Tort Law and the Nucleolus for Liability Problems with Rooted-tree Strcture

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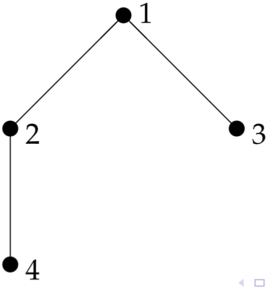
Liability problems

In this paper we consider situations in which an injured party suffers damages caused by wrongful acts performed subsequently by a sequence of injuring parties.

- d_i : the direct damage caused by agent i. (measured by money)
- How to apportion the total damage $d_1 + d_2 + d_3 + d_4$.



The background of our research

Historical background

- Common law
- The *Restatement of Torts* (Second, Third)

Related Literature (in Law and Economics/Game Theory)

- It is a central topic to clarify whether or not a legal compensation scheme for liability problems is useful.
- Landes and Posner (1980), Shavell (1983), and Parisi and Singh (2010): **Incentive matters**
- **Tort law** prescribes an award of damages to achieve **fair** compensation for injury, see Boston (1995-1996).
- Dehez and Ferey (2013), Fery and Dehez (2016):
- **Normative matters**
- **This topic needs further investigation.**

The purpose and the main result

The purpose is to analyze compensation schemes axiomatically under the situation described by a **rooted-tree graph**.

We show that there is a unique compensation scheme satisfying

- **lower bounds** *of individual compensations*,
- **upper bounds** *of individual compensations*, and
- **consistency associated with causal relation**.

This unique compensation scheme yields the **nucleolus** (Schmeidler 1969).

- The nucleolus is an important solution for coalitional games.
- The 'difference principle of social justice' à la Rawls (1971).

Our contribution

We propose an axiomatization of the **nucleolus for liability problems with rooted-tree structure**.

Besides the axioms, the nucleolus compensation scheme has appealing properties.

- It is likely that the injured party can receive compensation as soon as possible without facing with injuring parties' final appeal to the court.
- We can see the nucleolus as a prominent solution for a more modern legal rule than the Talmud investigated by Aumann and Maschler (1985).

TU games

- Let N be a finite set of agents.
- A TU game for N is a function $v\colon 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$.
- A TU game v for N is **convex** (Shapley, 1971) if for all $i \in N$ and all $S, T \subseteq N \setminus \{i\}$, $S \subseteq T$ implies

$$v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T).$$

- It is **concave** if these inequalities are reversed.
- Let $\mathcal{G}$ be the class of all TU games.
- A game, denoted v^d, is the **dual** of $v \in \mathcal{G}$ if for all $S \subseteq N$ $v^d(S) = v(N) - v(N \backslash S)$.

TU games (cont.)

- A **payoff vector** for game v for N is an element x of $\mathbb{R}^N$.
- A payoff vector x is **efficient** if $\sum_{i \in N} x_i = v(N)$.
- Given $(N, v) \in \mathcal{G}$, let $X(N, v)$ be the set of efficient payoff vectors.
- The **imputation set**, denoted $I(N, v)$, is the subset of payoff vectors in $X(N, v)$ that satisfy for every $i \in N$

$$x_i \geq v(\{i\}) \text{ (\textbf{individual rationality})}$$

- The **anti-imputation set**, denoted by $AI(N, v)$, is the subset of all vectors in $X(N, v)$ that satisfy for every $i \in N$

$$x_i \leq v(\{i\}) \text{ (\textbf{anti-individual rationality})}$$

- For a given subset $\mathcal{G}'$ of the class $\mathcal{G}$ of all TU-games, a **(single-valued) solution** is a function f that assigns to every game (N, v) in $\mathcal{G}'$ a payoff vector $f(N, v) \in X(N, v)$.

The nucleolus

- Given $(N, v) \in \mathcal{G}$, for all $x \in I(N, v) \neq \emptyset$ and $S \subseteq N$, the excess of S w.r.t. x is defined as

$$e(S, x; v) \equiv v(S) - \sum_{i \in S} x_i.$$

- Given $(N, v) \in \mathcal{G}$, let $\theta(x) \in \mathbb{R}^{2^N}$ be the vector obtaining arranging all the excesses in non-increasing order.
- For all $\theta(x) \in \mathbb{R}^{2^N}$, $\theta(x)$ is lexicographically smaller than $\theta(x')$ if $\theta_1(x) < \theta_1(x')$ or $[\theta_1(x) = \theta_1(x')$ and $\theta_2(x) < \theta_2(x')]$ or $[\theta_1(x) = \theta_1(x')$ and $\theta_2(x) = \theta_2(x')$ and $\theta_3(x) < \theta_3(x')]$, and so on.
- The **nucleolus** is defined as follows:

$$Nu(N, v) \equiv \left\{ x \in I(N, v) \;\middle|\; \begin{array}{c} \text{For all } y \in I(N, v) \backslash \{x\},\ \theta(x) \text{ is} \\ \text{lexicographically smaller than } \theta(y). \end{array} \right\}.$$

- The nucleolus is a *single-valued* solution.

The anti-nucleolus

- The **anti-nucleolus** is defined on the class of games with $AI(N,v) \neq \emptyset$.

$$ANu(N,v) \equiv \left\{ x \in AI(N,v) \;\middle|\; \begin{array}{l} \text{For all } y \in AI(N,v)\backslash\{x\}, -\theta(x) \text{ is} \\ \text{lexicographically smaller than } -\theta(y) \end{array} \right\}$$

- The anti-nucleolus is the nucleolus for cost games.

Additional damage

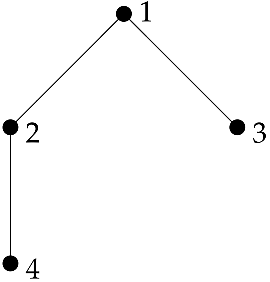


Figure:

Additional damage by i, denoted e_i : It is the sum of damages that would have been avoided if he did not exercise a wrongful act, e.g., $e_2 = d_2 + d_4$.

Additional damage by coalitions

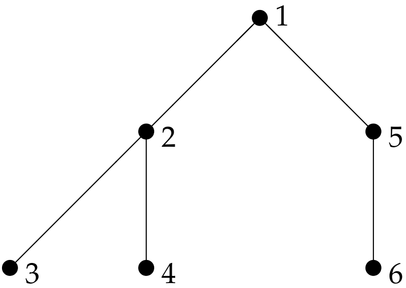


Figure:

Additional damage by S, denoted e_S : For $S = \{2, 6\}$,

$$e_S = d_2 + d_3 + d_4 + d_6.$$

Potential damage

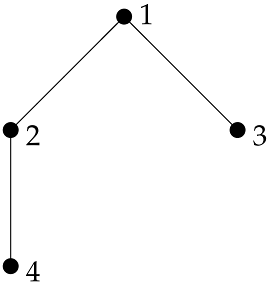


Figure:

Potential damage by i, denoted b_i : It is the sum of damages that agent i causes when the other agents do not behave wrongfully: $b_1 = d_1$; $b_i = 0$ for all $i \neq 1$.

Potentail damage by coalitions

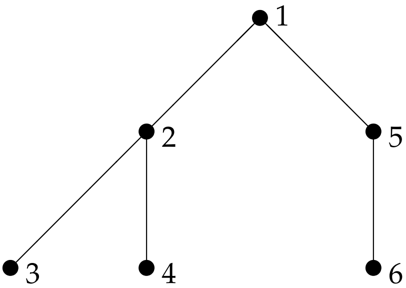


Figure:

Potential damage by S**, denoted** b_S : For $S = \{1, 2, 3, 6\}$,

$$b_S = d_1 + d_2 + d_3.$$

Liability problems, and their associated games

A **liability problem with rooted-tree structure** (shortly a **liability problem**) is a triple (N, T, d), where (N, T) is a **digraph**, and $d \in \mathbb{R}_+^N$ a **profile of direct damages**.

- We denote the **class of all liability problems** by $\mathcal{L}$.

We introduce two coalitional TU games:

- The **lower-bound liability game** assigns to every subset S of tortfeasors a worth $v_L(S)$ that is equal to the **potential damage** of S. This game is **convex**.
- The **upper-bound liability game** assigns to every subset S of tortfeasors a worth $v_U(S)$ that is equal to the **additional damage** of S. This game is **concave**.
- $v_L(S)$ and $v_U(S)$ are **dual** to each other.

The compensation scheme

Given $(N, T, d) \in \mathcal{L}$, an **allocation** for (N, T, d) is a non-negative vector $x \in \mathbb{R}^N_+$ such that $\sum_{i \in N} x_i = \sum_{j \in N} d_j$.

Definition

A **compensation scheme** for liability problems is a mapping φ on $\mathcal{L}$ that associates with every problem $(N, T, d) \in \mathcal{L}$ an allocation $\varphi(N, T, d) \in \mathbb{R}^N_+$.

The nucleolus compensation scheme

Definition

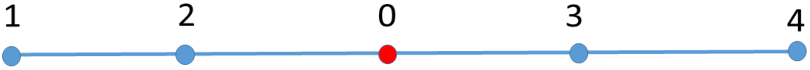
The **nucleolus compensation scheme** is the mapping Nuc on $\mathcal{L}$ that associates with every problem $(N, T, d) \in \mathcal{L}$ the nucleolus of its corresponding lower-bound liability game (N, v_L):

$$Nuc(N, T, d) \equiv Nuc(N, v_L).$$

Notice that $Nuc(N, v_L) = ANuc(N, v_U)$ (by duality).

- With respect to the additional damages of the coalitions, the smallest cost saving over all coalitions is made as large as possible, then the second smallest is made as large as possible, then the third smallest is made as large as possible, and so on.

An illustration of a multiple pile-up

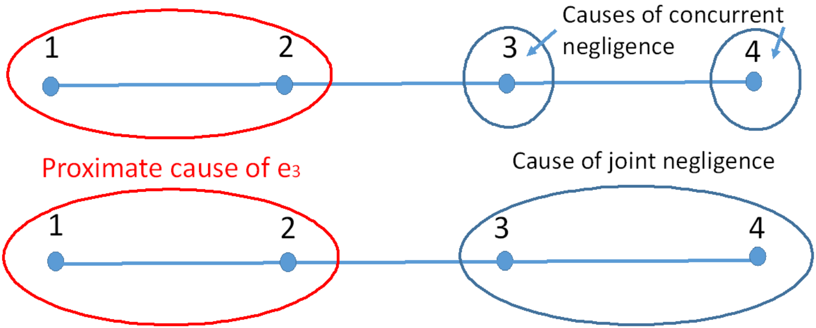


- A multiple pile-up among five drivers including a plaintiff (driver 0).
- A pile-up between derivers 1 and 2 occurred.
- As a result of her emergency stop, driver 0 suffered **mild whiplash**.
- Another pile-up between drivers 3 and 4 occurred. This latter crash moved the car of driver 3 into driver 0's car.
- As a result, driver 0 suffer **severe whiplash**. The other four drivers remain uninjured.

Legal notion of causes

- *Proximate cause* is a cause that directly produces an event and without which the event would not have occurred.
  - Proximate cause of 3's additional damage consists of drivers 1 and 2.
  - Because courts often recognize a requirement of proving proximate cause by a plaintiff, as stated in the syllabus of the Kansas Supreme Court for the case of *Hale v. Brown.*
- *Concurrent negligence* is negligence of two or more injuring parties acting independently but causing the same damage.
  - 3 and 4 do not know each other, and causes of their wrongful acts are independent.
- *Joint negligence* is negligence of two or more injuring parties acting together to cause an accident.
  - 3 and 4 are friends. Before the multiple pile-up, they enjoyed beer at a pub. Their drinking affects their wrongful acts crucially.

Legal notion of causes in the pile-up



- The proximate cause of e_3 (i.e. the additional damage by 3) : $\{1,2\}$
- Cause of "concurrent negligence" of e_3: $\{\{3\}\{4\}\}$
- Cause of "joint negligence" of e_3: $\{3,4\}$

Per capita criterion

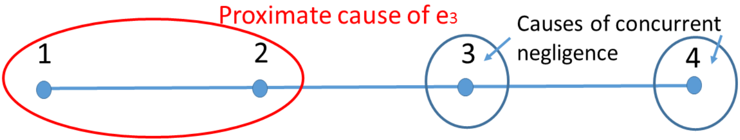
- We assume that the causal weights among the associated causes cannot be determined.
  - In several cases in practice, transaction cost for determination of causal weights may be very high.
- We adopt a 'per-capita criterion'.
- The criterion requires that the additional damage e_i should be divided *equally* among the corresponding *proximate cause* and the corresponding *causes of negligence*.
- The stylized fact
  - Courts in the USA applied the divided-damage rule for joint liability problems in Maritime Law
  - Courts in Japan also employed the notion of per-capita criterion before the 1990's.

Legal motivation of upper/lower bounds

- Every injuring party should pay at least his potential damage (Peaslee 1934).
    - Utilizing potential damage may make no sense in practice.
- Instead, we consider that every injuring party should pay at least the smallest per-capita contribution associated with additional damage.

- Every injuring party should pay at most his additional damage (Restatement of Torts, Third).
- Instead, we consider that every injuring party should pay at most the largest per-capita contribution associated with additional damage.

Case system: An illustration



- Case system (A legal system in the UK and USA)
- Given φ, imagine 3 and 4 pay their payments and leave.
  - Either drivers 1, 2, and 3 or drivers 1, 2 and 4 have no causal relation associated with the matter of changing driver 0's mild whiplash into a severe one.
- The compensation of drivers who make the proximate cause should be invariant.

Axiomatization of the nucleolus compensation scheme

Theorem

A compensation scheme φ on the class $\mathcal{L}$ of liability problems satisfies uniform lower bound, individual upper bounds, and causal consistency if and only if

$$\varphi(N, T, d) = Nuc(N, T, d).$$

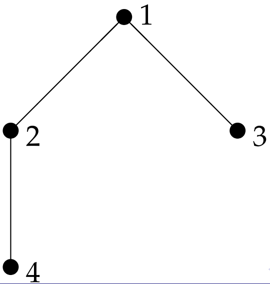
Each axiom is derived from legal observations on tort law

- **Uniform lower bound** ← Keys: **per-capita criterion** / **concurrent negligence**
- **Individual upper bounds** ← Keys: **per-capita criterion** / **joint negligence**
- **Causal consistency** ← Key: **Case system**

Uniform lower bound: an illustration

$$\forall i \in N,\ \varphi_i(N,T,d) \geq \min\left\{\frac{e_4}{|\{1,2\},\{4\}|}, \frac{e_3}{|\{1\},\{3\}|}, \frac{e_2}{|\{1\},\{2\},\{4\}|}, \frac{e_1}{4}\right\},$$

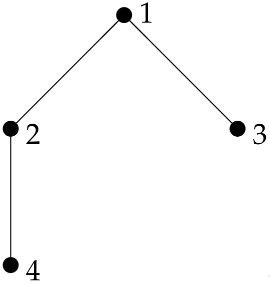
where the denominator for e_i is given by the number of its proximate cause and its causes of purely concurrent negligence.



Individual upper bounds: an illustration

$$\varphi_4 \leq \frac{e_4}{|\{1,2\},\{4\}|}, \quad \varphi_3 \leq \frac{e_3}{|\{1\},\{3\}|}, \quad \varphi_2 \leq \frac{e_2}{|\{1\},\{2,4\}|}, \quad \varphi_1 \leq e_1,$$

where the denominator for e_i is given by the number of its proximate cause and its cause of purely joint negligence.



Consistency

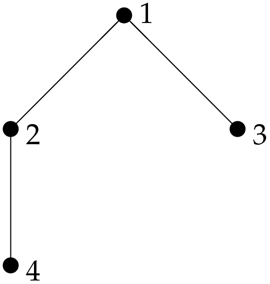
Causal consistency requires that the outcome of agents who make proximate cause should be invariant under the removal of causes of concurrent/joint negligence of any additional damage.

- Causal consistency is equivalent to **leaf consistency**.
- For simplicity, my talk will deal with leaf consistency.

Leaf consistency: the case of leaf 4: an illustration

Let $d'_1 = d_1,\ d'_2 = d_2 + d_4 - \varphi_4$ (by **last clear chance**) and $d'_3 = d_3$.
Let $N' = \{1, 2, 3\}$, and $T(N') = \{(12), (13)\}$.

$$\forall i \in N',\ \varphi_i(N', T(N'), d') = \varphi_i(N, T, d).$$



Appendix: Nucleolus vs Shapley value

On the domain of liability games, we can compare the **Shapley value** (Katsev 2009) and the nucleolus by their set of axioms.

Properties	Nu	Sh
Uniform Lower Bound	$+^*$	$-$
Individual Upper Bound	$+^*$	$+$
Leaf (Causal) Consistency	$+^*$	$+^*$
Weak veto property	$+$	$+^*$
Top monotonicity	$+$	$+^*$
Independence of non-subordinates	$-$	$+^*$

$+^*$: used as an axiom $+$: satisfied $-$: not satisfied

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