

where $x = \frac{\lambda_0-\lambda_1}{2\sigma}$.

Next, consider the case that $1 - \mu > \underline{P}$. In this case, at $t = T$, only H type purchases the product.

By assumption, $K > \frac{\mu-\kappa}{2}$ implies that $\overline{P} > \mu > 1 - \mu$, and as $1 - \mu > K + \kappa$, Proposition 1 implies that an equilibrium exists and it is necessarily informative at $p = 1 - \mu$. Further, the expected profit is

$$\begin{aligned} &\frac{1 - \mu - \kappa}{2} \times [\mu\Phi(x + z') + (1 - \mu)\Phi(x) + (1 - \mu)\Phi(-x + z') + \mu\Phi(-x)] \\ &\qquad -[\mu\Phi(x + z') + (1 - \mu)\Phi(-x + z')]K \end{aligned} \tag{1}$$

where $z' = \frac{\sigma}{4x} \ln\left(\frac{\mu - \frac{K}{p-\kappa}}{\frac{K}{p-\kappa} - (1-\mu)}\right)$.

As $T > T_L(p) = \frac{\lambda_0+\lambda_1}{2}$, the expected profit (1) is greater than $= \frac{1-\mu-\kappa-K}{2} > 0$. ■

Proof of Proposition 3. By Claims 2 and 3, and assumption $1 - \mu - \kappa > K$, a price that leads to an informative equilibrium yields a positive profit, which is larger than the profit of any uninformative equilibrium. Then, the optimal price leads to an informative equilibrium. ■

A.4. Proofs in Section 5

Proof of Lemma 3. First note that for sufficiently small t, $\Pi_L^*(t) > \Pi_H^*(t)$. This is because for sufficiently small $t \approx -\infty$, $P_\theta(t) \approx 1$ and therefore $\Pi_L^*(t) \approx 1 > \mu \approx \Pi_H^*(t)$.

Let $a = \exp\left(\frac{\lambda_0-\lambda_1}{\sigma^2}\left(t - \frac{\lambda_0+\lambda_1}{2}\right)\right)$. Then, $P_\theta(t)$ is written as

$$P_\theta(t) = \frac{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta}{\varphi\left(\frac{t-\lambda_1}{\sigma}\right)\theta + \varphi\left(\frac{t-\lambda_0}{\sigma}\right)(1-\theta)} = \frac{1}{1 + \frac{1-\theta}{\theta}a},$$

and also

$$\Pr(v = 1 \mid t) = \frac{1}{1+a}.$$

To simplify the notation, let denote $P(t) = \Pr(v = 1 \mid t)$. By differentiation, if $\lambda_0 > \lambda_1$, we have

$$P'_\theta(t) = -\frac{\lambda_0 - \lambda_1}{\sigma^2}\frac{1-\theta}{\theta}a(P_\theta(t))^2 < 0,$$

and each profit has the following derivative, and the sign is negative as $\mu > \frac{1}{2}$ and $P_H(t) > P_L(t) > \kappa$.

Note that we can write $P_H(t) = \frac{1}{1+\frac{1-\mu}{\mu}a}$ and $P_L(t) = \frac{1}{1+\frac{\mu}{1-\mu}a}$. This implies that $\frac{P_H(t)}{P_L(t)}$ and $\frac{P(t)}{P_L(t)}$ are increasing in t.

Now we calculate the ratio of the profits:

$$\begin{aligned} \frac{\Pi^*_H(t)}{\Pi^*_L(t)} &= (P(t)(2\mu-1) + (1-\mu))\frac{P_H(t)}{P_L(t)} > 1 \iff \\ Z(a) &:= (1-\mu)\mu^2a^2 - (2\mu-1)(1-(1-\mu)\mu)a - \mu(1-\mu)^2 > 0 \end{aligned} \tag{2}$$

Note that the left-hand side is a convex quadratic function of a, and $Z(0) < 0$. Also note that as $P_L(t) > K$, $a < \left[\frac{1}{K} - 1\right]\frac{1-\mu}{\mu}$. A calculation shows that $Z\left(\left[\frac{1}{K} - 1\right]\frac{1-\mu}{\mu}\right)$ is decreasing in μ, and let μ^{**} be the solution that $Z\left(\left[\frac{1}{K} - 1\right]\frac{1-\mu}{\mu}\right) = 0$. This implies that if $\mu < \mu^{**}$, $\Pi^*_H(t) < \Pi^*_L(t)$ for each t. On the contrary, if $\mu > \mu^{**}$, there is $\hat{t} < T_L(K)$ such that $\Pi^*_H(t) < \Pi^*_L(t)$ if and only if $t < \hat{t}$.

(c) As $T_L(K)$ is the solution of $P_L(t) = K$, That is,

$$T_L(K) = \frac{\lambda_0 + \lambda_1}{2} + \frac{\sigma^2}{\lambda_0 - \lambda_1}\ln\left(\left[\frac{1}{K} - 1\right]\frac{1-\mu}{\mu}\right)$$

Let μ^* be the solution to $\left(\frac{1}{K} - 1\right)\frac{1-\mu}{\mu} = 1$. This exists if $K < \frac{1}{2}$. Then, if $\mu < \mu^*$, $T_L(K) \to \infty$. In contrast, if $\mu > \mu^*$, $T_L(K) \to -\infty$. We can also verify that $Z\left(\left[\frac{1}{K} - 1\right]\frac{1-\mu^*}{\mu^*}\right) < 0$, that is, $Z(1) < 0$. This implies that $\mu^* < \mu^{**}$. ■

Proof of Proposition 4. Let $a = \exp\left(\frac{\lambda_0-\lambda_1}{\sigma^2}\left[t - \frac{\lambda_1+\lambda_0}{2}\right]\right)$. Note that

$$\begin{aligned} P_\theta &= \frac{1}{1 + \frac{1-\theta}{\theta}a} \\ P_H(t) - P_L(t) &= \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\left[1 + \frac{\mu}{1-\mu}a\right]\left[1 + \frac{1-\mu}{\mu}a\right]}a, \\ -P'_\theta(t) &= \frac{\lambda_0 - \lambda_1}{\sigma^2}\frac{1-\theta}{\theta}a[P_\theta(t)]^2 > 0. \\ \xi_L(t) - \xi_H(t) &= \frac{2\mu-1}{\sigma}\left[\varphi\left(\frac{t-\lambda_0}{\sigma}\right) - \varphi\left(\frac{t-\lambda_1}{\sigma}\right)\right] \\ &= \frac{2\mu-1}{\sigma}\varphi\left(\frac{t-\lambda_0}{\sigma}\right)\left[1 - \exp\left(-\frac{\lambda_0-\lambda_1}{\sigma}\left[t - \frac{\lambda_0+\lambda_1}{2}\right]\right)\right] \\ \xi_H(t) &= \frac{1}{\sigma}\left[\mu\varphi\left(\frac{t-\lambda_1}{\sigma}\right) + (1-\mu)\varphi\left(\frac{t-\lambda_0}{\sigma}\right)\right]. \end{aligned}$$

As shown in the proof of Lemma 3, $\Pi^*_H(t) = \Pi^*_L(t)$ if and only if $Z(a) = 0$, which is defined in (2) . Therefore, when we write $\hat{a} = \exp\left(\frac{\lambda_0-\lambda_1}{\sigma^2}\left(\hat{t} - \frac{\lambda_1+\lambda_0}{2}\right)\right)$, $Z(\hat{a}) = 0$. Note also that Z is independent of t other than a. Then, even $\sigma \to \infty$, $\hat{a}$ is a finite value, which implies that $\hat{t} \in \Theta\big(\sigma^2\big)$.[10] Further,

$$\frac{\xi_H(t)}{\xi_H(\hat{t})} = \frac{\mu \exp\Big((\hat{t}-t)\frac{\hat{t}+t-2\lambda_1}{2\sigma^2}\Big) + (1-\mu)\exp\Big((\hat{t}-t+\lambda_0-\lambda_1)\frac{\hat{t}+t-(\lambda_0+\lambda_1)}{2\sigma^2}\Big)}{\mu + (1-\mu)\exp\Big(\frac{1}{\sigma^2}(\lambda_0-\lambda_1)\Big(\hat{t}-\frac{\lambda_0+\lambda_1}{2}\Big)\Big)}.$$

As $\frac{\hat{t}}{\sigma^2}$ converges to a real number, for each t with $|t| < |\hat{t}| + z$ for some constant z, $\frac{\xi_H(t)}{\xi_H(\hat{t})} \to \infty$. In contrast, if $t < \hat{t}$, $\frac{\xi_H(t)}{\xi_H(\hat{t})} \to 0$ as $\hat{t} \to -\infty$. By Lemma 3, $\hat{t} \to -\infty$ as $\sigma \to \infty$. Then, dividing D by $\xi_H(\hat{t})$ yields that

$$\int^{\hat{t}} [-P_L'(t)] \frac{\xi_L(t)-\xi_H(t)}{\xi_H(\hat{t})}\,\mathrm{d}t - \int_{\hat{t}}^{T^*} [-P_H'(t)] \frac{\xi_H(t)}{\xi_H(\hat{t})}\,\mathrm{d}t + [P_H(\hat{t}) - P_L(\hat{t})]. \tag{3}$$

Note also that $\xi_L(t) < \xi_H(t)$ for sufficiently small t when $\lambda_0 > \lambda_1$. Also note that by $\frac{\mu}{2} > K$, $T^* \to \infty$ as $\sigma \to \infty$. Therefore, as $\sigma \to \infty$, the value of (3) diverges to $-\infty$. This shows that $D < 0$. ■

Proof of Proposition 5. Dividing $\tilde{D}$ by $\xi_H(T_*)$ yields that

$$\frac{\int^{T_*}[-P_L'(t)](\xi_L(t)-\xi_H(t))\,\mathrm{d}t}{\xi_H(T_*)} + [P_H(T_*) - P_L(T_*)] \tag{4}$$

As $\mu > \mu^*$, $\lim_{\sigma\to\infty} T_* = -\infty$ by Lemma 3. Therefore, the first term of (4) converges to

$$-P_L'(T_*)\frac{\xi_L(T_*)-\xi_H(T_*)}{\xi_H'(T_*)}.$$

Note that

$$\begin{aligned}\frac{\xi_L(T_*)-\xi_H(T_*)}{\xi_H'(T_*)} &= \frac{-(2\mu-1)\Big(\exp\Big(-\frac{1}{2}\Big(\frac{T_*-\lambda_0}{\sigma}\Big)^2\Big) - \exp\Big(-\frac{1}{2}\Big(\frac{T_*-\lambda_1}{\sigma}\Big)^2\Big)\Big)}{\mu\frac{T_*-\lambda_1}{\sigma^2}\exp\Big(-\frac{1}{2}\Big(\frac{T_*-\lambda_1}{\sigma}\Big)^2\Big) + (1-\mu)\frac{T_*-\lambda_0}{\sigma^2}\exp\Big(-\frac{1}{2}\Big(\frac{T_*-\lambda_0}{\sigma}\Big)^2\Big)}\\ &= \frac{(2\mu-1)\Big(1-\exp\Big(\frac{\lambda_0-\lambda_1}{\sigma^2}\Big(T_*-\frac{\lambda_1+\lambda_0}{2}\Big)\Big)\Big)}{\mu\frac{T_*-\lambda_1}{\sigma^2} + (1-\mu)\frac{T_*-\lambda_0}{\sigma^2}\exp\Big(\frac{\lambda_0-\lambda_1}{\sigma^2}\Big(T_*-\frac{\lambda_1+\lambda_0}{2}\Big)\Big)}\end{aligned}$$

As shown in Lemma 3, $\frac{T_*}{\sigma^2}$ converges to a finite value. Then, the above value converges to a finite value.

Note also that

$$-P_L'(t) = \frac{\lambda_0-\lambda_1}{\sigma^2}\frac{\mu}{1-\mu} \times \frac{1}{\frac{1}{a} + 2\frac{\mu}{1-\mu} + \Big(\frac{\mu}{1-\mu}\Big)^2 a} > 0.$$

$$a = \exp\Big(\frac{\lambda_0-\lambda_1}{\sigma^2}\Big(t-\frac{\lambda_1+\lambda_0}{2}\Big)\Big)$$

As $\frac{T_*}{\sigma^2}$ converges to a finite value, a is also a finite value. Then, $-P_L'(T_*) \to 0$. Then, (4) reduced to $[P_H(T_*) - P_L(T_*)]$, which is calculated as

$$P_H(t) - P_L(t) = \frac{\frac{\mu}{1-\mu} - \frac{1-\mu}{\mu}}{\Big(1+\frac{\mu}{1-\mu}a\Big)\Big(1+\frac{1-\mu}{\mu}a\Big)}a,$$

[10] Θ is Landau symbol; that is $g(t) \in \Theta(f(t))$ implies that $\lim_{t\to\infty}\frac{g(t)}{f(t)} \in \mathbb{R}\setminus\{0\}$

As a is a finite value, $\lim_{\sigma^2 \to \infty} P_H(T_*) - P_L(T_*) > 0$. This concludes that $\tilde{D} > 0$. ■

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