

# Mixed Risk Apportionment and Non-financial Correlated Background Uncertainty\*

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This Version: September 7, 2023

## Abstract

This paper considers a portfolio problem with one safe asset and one risky asset in the presence of background risk. We assume that the background risk is a non-financial variable and it is correlated to financial risk. The aim of this paper is to investigate the effect of correlation on portfolio choice. While we find that an increase in correlation lowers (raises) the expected utility for mixed correlation averse (seeking) individuals, contrary to intuition, it does not necessarily decrease (increase) investment in the risky asset. We determine that the conditions needed to reduce (increase) investment can be instead related to preferences for the combination of good and bad. We also introduce ambiguity into the correlation and investigate its effects on the optimal portfolio.

JEL classification numbers: D81, D91, G11

Key Words: Ambiguity, Bivariate Utility Function, Linear Payoff, Mixed Correlation Aversion (Seekingness), Background Uncertainty, Portfolio Choice

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\*We are grateful to Toshiki Honda, the seminar participants at Nagoya City University, and the participants of the 2023 East Japan Section of the Japan Finance Association for their helpful comments. This research is financially supported by the JSPS KAKENHI Grant Numbers 20K01745 and 20H01519, and the Joint Research Program of KIER.

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# 1 Introduction

People need to make decisions in the face of multidimensional consequences that are risky but also interdependent.<sup>1</sup> Let us illustrate such a situation with two examples. To start, imagine the situation where you are considering purchasing a new health insurance policy. In this case, you need to consider not only your future financial conditions, but also your future health status.<sup>2</sup> Now imagine that you are the general manager of a multinational firm in the East Asian region, where you are concerned about the profits from other areas, such as the North American region, which may affect the management strategy of your region.<sup>3</sup> These illustrative examples highlight the need to incorporate multidimensional and interdependent risks into a model and to investigate the effects of their interdependence on decision making.

The above cases are examples of the linear payoff model in Dionne et al. (1993) in the presence of non-financial background risk.<sup>4</sup> Here, background risk is a type of risk that is exogenous and cannot be controlled. For clarity, we specify the linear payoff model as a portfolio problem choosing between one safe asset and one risky asset in the presence of a health risk.<sup>5</sup> The reasons for choosing the above specification are: (i) the portfolio choice problem is representative of the linear payoff problem, and (ii) empirical analysis is making progress on the properties of

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<sup>1</sup>This analysis focuses on bivariate consequences, but can also be applied to higher-order dimensions when all variables are exogenous except for two variables.

<sup>2</sup>There is now consensus about the positive relationship between health status and income (e.g., Bloom and Canning, 2000), and the underlying mechanism is now explored in detail in, for example, the importance of health status in childhood (Case et al., 2002) and the long-run effects of health improvements (Kotschy, 2021).

<sup>3</sup>He and Ng (1998) and Allayannis and Ihrig (2001) empirically find that correlations between returns and exchange rates may be positive or negative, depending on the company's characteristics. He and Ng (1998) investigate Japanese multinational firms and demonstrate a positive correlation between firm returns and exchange rates, while Allayannis and Ihrig (2001) analyze US manufacturing industries and find a negative correlation.

<sup>4</sup>The former is like the analysis of the demand for insurance by Mossin (1968), where the bivariate variables are wealth level and health status. The latter is like the analysis of production under price risk by Sandmo (1971), where the bivariate variables are the profits of one own region and another region.

<sup>5</sup>This model is justifiable as the portfolio problem of a representative investor by empirical observations of a positive relationship between health status and income.

the bivariate utility function of wealth level and health status. The research question of this paper is how the correlation between financial and health risks affects portfolio choice.

Background risk has been extensively studied in the literature under the assumption of independence from endogenous risks. For the portfolio choice problem, the effects of background risk are analyzed in the univariate setting by Weil (1992) and the bivariate setting by Crainich et al. (2017). Apart from specific contexts, the analysis of background risk can be made using the derived utility function by Kihlstrom et al. (1981) and Nachman (1982). This approach requires the assumption of independence and cannot be applied to the case where nonfinancial background risk depends on endogenous financial risk.

However, independent background risk contradicts reality in many situations. This leads to the need to relax the independence assumption and introduce dependent background risk. In the literature, the notion of stochastic dependence by Lehmann (1966) is widely used to describe dependent background risk. This stochastic dependence and its variants are stronger than correlation because these notions imply correlation (Li, 2011). Some interesting results are obtained under dependent background risk. For example, investment is not undertaken, even with positive excess returns. This relates to the stock market participation puzzle by Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995). However, because of the analytical difficulty, we have obtained few results about dependent background risk when compared with independent background risk. We also note that previous studies demonstrate the effects of stochastic dependence for the comparison with independence. Thus, more research is needed on the effect of changes in stochastic dependence on portfolio choices. In this paper, we adopt a different description of stochastic dependence and undertake further analyses.

This paper describes stochastic dependence following Doherty and Schlesinger (1983) that modelled the relationship between insurable and contract nonperformance risks. This can capture correlation through a single parameter. Moreover,

in addition to facilitating the analysis, this description has several advantages: (i) the structure of mixed risk apportionment is embedded into the model to analyze an increase in correlation,<sup>6</sup> (ii) the effect of correlation on portfolio choices can be investigated, and (iii) ambiguity can be incorporated into correlation.

This paper employs the term uncertainty in a broad sense, whereby uncertainty is an umbrella term that includes the notions of risk and ambiguity.<sup>7</sup> Whereas ambiguity about asset returns in a univariate framework has been analyzed in the framework of portfolio choice,<sup>8</sup> ambiguity about correlation in a bivariate framework remains to be investigated.<sup>9</sup> In general, it is difficult to pin down the correlation between two variables, for example, financial and health risks. We overcome this difficulty through ambiguous correlation and investigate its effects on the optimal portfolio in a bivariate framework. By adopting the smooth ambiguity model by Klibanoff et al. (2005), which is the primary ambiguity model used in past applications, we are able to differentiate between ambiguity and ambiguity attitude.<sup>10</sup>

The contribution of our paper is threefold. First, we provide a framework for analyzing the effects of correlation between financial and health background risks on optimal portfolio choices under a bivariate setting. We use this to determine the conditions under which an increase in correlation monotonically affects the optimal portfolio choice. Second, we relate these conditions to mixed risk apportionment. Finally, we introduce ambiguity in the correlation and show that ambiguous correlation decreases the optimal investment in the risky asset.

The organization of this paper is as follows. Section 2 provides a literature review, Section 3 introduces a formal model, and Section 4 investigates the effects

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<sup>6</sup>The notion of risk apportionment is introduced by Eeckhoudt and Schlesinger (2006), and is characterized by a specific pattern of the preference for a combination of “good” and “bad”. To apply to bivariate utility functions, “mixed” is added to the beginning of the term.

<sup>7</sup>Since seminal works by Gilboa and Schmeidler (1989) and Schmeidler (1989), the effects of ambiguity on decision-making have been extensively investigated. For textbook explanations, see Gilboa (2009).

<sup>8</sup>See Guidolin and Rinaldi (2013) for a review.

<sup>9</sup>Jiang et al. (2022) tackle ambiguous correlation in the portfolio problem. Using the mean-variance approach, they show that portfolio concentration occurs under the max-min expected utility in Gilboa and Schmeidler (1989).

<sup>10</sup>For applications of the smooth ambiguity model to finance, see Gollier (2011).

of correlation on portfolio choices. Section 5 introduces ambiguity into correlation and investigates its effects on portfolio choices. Section 6 relates the conditions determined in Section 4 to mixed risk apportionment, which is a specific type of preference for the combination of good and bad. Section 7 presents higher-order generalizations of the results in previous sections. Section 8 considers correlation seeking and mixed correlation seeking individuals. Section 9 concludes.

## 2 Literature review

The extant literature has investigated the effects of independent background risk. For example, as early references for specific contexts, we refer to Eeckhoudt and Kimball (1992) for insurance demand and Bryis et al. (1993) for hedging behavior. Apart from specific contexts, Gollier and Pratt (1996) introduce the notion of risk vulnerability under which the derived utility function is more risk averse than the underlying utility function. That is, background risk leads to more risk averse behaviors under risk vulnerability. Malevergne and Rey (2009) apply the notion of risk vulnerability to the bivariate utility function and term it as cross risk vulnerability. Even though these previous works have obtained fruitful results under independent background risk, as discussed earlier, the results cannot necessarily be obtained under dependent background risk.

There have been several attempts to investigate the effects of dependent background risk. The main idea of dependent background risk draws on that of stochastic dependence by Lehmann (1966) in the statistical literature and Wright (1987) for economic applications.<sup>11</sup> Tsetlin and Winkler (2005), Li (2011), and Chiu (2020) align with this description. Unlike independent background risk, these studies reveal the possibility that investment is not made even with positive excess returns. Even though their findings have some interest in particular problems, we face difficulty in obtaining clear comparative static predictions within this framework. For exam-

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<sup>11</sup>We can find other ways to describe dependence. For example, bivariate stochastic dominance by Denuit et al. (2011).

ple, previous studies have not obtained the conditions under which an increase in stochastic dependence leads to more risk averse behaviors.

This paper adopts a different notion of dependence as introduced by Doherty and Schlesinger (1983, 1990) to describe dependent background risk. This dependence is widely adopted in various models, for example, index insurance (Clarke, 2016) comorbidity risk (Bleichrodt et al., 2003; Fujii and Osaki, 2019), and precautionary savings (Courbage et al., 2020; Asano and Osaki, 2022). Using this notion of stochastic dependence, the present study determines the conditions under which an increase in correlation raises or lowers investment in risky assets. We also investigate the effect of ambiguous correlation on the optimal portfolio.

The signs of the cross derivatives of the bivariate utility functions play a crucial role in determining the effects of correlated background uncertainty. In a univariate framework, Caballé and Pomansky (1996) provide the notion of mixed risk aversion (seekingness). Eeckhoudt and Schlesinger (2006) characterize the notion of mixed risk aversion (seekingness) by the preference for the combination of good and bad as constructed using two building blocks: a sure reduction and a zero-mean risk. This idea is extended to the bivariate utility setting by Eeckhoudt et al. (2007) that characterize the notions of correlation aversion and cross prudence. In a univariate setting, Eeckhoudt et al. (2009) adopt a stochastic dominance relation to rank good and bad, and characterize mixed risk aversion (seekingness). This framework is extended to the bivariate setting by Jokung (2011). These works present useful characterizations of individuals' attitudes toward risk and correlation using the signs of the utility functions. A few studies also investigate the signs of the cross derivatives in experimental settings using the framework in Eeckhoudt et al. (2009), e.g., Attema et al., (2019).

### 3 The model

In this section, we provide a framework for analyzing the effects of the correlation between two risks, namely, financial risk and non-financial background risk, on port-

folio choice. As discussed earlier, for convenience, we interpret background risk as a health risk. We suppose that an individual faces a portfolio choice problem with one safe asset and one risky asset in the presence of a health risk. We consider a static model in which there are two dates,  $t = 0$  and  $t = 1$ . The individual makes a portfolio choice at  $t = 0$ , and its return is realized at  $t = 1$ . Utility is determined by the level of wealth and health status  $(w, h) \in W \times H \subseteq \mathbb{R}_+^2$ . Let  $u : W \times H \rightarrow \mathbb{R}$  denote a bivariate utility function. We denote  $u_{(1,0)}(w, h)$  as  $\partial u / \partial w$ ,  $u_{(0,1)}(w, h)$  as  $\partial u / \partial h$ , and  $u_{(1,1)}(w, h)$  as  $\partial^2 u(w, h) / \partial w \partial h$ . Similarly,  $u_{(i,j)}(w, h)$  stands for  $\partial^{i+j} u(w, h) / \partial w^i \partial h^j$ . All higher-order partial and cross derivatives are assumed to exist if necessary for the analysis. We impose the standard assumptions on the utility function for wealth level and health status, that is,  $u(w, h)$  is increasing and concave in wealth level  $w$  and health status  $h$ . We consider the two cases of the signs of cross derivatives,  $u_{(1,1)}(w, h) \leq 0$  and  $u_{(1,1)}(w, h) \geq 0$ , which capture individuals' correlation aversion and correlation seekingness, respectively.<sup>12</sup> Because the analysis of correlation seekingness is the same as correlation aversion, we focus on the analysis of correlation aversion and present the results for correlation seekingness in Section 8. Note that we do not make any claims about the validity of correlation aversion and seekingness. Indeed, both cases are possible. It then depends on situations whether correlation aversion or seekingness is compelling based on observations from empirical studies.

The individual has initial wealth  $w_0$  to invest in either safe or risky assets. The net return of the safe asset is  $r_f$ . There are two possible returns of the risky asset: “good”,  $\tilde{r}_G$ , and “bad”,  $\tilde{r}_B$  with probability  $p$  and  $1 - p$ . The excess returns are denoted by  $\tilde{x}_G = \tilde{r}_G - r_f$  and  $\tilde{x}_B = \tilde{r}_B - r_f$ . We omit “excess” for the remainder of this analysis because only excess returns are considered. We assume that these random variables are defined over the same compact support  $[\underline{x}, \bar{x}]$  with  $\underline{x} < 0 < \bar{x}$ .

<sup>12</sup>Eckhoudt et al. (2007) show that the notions of correlation aversion and correlation seekingness are captured by the signs of the cross derivatives of a bivariate utility function. See also Epstein and Tanny (1980). In empirical studies, we obtain mixed evidence of correlation aversion and correlation seekingness (for example, Edwards, 2008; Sloan et al., 1990; Viscusi and Evans, 1990). We note that our analytical method can be applied to both cases of correlation aversion and correlation seekingness.

The two types of risks are also involved in health status: “good”,  $\tilde{h}_G$ , and “bad”,  $\tilde{h}_B$  with probability  $q$  and  $1 - q$ , which are defined over the same compact support  $[\underline{h}, \bar{h}]$  with  $\underline{h} < \bar{h}$ . We use the terms “good” and “bad” in the ranking of expected utility; that is, good risk leads to higher expected utility than bad risk.

We introduce the stochastic dependence between risky return and health risks. There are four combinations: two types of risky returns and two types of health risks,

- $\tilde{x}_G$  and  $\tilde{h}_G$  with probability  $kpq$ ;
- $\tilde{x}_B$  and  $\tilde{h}_G$  with probability  $(1 - kp)q$ ;
- $\tilde{x}_G$  and  $\tilde{h}_B$  with probability  $p(1 - kq)$ ;
- $\tilde{x}_B$  and  $\tilde{h}_B$  with probability  $1 - p - q + kpq$ .

We calculate the probability of risky returns and health risks such that, for example, the good risky return occurs with probability  $p = kpq + p(1 - kq)$ . Other probabilities of the risky returns and health risks can be calculated similarly. A value of  $k$ , taking a positive value, is chosen so that all probabilities are strictly positive and the sum of all probabilities is unity. This  $k$  captures the stochastic dependence between the risky return and the health risk. When the value of  $k$  is unity, the risky returns and health risks are independent. Indeed, if  $k = 1$ , then the probability that  $\tilde{x}_G$  and  $\tilde{h}_G$  occur simultaneously is equal to  $pq$ . A value of  $k$  greater (less) than unity indicates a positive (negative) correlation. Because the correlation increases in  $k$ , correlation and  $k$  have a one-to-one correspondence. Thus, we can treat  $k$  to indicate correlation.

Under this setting, the individual determines how much to invest in the risky and safe assets, which is denoted by  $(\alpha, w_0 - \alpha)$ . The value of this portfolio at  $t = 1$  is written as

$$\begin{aligned} \alpha(1 + \tilde{r}) + (w_0 - \alpha)(1 + r_f) &= w_0(1 + r_f) + \alpha(\tilde{r} - r_f) \\ &= w_1 + \alpha\tilde{x}, \end{aligned}$$



where  $\tilde{r}$  denotes the random return of the risky asset,  $w_1 = w_0(1+r_f)$ , and  $\tilde{x} = \tilde{r} - r_f$ . Then, the individual determines the investment of the risky asset  $\alpha$  to maximize the following expected utility:

$$\begin{aligned} V(\alpha) &= kpqE[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_G)] + p(1-kq)E[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_B)] \\ &\quad + (1-kp)qE[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_G)] + (1-p-q+kpq)E[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_B)]. \end{aligned}$$

The first-order condition for (1) is

$$\begin{aligned} V'(\alpha^*) &= kpqE[\tilde{x}_G u_{(1,0)}(w_1 + \alpha^* \tilde{x}_G, \tilde{h}_G)] + p(1-kq)E[\tilde{x}_G u_{(1,0)}(w_1 + \alpha^* \tilde{x}_G, \tilde{h}_B)] \\ &\quad + (1-kp)qE[\tilde{x}_B u_{(1,0)}(w_1 + \alpha^* \tilde{x}_B, \tilde{h}_G)] + (1-p-q+kpq)E[\tilde{x}_B u_{(1,0)}(w_1 + \alpha^* \tilde{x}_B, \tilde{h}_B)] = 0. \end{aligned} \tag{1}$$

Because  $V(\alpha)$  is concave by  $u_{(2,0)}(w, h) \leq 0$ , the second-order condition for a maximum is satisfied. For simplicity, we assume that the optimal investment in the risky asset is interior,  $0 < \alpha^* < w$ , and is unique throughout the paper. We sometimes omit the asterisk when there is no possibility of confusion and use the notation  $\alpha(k)$  to represent the optimal portfolio explicitly under the correlation of  $k$ .

Let us consider that the correlation increases by  $\Delta k$  from  $k$  to  $k + \Delta k$ . This increases the probability of the combination of  $\tilde{x}_G$  and  $\tilde{h}_G$  and that of  $\tilde{x}_B$  and  $\tilde{h}_B$  by  $\Delta k \times pq$ . However, it also decreases the probability of the combination of  $\tilde{x}_G$  and  $\tilde{h}_B$  and that of  $\tilde{x}_B$  and  $\tilde{h}_G$  by  $\Delta k \times pq$ . In other words, an increase in correlation enhances the possibility of the combination of good and good,<sup>13</sup> but reduces the possibility of the combination of good and bad, which suggests that the structure of risk apportionment by Eeckhoudt and Schlesinger (2006) is incorporated into the effects of correlation on portfolio choice.

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<sup>13</sup>Following the literature, we simply say the combination of good and good instead of bad and bad in the remainder of the paper. Similarly, we apply the same convention for good and bad by omitting bad and good.

We can confirm this structure with the marginal expected utility for  $k$ . By differentiating (1) with respect to  $k$ , it follows that

$$\begin{aligned} & \frac{\partial V(\alpha)}{\partial k} \\ = & pq\{E[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_G)] + E[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_B)] - (E[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_B)] + E[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_G)])\}. \end{aligned}$$

The effects of an increase in correlation can be classified into two parts: the former two terms represent gains in utility, and the latter two terms represent losses in utility or disutility. The former is the combination of good and good (and bad and bad), and the latter is the combination of good and bad (bad and good).

## 4 Correlation and portfolio choice

We suppose that good and bad risks are ranked via first-order stochastic dominance (FSD). In the remainder of this paper, the cumulative distribution functions of good and bad risks are denoted by  $G$  and  $B$ , respectively. Distribution function  $G_{\tilde{x}}$  dominates  $B_{\tilde{x}}$  in the sense of FSD if  $G_{\tilde{x}}(x) \leq B_{\tilde{x}}(x)$  for all  $x \in [\underline{x}, \bar{x}]$ . The subscript  $\tilde{x}$  represents the cumulative distribution functions of random variable  $\tilde{x}$ . We also say that random variable  $\tilde{x}_G$  dominates random variable  $\tilde{x}_B$  in the sense of FSD when  $\tilde{x}_G$  and  $\tilde{x}_B$  have the distribution functions  $G_{\tilde{x}}$  and  $B_{\tilde{x}}$ . For all  $\alpha \in (0, w)$  and  $h \in [\underline{h}, \bar{h}]$ ,

$$E[u(w + \alpha\tilde{x}_G, h)] \geq E[u(w + \alpha\tilde{x}_B, h)]$$

for all utility function satisfying  $u_{(1,0)}(w, h) \geq 0$  when  $\tilde{x}_G$  dominates  $\tilde{x}_B$  in the sense of FSD. Because good risky returns lead to higher expected utility than do bad risky returns for individuals with  $u_{(1,0)}(w, h) \geq 0$ , FSD can identify the good and bad risky returns. Similarly, good health risk  $\tilde{h}_G$  dominates bad health risk  $\tilde{h}_B$  in the sense of FSD. Individuals with  $u_{(0,1)}(w, h) \geq 0$  then obtain higher expected utility from a good health risk than a bad health risk.

Before starting the analysis, we prepare the following lemma. The proof can be

found in Appendix.

**Lemma 1.** *Let  $\pi(w, h)$  be the payoff function and define*

$$\begin{aligned} E[\pi(\tilde{w}, \tilde{h})] &= kpqE[\pi(\tilde{w}_G, \tilde{h}_G)] + (1 - kpq)E[\pi(\tilde{w}_G, \tilde{h}_B)] \\ &\quad + p(1 - kq)E[\pi(\tilde{w}_B, \tilde{h}_G)] + (1 - p - q + kpq)E[\pi(\tilde{w}_B, \tilde{h}_B)]. \end{aligned}$$

*Suppose that*

- $\tilde{w}_G$  dominates  $\tilde{w}_B$  in the sense of FSD;
- $\tilde{h}_G$  dominates  $\tilde{h}_B$  in the sense of FSD.

*The following two conditions are also equivalent:*

- (i)  $\pi_{(1,1)}(w, h) \leq 0$ .
- (ii)  $E[\pi(\tilde{w}, \tilde{h})]$  is decreasing in  $k$ .

By replacing  $\pi$  with  $u$  in Lemma 1, we ascertain that the expected utility for correlation averse individuals decreases in correlation, that is,  $\partial V(\alpha)/\partial k \leq 0$  for all  $\alpha \in (0, w)$ . The result is summarized as the following proposition.

**Proposition 1.** *The following two conditions are equivalent:*

- (i) *Individuals are correlation averse.*
- (ii) *An increase in correlation lowers expected utility.*

Proposition 1 is a result consistent with the notion of correlation aversion. By reducing investment in the risky asset, individuals can mitigate the effect of correlation because the safe asset does not correlate with background health risk. In the case of correlation aversion, we expect that an increase in correlation reduces investment in the risky asset,  $\partial \alpha(k)/\partial k \leq 0$ , because an increase in correlation lowers expected utility.

This intuition is only correct in the limited case where correlation averse individuals are *cross prudent neutral*, which is defined by  $u_{(2,1)}(w, h) = 0$ . We have

that

$$\begin{aligned}
& \frac{\partial V'(\alpha)}{\partial k} \leq 0 \\
& \Leftrightarrow E[\tilde{x}_G(u_{(1,0)}(w + \alpha\tilde{x}_G, \tilde{h}_G) - u_{(1,0)}(w + \alpha\tilde{x}_G, \tilde{h}_B))] \\
& \leq E[\tilde{x}_B(u_{(1,0)}(w + \alpha\tilde{x}_B, \tilde{h}_G) - u_{(1,0)}(w + \alpha\tilde{x}_B, \tilde{h}_B))]. \tag{2}
\end{aligned}$$

Because  $\tilde{x}_G$  dominates  $\tilde{x}_B$  in the sense of FSD, (2) holds under the following condition:

$$E[x(u_{(1,0)}(w + \alpha x, \tilde{h}_G) - u_{(1,0)}(w + \alpha x, \tilde{h}_B))]$$

is decreasing in  $x$ , that is,

$$\begin{aligned}
& \frac{\partial}{\partial x} E[x(u_{(1,0)}(w + \alpha x, \tilde{h}_G) - u_{(1,0)}(w + \alpha x, \tilde{h}_B))] \leq 0 \\
& \Leftrightarrow E[u_{(1,0)}(w + \alpha x, \tilde{h}_G)] + E[\alpha x u_{(2,0)}(w + \alpha x, \tilde{h}_G)] \\
& \leq E[u_{(1,0)}(w + \alpha x, \tilde{h}_B)] + E[\alpha x u_{(2,0)}(w + \alpha x, \tilde{h}_B)].
\end{aligned}$$

By  $u_{(2,1)}(w, h) = 0$ , we have that

$$E[\alpha x u_{(2,0)}(w + \alpha x, \tilde{h}_G)] = E[\alpha x u_{(2,0)}(w + \alpha x, \tilde{h}_B)].$$

This leads to the following:

$$E[u_{(1,0)}(w + \alpha x, \tilde{h}_G)] \leq E[u_{(1,0)}(w + \alpha x, \tilde{h}_B)] \Leftrightarrow u_{(1,1)}(w, h) \leq 0.$$

We have confirmed that an increase in correlation reduces investment in the risky asset for correlation averse individuals who are also cross prudent neutral. From the above analysis, we know that correlation aversion is not enough to obtain the intuitive comparative static results, and we can guess that the sign of  $u_{(2,1)}(w, h)$  plays a crucial role in obtaining the intuitive results. We say that individuals are *cross prudent* if  $u_{(2,1)}(w, h) \geq 0$ , and are *cross imprudent* if  $u_{(2,1)}(w, h) \leq 0$ .<sup>14</sup>

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<sup>14</sup>In Eeckhoudt et al. (2007), individuals with  $u_{(2,1)}(w, h) \geq 0$  is referred to as cross prudent

We introduce the two intensity measures of cross prudence to obtain definitive comparative static results without assuming cross prudent neutrality.

**Definition 1.** We call the following two measures *the degree of partial cross prudence* and *the degree of relative cross prudence*, respectively.

- the degree of partial cross prudence:

$$-\frac{xu_{(2,1)}(w+x, h)}{u_{(1,1)}(w+x, h)};$$

- the degree of relative cross prudence:

$$-\frac{wu_{(2,1)}(w, h)}{u_{(1,1)}(w, h)}.$$

The terms “partial” and “relative” are based on the terminology in a univariate setting, partial risk aversion (Menezes and Hanson, 1970) and relative risk aversion (Pratt, 1964).<sup>15</sup>

Because it follows from (1) that  $V'(\alpha) = E[\pi(\tilde{w}, \tilde{h})]$  by setting  $\pi(w, h) = xu_{(1,0)}(w + \alpha x, h)$  and applying Lemma 1, we obtain that  $V'(\alpha)$  decreases in  $k$  if and only if

$$\pi_{(1,1)}(w, h) \leq 0 \Leftrightarrow -\frac{\alpha(k)xu_{(2,1)}(w + \alpha(k)x, h)}{u_{(1,1)}(w + \alpha(k)x, h)} \leq 1. \quad (3)$$

Using the terminology of Definition 1, the condition can be said that the partial cross prudence is less than unity. Because  $V'(\alpha)$  decreases in  $k$ , we know that the optimal investment in the risky asset decreases in  $k$ , which is obtained by the following:

$$V'(\alpha(k); k) = 0 \Rightarrow V'(\alpha(k); k') \leq 0$$

for  $k \geq k'$ . The value  $k$  after the semicolon means that the correlation is equal to  $k$ .

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in health to distinguish them from individuals with  $u_{(1,2)}(w, h) \geq 0$ , who are cross prudent in wealth. Because  $u_{(2,1)}(w, h)$  only appears in this paper, we can omit “health” without confusion.

<sup>15</sup>These intensity measures are used in the context of the consumption–saving model by Courbage et al. (2022).

Assuming cross prudence,  $u_{(2,1)}(w, h) \geq 0$ , a sufficient condition for (3) is that

$$-\frac{wu_{(2,1)}(w, h)}{u_{(1,1)}(w, h)} \leq 1. \quad (4)$$

This condition means that the degree of relative cross prudence is less than unity.

Applying a similar argument, we can prove that (4) is a sufficient condition under which an increase in correlation decreases the optimal portfolio for correlation seeking and cross imprudent individuals. We summarize the above argument as the following proposition and provide interpretations of these conditions in the next section.

**Proposition 2.** *Suppose that individuals are correlation averse. Then,*

- (i) the degree of partial cross prudence is less than unity if and only if an increase in correlation reduces the investment in the risky asset, and*
- (ii) if individuals are cross prudent and the degree of relative cross prudence is less than unity, then an increase in correlation reduces the investment in the risky asset.*

As discussed in Section 3, an increase in correlation has a greater likelihood of providing the combination of good and good than good and bad. The combination of good and good is undesirable for correlation averse individuals because an increase in correlation lowers the expected utility as in Proposition 1.<sup>16</sup> There are two effects concerning the FSD shift of risky returns: the substitution effect and the wealth effects. In a univariate setting, it is known that the wealth effect dominates the substitution effect when the degree of relative risk aversion is less than unity (e.g., Fishburn and Porter, 1976). Correlation averse individuals should then reduce their investment in the risky asset in response to an increase in correlation to avoid the undesirable combination when the wealth effect dominates the substitution effect. To determine the condition, we need to incorporate the effect of health risks. Combining these two effects, the condition is determined by comparing the degrees of relative or partial cross prudence with the threshold value being equal to unity.

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<sup>16</sup>We relate the preference for combining good with bad (good) to the signs of the cross derivatives of the utility function in the following section.

**Example 1.** As a concrete example, we consider the following Cobb–Douglas utility function:

$$u(w, h) = \frac{(w^\psi h^{1-\psi})^{1-\gamma}}{1-\gamma}.$$

Here,  $\psi$  takes a value in  $[0, 1]$  for  $u_{(1,0)}(w, h)$ ,  $u_{(0,1)}(w, h) \geq 0$  and  $u_{(2,0)}(w, h)$ ,  $u_{(0,2)}(w, h) \leq 0$ . The sign of  $u_{(1,1)}(w, h)$  depends on the parameter value of  $\gamma$ . When  $\gamma = 1$ , we have that  $u_{(1,1)}(w, h) = 0$ , that is,  $\gamma = 1$  is the threshold to determine the signs of  $u_{(1,1)}(w, h)$ . We have that  $u_{(1,1)}(w, h) > (<)0$  for  $\gamma < (>)1$ , which represents correlation seekingness (aversion). For  $\gamma < 1$ , it holds that

$$-\frac{wu_{(2,1)}(w, h)}{u_{(1,1)}(w, h)} = 1 - (1 - \gamma)\psi \leq 1.$$

Thus, from Proposition 8 in Section 8, we conclude that an increase in correlation raises the investment in the risky asset for investors who have the Cobb–Douglas utility function with  $\gamma < 1$ . For  $\gamma > 1$ , it holds that

$$-\frac{wu_{(2,1)}(w, h)}{u_{(1,1)}(w, h)} = 1 - (1 - \gamma)\psi \geq 1. \quad (5)$$

For correlation averse individuals, we cannot obtain clear comparative statics results. We note that an increase in correlation lowers the investment in the risky asset even though (5) holds but (3) is satisfied.

It is known that the sign of the investment in the risky asset is the same as that of the risky asset return in a univariate setting, that is, individuals invest in a positive amount of risky assets when their expected returns are positive. This result can be extended to the portfolio problem in the presence of independent background risk. For our setting,  $k = 1$  corresponds to independent background risk. We have that

$$\begin{aligned} V'(0) &= E[\tilde{y}u_{(1,0)}(w_1, \tilde{z})] \\ &= E[\tilde{y}]E[u_{(1,0)}(w_1, \tilde{z})], \end{aligned} \quad (6)$$

where  $\tilde{y} = p\tilde{x}_G + (1-p)\tilde{x}_B$  and  $\tilde{z} = q\tilde{h}_G + (1-q)\tilde{h}_B$ .

Let us consider a risky asset  $\tilde{y}$  whose expected return is equal to zero,  $E[\tilde{y}] = pE[\tilde{x}_G] + (1-p)E[\tilde{x}_B] = 0$ . When  $k$  is more than unity, correlation averse individuals make negative investments in the risky asset when (4) holds by Proposition 2 and short selling is allowed. When short selling is not allowed, the negative investment means that individuals do not hold the risky asset and hold only the safe asset. By a continuity argument, this means that correlation averse individuals may make negative investments in risky assets with positive expectations. This observation can be related to the stock market participation puzzle by Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995). The puzzle involves empirical observations that many households do not own any equities and hold only safe assets despite positive excess returns.

## 5 Mixed risk apportionment

Eeckhoudt and Schlesinger (2006) introduce a notion of risk apportionment as particular types of preferences between two lotteries. This preference can be related to the signs of higher-order derivatives of a utility function in a univariate setting within an expected utility framework. A sure reduction and noise risk are two building blocks used to construct the pairs of lotteries to characterize risk apportionment. By comparing zero, these two building blocks are bad for  $u'(w) \geq 0$  and  $u''(w) \leq 0$ .<sup>17</sup> The two lotteries are constructed by the combination of good and bad: combining good with bad and good with good. Risk apportionment can be classified into two types: preferences for combining good with bad and good with good. Eeckhoudt and Schlesinger (2006) show that the former is related to  $(-1)^{n+1}u^n(w) \geq 0$ <sup>18</sup> and Crainich et al. (2013) show that the latter is related to  $u^n(w) \geq 0$ . In this section,

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<sup>17</sup>A sure reduction  $-k$  with  $k > 0$  lowers utility for every increasing utility function because  $u(w-k) \leq u(w)$  given intimal wealth  $w$ . Now recall that the term of good and bad is related to the ranking of expected utility. Comparing  $[-k]$  with  $[0]$ , we can say that  $[-k]$  is bad and  $[0]$  is good. Similarly, we can say that a noise risk  $[\tilde{\varepsilon}]$  is bad for every concave utility function by comparing  $[0]$ .

<sup>18</sup>We denote  $u^n(w) = \partial^n u(w) / \partial w^n$ .



we show that all the second conditions in Proposition 2 are related to the particular type of preference for the combination of good and bad in a bivariate setting. In the present study, this type of preference is called *mixed risk apportionment*, following the term in a univariate setting. As in risk apportionment, mixed risk apportionment can be classified into two types: the preference for combining good with bad or good with good. Mixed risk apportionment can be considered a consistent preference because it exhibits the preference for the same pattern of the lottery preference.

First, we consider the two conditions in Proposition 2: correlation aversion and cross prudence. Let  $(w, h) \in \mathbb{R}_+^2$  denote a nonnegative vector of wealth level and health status, and let  $[A; B]$  denote a lottery where outcomes  $A$  and  $B$  occur with probability one-half. For positive constants  $k$  and  $c$  with  $k < w$  and  $c < h$ , we define two lotteries:  $A_{(1,1)} = [(w, h); (w - k, h - c)]$  and  $B_{(1,1)} = [(w - k, h); (w, h - c)]$ .<sup>19</sup> Similarly, for a positive constant  $c$  and a noise risk  $\tilde{e}$ , we define two lotteries:  $A_{(2,1)} = [(w, h); (w + \tilde{e}, h - c)]$  and  $B_{(2,1)} = [(w + \tilde{e}, h); (w, h - c)]$ . The lotteries have the same pattern for the combination of good and bad. Lotteries  $A_{(1,1)}$  and  $A_{(2,1)}$  are constructed by combining good with good (and bad with bad). Conversely, lotteries  $B_{(1,1)}$  and  $B_{(2,1)}$  are constructed by combining good with bad (and bad with good). Following Eeckhoudt et al. (2007), particular types of lottery preferences between the above pairs can be related to the signs of the cross derivatives of a bivariate utility function.

**Result 1** (Eeckhoudt et al. (2007)). *Eeckhoudt et al. (2007) show the following:*

- (i) *Individuals are correlation averse,  $u_{(1,1)}(w, h) \leq 0$  if and only if they prefer  $B_{(1,1)}$  over  $A_{(1,1)}$ .*
- (ii) *Individuals are cross prudent,  $u_{(2,1)}(w, h) \geq 0$  if and only if they prefer  $B_{(2,1)}$  over  $A_{(2,1)}$ .*

Next, we consider another condition in Proposition 2: the degree of relative cross prudence is less than unity. We introduce a multiplicative version of lot-

<sup>19</sup>Because this pair of lotteries is used to characterize the sign of  $u_{(1,1)}(w, h)$ , the subscript of, for example,  $A_{(1,1)}$ , corresponds to the subscript of utility function  $u_{(1,1)}(w, h)$ . Similar notations are applied to other lottery pairs.

tery pairs. For  $0 < k < 1$  and  $0 < r < 1$ , we define the two lotteries:  $\mathcal{A}_{(2,1)} = [(w, h); (w(1-k)(1-r), h)]$  and  $\mathcal{B}_{(2,1)} = [(w(1-k), h); (w(1-r), h)]$ . The lotteries are the same as Eeckhoudt et al. (2009a), except that health status is included. Next, we consider preferences between two lotteries for marginal utility in health,  $u_{(0,1)}(w, h)$ , to incorporate the effect of health risks into a bivariate utility function. This idea is in the same spirit as Kimball (1990), where the precautionary premium is defined for marginal utility. In other words, precautionary effects against health risks are incorporated into the lottery preferences. Applying an argument of Eeckhoudt et al. (2009a), we obtain the following proposition.

**Proposition 3.** *When individuals are correlation averse, the following conditions are equivalent:*

- (i) *The degree of relative cross prudence is less than unity.*
- (ii) *Lottery  $\mathcal{B}_{(2,1)}$  is preferred to  $\mathcal{A}_{(2,1)}$ .*

*Proof.* See Appendix. □

In Proposition 2, there are three conditions for obtaining intuitive comparative static results:

- correlation aversion, cross prudence, and the degree of relative cross prudence being less than unity;

These conditions are related to mixed risk apportionment that corresponds to the preference for combining good with bad. In other words, the second conditions in Proposition 2 are consistent with the preference for the combination of good and bad.

## 6 Ambiguous correlation

In general, understanding the relationship of stochastic dependence between financial and health risks is a more difficult task when compared with grasping the financial and health risks themselves. Applying to our context, it is more difficult

to determine the correlation between risky returns and health risks compared with figuring out risky returns and health risks themselves. To overcome this difficulty, we introduce ambiguity into correlation through the parameter  $k$ . We assume that a plausible set of possible  $k$  is the set  $\{k_1, k_2, \dots, k_\Theta\}$ . Without loss of generality,  $k_\theta$  is arranged in ascending order,  $k_1 < k_2 < \dots < k_\Theta$ . The individual attaches subjective probability  $q_\theta$  to the parameter of  $k_\theta$  for  $\theta = 1, 2, \dots, \Theta$ . We assume that the individual employs the smooth ambiguity model in Klbanoff et al. (2005). Define a concave and strictly increasing function  $\phi$  whose variable takes expected utility. The function  $\phi$  is assumed to be twice differentiable. The concavity of  $\phi$  captures ambiguity aversion. The convexity and linearity of  $\phi$  correspond to ambiguity seeking and ambiguity neutral attitudes, respectively.

Given  $\alpha$ , the objective function is written as

$$V(\alpha) = \sum_{\theta=1}^{\Theta} q_\theta \phi(U(\alpha, k_\theta)). \quad (7)$$

Here,  $U(\alpha, k)$  is the expected utility given  $\alpha$  and  $k$ , that is,

$$\begin{aligned} U(\alpha, k_\theta) = & kpqE[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_G)] + p(1 - kq)E[u(w_1 + \alpha\tilde{x}_G, \tilde{h}_B)] \\ & + (1 - kp)qE[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_G)] + (1 - p - q + kpq)E[u(w_1 + \alpha\tilde{x}_B, \tilde{h}_B)]. \text{ for } \theta = 1, 2, \dots, n. \end{aligned}$$

As in the previous sections, good and bad are ranked via FSD, that is,  $\tilde{x}_G$  dominates  $\tilde{x}_G$  in the sense of FSD, and  $\tilde{h}_G$  dominates  $\tilde{h}_G$  in the sense of FSD,

The first-order condition for (7) is

$$V'(\alpha^*) = \sum_{\theta=1}^{\Theta} q_\theta \phi'(U(\alpha^*, k_\theta)) U_\alpha(\alpha^*, k) = 0. \quad (8)$$

The second-order condition is easily verified by the concavity of  $u$  and  $\phi$ .

We define  $k_O$  by  $k_O = \sum_{\theta} q_\theta k_\theta$  and find  $i \in \{1, 2, \dots, n\}$  such that  $k_i \leq k_O \leq k_{i+1}$ . The level of the optimal portfolio is denoted by  $\alpha^O$  under  $k_O$ . Because linear

$\phi$  corresponds to expected utility, we have

$$U_\alpha(\alpha_O, k_O) = \sum_{\theta=1}^n q_\theta U(\alpha_O, k_\theta) = 0.$$

We assume that  $u$  exhibits correlation aversion. From Proposition 1,  $U(\alpha, k)$  decreases in  $k$ . This leads to the following:

- $U(\alpha_O, k_\theta) \leq U(\alpha_O, k_O) \Leftrightarrow \phi'(U(\alpha_O, k_\theta)) \leq \phi'(U(\alpha_O, k_O))$  for  $\theta = 1, \dots, i$ , and
- $U(\alpha_O, k_\theta) \geq U(\alpha_O, k_O) \Leftrightarrow \phi'(U(\alpha_O, k_\theta)) \geq \phi'(U(\alpha_O, k_O))$  for  $\theta = i+1, \dots, n$ .

From Proposition 2, the optimal portfolio decreases in  $k$  when the individual is correlation averse, cross prudent, and

$$-\frac{wu_{(2,1)}(w, h)}{u_{(1,1)}(w, h)} \leq 1.$$

This leads to the following:

- $U_\alpha(\alpha_O, k_\theta) \leq 0$  for  $\theta = 1, \dots, i$ , and
- $U_\alpha(\alpha_O, k_\theta) \geq 0$  for  $\theta = i+1, \dots, n$ .

From the above two relations, we have the following:

- $\phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) \leq \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O) \leq 0$  for  $\theta = 1, \dots, i$ , and
- $0 \leq \phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) \leq \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O)$  for  $\theta = i+1, \dots, n$ .

Finally, we obtain the following:

$$\begin{aligned} V'(\alpha^*) &= \sum_{\theta=1}^i q_\theta \phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) + \sum_{\theta=i+1}^n q_\theta \phi'(U(\alpha_O, k_\theta))U_\alpha(\alpha_O, k_\theta) \\ &\leq \sum_{\theta=1}^i q_\theta \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O) + \sum_{\theta=i+1}^n q_\theta \phi'(U(\alpha_O, k_O))U_\alpha(\alpha_O, k_O) \quad (9) \\ &= \phi'(U(\alpha_O, k_O)) \sum_{\theta=1}^n U_\alpha(\alpha_O, k_\theta) = 0. \end{aligned}$$

Inequality (9) shows that ambiguous correlation decreases the optimal portfolio. We summarize the above argument into the following proposition.

**Proposition 4.** *Suppose that risky return and health risk are ranked by FSD. If  $u$  exhibits correlation aversion, cross prudence, and*

$$-\frac{wu_{(2,1)}(w, h)}{u_{(1,1)}(w, h)} \leq 1,$$

*then ambiguous correlation reduces the investment in the risky asset.*

Ambiguity aversion puts more weight on worse expected utilities. For correlation averse individuals, worse expected utilities correspond to higher correlation. From Proposition 2, the conditions on  $u$  guarantee that the demand for the risky asset is lower when individuals face higher correlation. Combining the above arguments, ambiguous correlation reduces the optimal portfolio.

## 7 Higher-order generalization

This section considers higher-order stochastic dominance relations to rank good and bad risks. The basic idea can be shared with the case of FSD, and the analysis can be extended from FSD to higher-order stochastic dominance relations. We consider two types of higher-order stochastic dominance,  $N$ -th order stochastic dominance (NSD, e.g., Ingersoll, 1987) and an  $N$ -th degree increase in risk (Ekern, 1980). Let  $G_{\tilde{w}}$  and  $B_{\tilde{w}}$  denote two cumulative distribution functions of good and bad wealth risks,  $\tilde{w}_G$  and  $\tilde{w}_B$ , that are defined on the same compact support,  $[\underline{w}, \bar{w}]$ . Let us define  $G_{\tilde{w}}^1(w) = G_{\tilde{w}}(w)$  and  $G_{\tilde{w}}^{n+1}(w) = \int_{\underline{w}}^w G_{\tilde{w}}^n(z)dz$  for  $n = 1, 2, \dots, N$  and  $B_{\tilde{w}}^1(w) = B_{\tilde{w}}(w)$  and  $B_{\tilde{w}}^{n+1}(w) = \int_{\underline{w}}^w B_{\tilde{w}}^n(z)dz$  for  $n = 1, 2, \dots, N$ . Distribution function  $G_{\tilde{w}}$  dominates distribution function  $B_{\tilde{w}}$  in the sense of NSD if  $G_{\tilde{w}}^N(w) \leq B_{\tilde{w}}^N(w)$  for all  $w \in [\underline{w}, \bar{w}]$  and  $G_{\tilde{w}}^n(\bar{w}) \leq B_{\tilde{w}}^n(\bar{w})$  for  $n = 1, 2, \dots, N-1$ . As in Section 4, we also say that  $\tilde{w}_G$  dominates  $\tilde{w}_B$  in the sense of NSD when random variables  $\tilde{w}_G$  and  $\tilde{w}_B$  have the distribution functions  $G_{\tilde{w}}$  and  $B_{\tilde{w}}$ , respectively. Similarly, we can

define NSD for health risks  $\tilde{h}_G$  and  $\tilde{h}_B$  whose distribution functions are  $G_{\tilde{h}}$  and  $B_{\tilde{h}}$ , respectively.

We assume that the good risky return  $\tilde{x}_G$  dominates the bad risky return  $\tilde{x}_B$  in the sense of NSD, and that the good health risk  $\tilde{h}_G$  dominates the bad health risk  $\tilde{h}_B$  in the sense of MSD. We also assume that individuals are mixed risk averse in both wealth and health, that is,  $(-1)^{i+1}u_{(i,0)}(w, h) \geq 0$  for  $i = 1, 2, \dots, N$  and  $(-1)^{j+1}u_{(0,j)}(w, h) \geq 0$  for  $j = 1, 2, \dots, M$ .<sup>20</sup> By this assumption, stochastic dominance relations of wealth and health risks can be related to the ranking of expected utility. Thus, good risky return (health risk) leads to higher expected utility than does bad risky return (health risk). We say that individuals are mixed correlation averse if

$$(-1)^{i+j+1}u_{(i,j)}(w, h) \geq 0 \text{ for } i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, M.$$

We define the two intensity measures for mixed correlation aversion as follows.

**Definition 2.** We call the following two measures *the degree of partial mixed correlation aversion* and *the degree of relative mixed correlation aversion*, respectively.

- the degree of partial mixed correlation aversion in  $(n, m)$ :

$$-\frac{xu_{(n+1,m)}(w+x, h)}{u_{(n,m)}(w+x, h)}.$$

- the degree of relative mixed correlation aversion in  $(n, m)$ :

$$-\frac{wu_{(n+1,m)}(w, h)}{u_{(n,m)}(w, h)}.$$

These measures are higher-order versions of the degrees of partial and relative cross prudence. From the above preparation, we provide the following results about correlation and portfolio choices.

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<sup>20</sup>The term for mixed risk aversion is borrowed from Caballé and Pomansky (1996) in a univariate setting.

**Proposition 5.** *Suppose that good risky return dominates bad risky return in the sense of NSD and good health risk dominates bad health risk in the sense of MSD, and individuals are mixed correlation averse. Then,*

- (i) *the degree of partial mixed correlation aversion in  $(n, m)$  is less than  $n$  for  $n = 1, 2, \dots, N$  and  $m = 1, 2, \dots, M$  if and only if an increase in correlation reduces the investment in the risky asset, and*
- (ii) *if the degree of relative mixed correlation aversion in  $(n, m)$  is less than  $n$  for  $n = 1, 2, \dots, N$  and  $m = 1, 2, \dots, M$ , then an increase in correlation reduces the investment in the risky asset.*

*Proof.* See Appendix. □

Next, we consider whether  $N$ -th and  $M$ -th degree increases in risk introduced by Ekern (1980) can be related to the signs of the higher-order cross derivatives of the utility function. Random variable  $\tilde{w}_G$  is an  $N$ -th degree riskier than random variable  $\tilde{w}_B$  if  $G_{\tilde{w}}^N(w) \leq B_{\tilde{w}}^N(w)$  for all  $w \in [\underline{w}, \bar{w}]$  and  $G_{\tilde{w}}^n(\bar{w}) = B_{\tilde{w}}^n(\bar{w})$  for  $n = 1, 2, \dots, N-1$ . The latter condition means that the first  $N-1$  moments have the same values. Individuals with  $(-1)^{N+1}u_{(N,0)}(w, h) \geq 0$  prefer  $\tilde{w}_G$  over  $\tilde{w}_B$ . Similarly, we can define good and bad health risks in which  $\tilde{h}_G$  is an  $M$ -th degree riskier than  $\tilde{h}_B$ . For example,  $N = 2$  corresponds to the increase in risk introduced by Rothschild and Stiglitz (1970). It is constructed using a series of mean-preserving spreads. Good risky return (health risk) leads to higher expected utility than does bad risky return (health risk) when individuals are averse to wealth (health) risk.<sup>21</sup>

From Proposition 5, we can obtain the following corollary.

**Corollary 1.** *Suppose that bad risky return is an  $N$ -th degree riskier than good risky return and bad health risk is an  $M$ -th degree riskier than good health risk, and that  $(-1)^{N+M+1}u_{(N,M)}(w, h) \geq 0$ . Then,*

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<sup>21</sup>  $N = 3$  corresponds to the increase in downside risk in Menezes et al. (1980) that is ranked by skewness with the same means and variances. The preference for an increase in downside risk can be related to the positive third derivative of the utility function and is called prudence.  $N = 4$  corresponds to the increase in outer risk by Menezes and Wang (2005) that is ranked by kurtosis with the same means, variances, and skewnesses. The preference for an increase in outer risk can be related to the negative fourth derivative of the utility function and is called temperance.

- (i) the degree of partial mixed correlation aversion in  $(N, M)$  is less than  $N$  if and only if an increase in correlation reduces the investment in the risky asset, and
- (ii) if the degree of relative mixed correlation aversion in  $(N, M)$  is less than  $N$ , then an increase in correlation reduces the investment in the risky asset.

All the conditions in Proposition 2 can be related to mixed risk apportionment and the specific type of lottery preferences for the combination of good and bad. Following Jokung (2011), we relate the signs of the higher-order cross derivatives to mixed risk apportionment. Suppose that  $\tilde{w}_G$  dominates  $\tilde{w}_B$  in the sense of NSD and  $\tilde{h}_G$  dominates  $\tilde{h}_B$  in the sense of MSD. Let us consider two lotteries:  $A_{(N,M)} = [(\tilde{w}_G, \tilde{h}_G); (\tilde{w}_B, \tilde{w}_B)]$  and  $B_{(N,M)} = [(\tilde{w}_G, \tilde{h}_B); (\tilde{w}_B, \tilde{w}_G)]$ . The lottery  $A_{(N,M)}$  is constructed by combining good with bad, and the lottery  $B_{(N,M)}$  is constructed by combining good with good. Using the lottery preference, mixed correlation averse individuals prefer  $B_{(N,M)}$  to  $A_{(N,M)}$ . The following result is a natural extension of Eeckhoudt et al. (2009) from the univariate to bivariate setting.

**Result 2** (Jokung (2011)). *Individuals are mixed correlation averse if and only if  $(-1)^{i+j+1}u_{(i,j)}(w, h) \geq 0$  for  $i = 1, 2, \dots, N$  and  $j = 1, 2, \dots, M$ .*

For  $N = M = 1$ , the condition degenerates into correlation aversion.

Finally, we consider the condition on the degree of relative mixed risk aversion. We now apply the lottery preference to  $u_{(0,m)}(w, h)$  for  $m = 1, 2, \dots, M$  to incorporate the effect of changes in health risks. Let us consider two lotteries that are multiplicative versions of the above lotteries:  $\mathcal{A}_{(N,M)} = [(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_B), h); (w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_G), h)]$  and  $\mathcal{B}_{(N,M)} = [(w(1+\tilde{\epsilon}_G)(1+\tilde{\delta}_G), h); (w(1+\tilde{\epsilon}_B)(1+\tilde{\delta}_B), h)]$ . Here, random variables  $\tilde{\epsilon}_G$  and  $\tilde{\epsilon}_B$  are defined over the same compact support  $[-1, \bar{\epsilon}]$ , and random variables  $\tilde{\delta}_G$  and  $\tilde{\delta}_B$  are defined over the same compact support  $[-1, \bar{\delta}]$ .  $\tilde{\epsilon}_G$  and  $\tilde{\epsilon}_B$  are ranked via NSD, and  $\tilde{\delta}_G$  and  $\tilde{\delta}_B$  are ranked via FSD. The four risks  $(\tilde{\epsilon}_G, \tilde{\epsilon}_B, \tilde{\delta}_G$  and  $\tilde{\delta}_B)$  in the lotteries  $\mathcal{A}_{(N,M)}$  and  $\mathcal{B}_{(N,M)}$  are mutually independent. The following characterizations can be viewed as an extension of Eeckhoudt et al. (2009) from a univariate setting with additive lotteries to a bivariate setting with



multiplicative lotteries.<sup>22</sup> The proof can be found in Appendix.

**Proposition 6.** *When individuals are mixed correlation averse, the following two conditions are equivalent:*

- (i) *The degree of relative mixed correlation aversion of  $(n, m)$  is less than  $n$  for  $n = 1, 2, \dots, N$  and  $m = 1, 2, \dots, M$ .*
- (ii) *Lottery  $\mathcal{B}_{(N, M)}$  is preferred to  $\mathcal{A}_{(N, M)}$ .*

## 8 Correlation seekingness

Up until now, we focused on the analysis of the correlation averse or mixed correlation averse individuals. This section provides the results for correlation seeking individuals parallel to those of correlation averse individuals. Therefore, we only present the results and omit their proofs and interpretations.

Note that correlation seeking individuals and cross imprudence individuals are captured by  $u_{(1,1)}(w, h) \geq 0$  and  $u_{(2,1)}(w, h) \geq 0$ , respectively. In this section, good and bad are ranked via FSD, that is,  $\tilde{x}_G$  dominates  $\tilde{x}_B$  in the sense of FSD, and  $\tilde{h}_G$  dominates  $\tilde{h}_B$  in the sense of FSD. To obtain the first two results, we use the following lemma, which is a symmetric version of Lemma 1.

**Lemma 2.** *Let consider  $\pi(w, h)$  in Lemma 1. Suppose that*

- *$\tilde{w}_G$  dominates  $\tilde{w}_B$  in the sense of FSD;*
- *$\tilde{h}_G$  dominates  $\tilde{h}_B$  in the sense of FSD.*

*The following two conditions are also equivalent:*

- (i)  *$\pi_{(1,1)}(w, h) \geq 0$ .*
- (ii)  *$E[\pi(\tilde{w}, \tilde{h})]$  is increasing in  $k$ .*

Applying Lemma 2, Proposition 7 is obtained by setting  $\pi(x, h) = u(w + \alpha x, h)$  and Proposition 8 is obtained by setting  $\pi(x, h) = xu(w + \alpha x, h)$ .

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<sup>22</sup>Wang and Li (2010) provide a higher-order generalization of Eeckhoudt et al. (2009a) following Eeckhoudt and Schlesinger (2006). They construct the lottery pairs using the combination of a sure reduction and independent risks.

**Proposition 7.** *The following two conditions are also equivalent:*

- (i) *Individuals are correlation seeking.*
- (ii) *An increase in correlation raises expected utility.*

**Proposition 8.** *Suppose that individuals are correlation seeking. Then,*

- (i) *the degree of partial cross prudence is less than unity if and only if an increase in correlation increases the investment in the risky asset, and*
- (ii) *if individuals are cross imprudent and the degree of relative cross prudence is less than unity, then an increase in correlation increases the investment in the risky asset.*

Note that the conditions in Proposition 2 are the same as those in Proposition 8. This is because the signs of  $u_{(1,1)}(w, h)$  differ between correlation aversion and correlation seekingness. Because the sign is reversed twice, comparing correlation aversion and correlation seekingness results in the same conditions appearing in Propositions 2 and 8.

Applying a similar argument in Section 5, we have the following proposition.

**Proposition 9.** *Suppose that the risky return and health risk are ranked by FSD. If  $u$  exhibits correlation seekingness and cross imprudence, and*

$$-\frac{wu_{(2,1)}(w, h)}{u_{(1,1)}(w, h)} \leq 1,$$

*then ambiguous correlation decreases the investment in the risky asset.*

The conditions in Proposition 8 can be related to the lottery preference.

**Result 3** (Eeckhoudt et al. (2007)). *Eeckhoudt et al. (2007) show the following:*

- (i) *Individuals are correlation seeking,  $u_{(1,1)}(w, h) \geq 0$ , if and only if they prefer  $A_{(1,1)}$  over  $B_{(1,1)}$ .*
- (ii) *Individuals are cross imprudent,  $u_{(2,1)}(w, h) \leq 0$ , if and only if they prefer  $A_{(2,1)}$  over  $B_{(2,1)}$ .*

The following proposition corresponds to Proposition 3.

**Proposition 10.** *When individuals are correlation seeking, the following conditions are equivalent:*

- (i) *The degree of relative cross prudence is less than unity.*
- (ii) *Lottery  $\mathcal{A}_{(2,1)}$  is preferred to  $\mathcal{B}_{(2,1)}$ .*

We note that the signs of  $u_{(1,1)}(w, h)$  differ between correlation aversion and seekingness, and that the first condition in Proposition 3 and 10 is the same, that is, the degree of relative cross prudence is less than unity.

In Proposition 8, there are three conditions for obtaining intuitive comparative static results:

- correlation seekingness, cross imprudence, and the degree of relative cross prudence being less than unity.

Combining Result 3 and Proposition 10, these conditions are related to mixed risk apportionment, which corresponds to the preference for combining good with good. In other words, the conditions in Proposition 8 are consistent with the preference for the combination of good and good.

As in FSD, we can obtain the corresponding results of higher-order stochastic dominance for mixed correlation seeking individuals. Thus, we omit these to avoid repetition.

## 9 Conclusion

This paper considered the portfolio choice problem with one risk-free asset and one risky asset in the presence of background risk. We investigated the effects of correlation on the optimal portfolio. While increases in correlation reduce (raise) expected utility for mixed correlation averse (seeking) individuals, contrary to intuition, they do not necessarily reduce (raise) the optimal portfolio. We determined the conditions for a reduction (rise) in the optimal portfolio that can be related to the preference for the combination of good and bad. Furthermore, we introduced ambiguity into correlation and investigated its effects on the optimal portfolio.

There are several directions for future research that extend this study. First, because the linear payoff problem has versatility, this study can be applied to specific problems, such as the demand for insurance and a firm's production under price risk, by incorporating characteristics specific to each problem. Second, it is interesting to consider health risk as controllable. In this study, health risk is treated as background risk. However, it may be controlled by introducing, for example, insurance and prevention activities. Third, experimental studies are worth investigating. Only a few studies have experimentally investigated higher-order risk preferences based on bivariate utility functions (e.g., Attema et al., 2019). These studies are context-free and use the framework of Eeckhoudt et al. (2007). However, to the best of our knowledge, there is no experimental investigation of these under specific contexts. In addition to our results, experimental studies would illuminate the importance of higher-order risk preferences and the effects of correlation in multidimensional frameworks.

## Appendix

*Proof of Lemma 1.* We show that (i) implies (ii). By straightforward calculation, we have

$$\begin{aligned} \operatorname{sgn} \left( \frac{\partial}{\partial k} E[\pi(\tilde{w}, \tilde{h})] \right) &\leq 0 \\ \Leftrightarrow E[\pi(\tilde{w}_G, \tilde{h}_G)] - E[\pi(\tilde{w}_G, \tilde{h}_B)] &\leq E[\pi(\tilde{w}_B, \tilde{h}_G)] - E[\pi(\tilde{w}_B, \tilde{h}_B)]. \end{aligned} \quad (10)$$

Because  $\tilde{w}_G$  dominates  $\tilde{w}_B$  in the sense of FSD, (10) holds if  $E[\pi(w, \tilde{h}_G)] - E[\pi(w, \tilde{h}_B)]$  decreases in  $w$ , which is equivalent to  $E[\pi_{(1,0)}(w, \tilde{h}_G)] \leq E[\pi_{(1,0)}(w, \tilde{h}_B)]$ . Again applying the property of FSD, we have

$$E[\pi_{(1,0)}(w, \tilde{h}_G)] \leq E[\pi_{(1,0)}(w, \tilde{h}_B)] \Leftrightarrow \pi_{(1,1)}(w, h) \leq 0.$$

We conclude the proof that (i) implies (ii).

Next, we show by contradiction that (ii) implies (i). Assume that  $\pi_{(1,1)}(w^o, h^o) > 0$  for the neighborhood of  $w^o$  and  $h^o$ . Let us consider that  $\tilde{w}_G$  and  $\tilde{w}_B$  that are identical except for the neighborhood of  $w^o$  and  $h^o$ . In this case, we have that  $E[\pi(\tilde{w}, \tilde{h})]$  increases in  $k$ . Because this is a contradiction, we complete the proof.  $\square$

*Proof of Proposition 3.* We only provide a proof that (ii) implies (i). The opposite can be easily proven by contradiction, and it can be omitted.

Because we assume that  $\mathcal{B}_{(2,1)} \succeq \mathcal{A}_{(2,1)}$ , we have

$$\begin{aligned} \frac{1}{2}u_{(0,1)}(w(1-r), h) + \frac{1}{2}u_{(0,1)}(w(1-k), h) &\geq \frac{1}{2}u_{(0,1)}(w, h) + \frac{1}{2}u_{(0,1)}(w(1-r)(1-k), h) \\ \Leftrightarrow u_{(0,1)}(w(1-k), h) - u_{(0,1)}(w, h) &\geq u_{(0,1)}(w(1-r)(1-k), h) - u_{(0,1)}(w(1-r), h). \end{aligned} \quad (11)$$

From (11), it holds that  $u_{(0,1)}(w(1-r)(1-k), h) - u_{(0,1)}(w(1-r), h)$  decreases

in  $r$ , that is,

$$\begin{aligned} & \frac{\partial}{\partial r} u_{(0,1)}(w(1-r)(1-k), h) - u_{(0,1)}(w(1-r), h) \leq 0 \\ \Leftrightarrow & (1-k)u_{(1,1)}(w(1-k)(1-r), h) \geq u_{(1,1)}(w(1-r), h). \end{aligned} \quad (12)$$

From (12), it holds that  $u_{(1,1)}(w(1-r)(1-k), h)$  increases in  $k$ , that is,

$$\begin{aligned} & \frac{\partial}{\partial k} u_{(1,1)}(w(1-r)(1-k), h) \geq 0 \\ \Leftrightarrow & -u_{(1,1)}(w(1-k)(1-r), h) - w(1-k)(1-r)u_{(2,1)}(w(1-k)(1-r), h) \geq 0 \\ \Leftrightarrow & -\frac{w(1-k)(1-r)u_{(2,1)}(w(1-k)(1-r), h)}{u_{(1,1)}(w(1-k)(1-r), h)} \leq 1. \end{aligned} \quad (13)$$

This completes the proof. The proof is almost the same as Eeckhoudt et al. (2009a), but the inequality is reversed in the last step of (13) because of  $u_{(1,1)} \leq 0$  by correlation aversion.  $\square$

To show Proposition 5, we need the following lemma, which is a generalization of Lemma 1.

**Lemma 3.** *Suppose that*

- $\tilde{w}_G$  dominates  $\tilde{w}_B$  in the sense of NSD;
- $\tilde{h}_G$  dominates  $\tilde{h}_B$  in the sense of MSD.

*The following two conditions are equivalent:*

- (i)  $(-1)^{n+m}\pi_{(1,1)}(w, h) \leq 0$ .
- (ii)  $E[\pi(\tilde{w}, \tilde{h})]$  is decreasing in  $k$ .

*Proof of Lemma 3.* To show that (i) implies (ii), as in the proof of Lemma 1, we need to prove that

$$E[\pi(\tilde{w}_G, \tilde{h}_G)] - E[\pi(\tilde{w}_G, \tilde{h}_B)] \leq E[\pi(\tilde{w}_B, \tilde{h}_G)] - E[\pi(\tilde{w}_B, \tilde{h}_B)]. \quad (14)$$

Because  $\tilde{w}_G$  dominates  $\tilde{w}_B$  in the sense of NSD, (14) holds if

$$\begin{aligned} & (-1)^{n+1} \frac{\partial^n}{\partial w^n} \left\{ E[\pi(w, \tilde{h}_G)] - E[\pi(w, \tilde{h}_B)] \right\} \leq 0 \\ \Leftrightarrow & (-1)^{n+1} E[\pi_{(n,0)}(w, \tilde{h}_G)] \leq (-1)^{n+1} E[\pi_{(n,0)}(w, \tilde{h}_B)]. \end{aligned}$$

Applying the property of MSD, we have

$$\begin{aligned} & (-1)^{m+1} \frac{\partial^m}{\partial h^m} \left\{ (-1)^{n+1} \pi_{(n,0)}(w, h) \right\} \leq 0 \\ \Leftrightarrow & (-1)^{(n+m)} \pi_{(n,m)}(w, h) \leq 0. \end{aligned}$$

Similar to the proof of Lemma 1, we can show that (ii) implies (i).  $\square$

*Proof of Proposition 5.* Applying Lemma 3 to  $\pi(x, h) = xu_{(1,0)}(w + \alpha x, h)$ , we can obtain the condition. There are two cases where  $n + m$  is an even or odd number. Note that  $\pi_{(n,m)}(w, h) = n\alpha^{n-1}u_{(n,m)}(w + \alpha x, h) + \alpha^n xu_{(n+1,m)}(w + \alpha x, h)$ . For the case of  $n + m$  being even numbers, we have the following:

$$\begin{aligned} & \pi_{(n,m)}(x, h) \leq 0 \\ \Leftrightarrow & n\alpha^{n-1}u_{(n,m)}(w + \alpha x, h) + \alpha^n xu_{(n+1,m)}(w + \alpha x, h) \leq 0 \\ \Leftrightarrow & nu_{(n,m)}(w + \alpha x, h) + \alpha xu_{(n+1,m)}(w + \alpha x, h) \leq 0 \\ \Leftrightarrow & -\frac{\alpha xu_{(n+1,m)}(w + \alpha x, h)}{u_{(n,m)}(w + \alpha x, h)} \leq n, \end{aligned}$$

where the last equivalence holds because  $(-1)^{n+m+1}u_{(n,m)}$  is positive by mixed correlation aversion, and thus  $u_{(n,m)}$  is negative in the case of  $n+m$  being even numbers.

Because  $u_{(n+1,m)}(w, h) \geq 0$  for  $n + m$  being an even number, we have

$$-\frac{\alpha xu_{(n+1,m)}(w + \alpha x, h)}{u_{(n,m)}(w + \alpha x, h)} \leq -\frac{(w + \alpha x)u_{(n+1,m)}(w + \alpha x, h)}{u_{(n,m)}(w + \alpha x, h)}.$$

Thus, if the right-hand-side of the inequality is less than  $n$ , that is, the degree of relative mixed correlation aversion in  $(n, m)$  is less than  $n$ , then the degree of partial

mixed correlation aversion in  $(n, m)$  is less than  $n$ . Thus, an increase in correlation reduces the investment in the risky asset. We complete the proof for the case where  $n + m$  is an even number. For the case of  $(n + m)$  being odd numbers, we can apply a similar argument.  $\square$

*Proof of Proposition 6.* As in the proof of Proposition 3, we only provide a proof that (ii) implies (i). Because  $\mathcal{B}_{(n,m)} \succeq \mathcal{A}_{(n,m)}$ , it follows that

$$\begin{aligned}
& \frac{1}{2}E[u_{(0,m)}(w(1 + \tilde{\epsilon}_G)(1 + \tilde{\delta}_B), h)] + \frac{1}{2}E[u_{(0,m)}(w(1 + \tilde{\epsilon}_B)(1 + \tilde{\delta}_G), h)] \\
& \geq \frac{1}{2}E[u_{(0,m)}(w(1 + \tilde{\epsilon}_G)(1 + \tilde{\delta}_G), h)] + \frac{1}{2}E[u_{(0,m)}(w(1 + \tilde{\epsilon}_B)(1 + \tilde{\delta}_B), h)] \\
\Leftrightarrow & E[u_{(0,m)}(w(1 + \tilde{\epsilon}_G)(1 + \tilde{\delta}_B), h)] - E[u_{(0,m)}(w(1 + \tilde{\epsilon}_G)(1 + \tilde{\delta}_G), h)] \\
& \geq E[u_{(0,m)}(w(1 + \tilde{\epsilon}_B)(1 + \tilde{\delta}_B), h)] - E[u_{(0,m)}(w(1 + \tilde{\epsilon}_B)(1 + \tilde{\delta}_G), h)]. \quad (15)
\end{aligned}$$

By NSD, (15) holds when

$$\begin{aligned}
& \frac{\partial^n}{\partial \epsilon^n} \{E[u_{(0,m)}(w(1 + \epsilon)(1 + \tilde{\delta}_B), h)] - E[u_{(0,m)}(w(1 + \epsilon)(1 + \tilde{\delta}_G), h)]\} \geq 0 \\
\Leftrightarrow & E[w^n(1 + \tilde{\delta}_G)^n u_{(n,m)}(w(1 + \epsilon)(1 + \tilde{\delta}_G), h)] \geq E[w^n(1 + \tilde{\delta}_B)^n u_{(n,m)}(w(1 + \epsilon)(1 + \tilde{\delta}_B), h)]. \quad (16)
\end{aligned}$$

By FSD, (16) is equivalent to the following:

$$\begin{aligned}
& \frac{\partial}{\partial \delta} \{w^n(1 + \delta)^n u_{(n,m)}(w(1 + \epsilon)(1 + \delta), h)\} \geq 0 \\
\Leftrightarrow & nw^n(1 + \delta^{n-1})u_{(n,m)}(w(1 + \epsilon)(1 + \delta), h) - w^{n+1}(1 + \delta^n)(1 + \epsilon)u_{(n+1,m)}(w(1 + \epsilon)(1 + \delta), h) \geq 0 \quad (17)
\end{aligned}$$

(17) is equivalent to

$$-\frac{w(1 + \epsilon)(1 + \delta)u_{(n+1,m)}(w(1 + \epsilon)(1 + \delta), h)}{u_{(n,m)}(w(1 + \epsilon)(1 + \delta), h)} \leq n.$$

This completes the proof.  $\square$



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