Not just for the money, but...: the signaling role of wage offer in a shared-value project

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Abstract

This paper studies a workplace in which the worker shares the value of the project for the beneficiary and finds it rewarding if his work is helpful. The worker is not sure how much his work is helpful for the beneficiary, and we investigate how the beneficiary uses wage offer as a signaling device to convince or deceive that the project is rewarding for the worker.

In the simplest model, we show that equilibrium wage offer does not reveal the value, and there is a range of values in which the worker is manipulated to believe that the project is in expectation worth doing despite it is not. When the worker's effort is endogenous, we show that the unrevealing equilibrium still persists while there is a partially revealing equilibrium as well. Finally, when the worker has it as his private information how much he shares the value for the beneficiary, we show that this pressure leads the equilibrium wage offer to fully reveal the value.

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1 Introduction

The idea that work is not just for money seems prevalent, not only in the world of enthusiasts, but also in usual-life professions such as education, health, childcare, social welfare and even general service provisions. Google "not just for (the) money" or "not for (the) money," then you can find tons of manifestos using these phrases, while it is also the title of the well-known book by Frey [9]. Whether it is normatively desirable or not, once people do have such faith, or follow a norm based on such faith, or resisting such norm is difficult, it will necessarily affect economic performance of workplaces.

There may be various reasons for why people do have or apparantly accept such faith. The simplest one will be that work itself is a joy, regardless of its consequence. Another one will be that work realizes the worker's own value, being apart from the beneficiary's one. Or, it might be because work satisfies the worker's approval desire, or desire for a status. An economically more non-trivial story will be a dynamic one such as workers have career/reputation concerns over time.

In this paper, we look into a different aspect of the idea that work is not just for money: work is *rewarding*, in the sense that the worker finds the work worthwhile because he shares the beneficiary's value and wants to be helpful in achieving this value. In Japan, *yarigai* (rewardingness) is a common word for explaining why people believe that work is not just for money, while it is also pointed out that this word is being utilized for "brainwashing" or "mind-controlling" workers to engage themselves in underpaid/unpaid works. So-called *yarigai sakusyu* (exploitation of rewardingness), which became a prevalent term after Honda [13], refers to the practice that an employer keeps workers overworking and underpaid/unpaid by continuously motivating them that the work is rewarding, and that the workers find it hard to resist as they are manipulated to take it as the way for their "self-actualization." See Brasor [3] for an illustration of the discourses around *yarigai* in Japan. Not only in Japan. Google "work rewarding" then you will find that rewarding is a prevalent buzz word related to workplaces in English-speaking countries as well.

What makes rewardingness distinct from joy of working or satisfaction of worker's own independent value is that how much a work is rewarding commensurates with the value it brings to the beneficiary, and the worker cannot feel being rewarded when he knows he is just working in vain. This will be typically the case in in education, health, childcare and social welfare. Thus we would call such project a *shared-value project.*

We present a simple model in which the worker shares a proportion of the value being held by the beneficiary, while we could think of a more general interdependent-value setting. We take the shared-value nature of a project as a given element and do not question where it is coming from. It might be due to pure empathy, or it might be due to some sociological aspects, or again it might be due to rather dynamic economic aspects such as career concerns. Instead we focus on the implication of the shared-value nature to economic performance of workplaces.

But what's wrong with that, from the efficency viewpoint, if a worker is just happy to take a job for lower pay because he shares the beneficiary's value, although there would obviously be some problem with fairness? It becomes a problem about efficiency indeed, when the value of a project is private information of the beneficiary. *Highly motivated workers are not simply workers whose labour cost is cheaper,* while this equivalence is presumed in many studies of intrinsic motivation.

When the worker shares the beneficiary's value, the beneficiary will try to take advantage of it. Thus, *if* there is complete information between them, the beneficiary's wage offer is *decreasing* in the value. Such wage offer is supposed to convey the signal saying,

> "My offering lower wage means the work is more rewarding (so you should be happy to take it)."

However, as the value is private information of the beneficiary, he will never truthfully commit to such wage policy, because *he would say the same thing also when he is simply unwilling to pay.* We would say this is the key deception trick in *yarigai sakusyu.*

When the agents are rational and fully aware of this deception trick, it will no longer work as it is. Then where do we reach? Under incomplete information, the beneficiary, if he wants to implement the project, needs to *convince* the worker that the project is rewarding. This is the signalling role of wage offer here. On the othis hand, since the beneficiary tries to minimize the pay conditional on a given outcome, there is a natural trade-off between the two roles.

In the simplest model in which the worker's effort is exogenous, we show that equilibrium cannot be revealing. The equilibrium wage offer must be constant in value as far as the worker's participation constraint is met. Here the wage offer is equal to the minimal necessary value such that the worker is indiffernt between taking the offer knowing only that the true value is greater than that and declining that. Such threshold is lower than the value at which ex-post social surplus (including the worker's shared value) is zero, hence there is a range of values in which the project is undertaken, *despite it should not be* from the efficiency viewpoint.

When the worker's effort level is endogenous, there will be more incentive for the beneficiary to offer higher wage in order to convince the worker that the project is worth spending more effort. We show that there are two kinds of equilibria, one in which the beneficiary still refrains from revealing the value, the other in which the value is partially revealed when it is sufficiently high.

Finally, we consider that the degree of how much the worker shares the beneficiary's value is private information of the worker. This creates a risk for the beneficiary that lowering wage offer leads to lose the worker, and pressures the beneficiary to raise the wage offer, which is the case more when the value is larger. We show that such pressure leads the beneficiary to fully reveal the value.

Related literature

The importance of intrinsic motivation as compared to the extrinsic one has been widely recognized in psychology (see Deci and Ryan [8], Ryan and Deci [17] for overviews, among many). In the last two decades, intrinsic motivation and other non-pecuniary motivations for work have been widely studied in economics as well (see Frey [9], Robitzer and Taylor [16] for overviews). Although, what is rather more important for us here is how this psychological knowledge, either formal or informal, is perceived in the society.

One important issue here is the relationship between intrinsic and extrinsic motivation, as numbers of studies in experimental social psychology report that increasing extrinsic motivation crowds out the intrinsic one (see Deci [6], Pritchard, Campbell and Campbell [15], among many, and also Deci, Koestner and Ryan [7] for meta analysis). It is also confirmed in the experimental economics literature (see for example Gneezy and Rustichini [11], Frey and Jegen [10]), and a number studies attempt to give an explanation of it from the economic theory viewpoint (see for example Benabou and Tirole [2], Brekke and Nyborg [4]).

When intrinsic motivation is understood as rewardingness, our model predicts that intrinsic and extrinsic motivation cannot be negatively correlated (not positively either in the unrevealing equilibrium, though), because if they are it creates an incentive for the beneficiary to overstate his value by making a low wage offer.

Our result apparently contradicts to the confirmed negative relationship. We would say, however, that it is a story at a different level once people get aware that the negative relationship between intrinsic and extrinsic motivation is being utilized as an excuse for not paying, which is indeed the recent trend with *yarigai sakusyu*, while the psychologically and experimentally confirmed relationship is at an individual level in controlled environments. Our result will be understood as a prediction of where we reach if we are rational once we get fully aware of the deception trick.

Another significant issue is a screening problem when the employer cannot observe how much the worker is intrinsically motivated, which arises even when the combination of intrinsic and extrinsic motivation are fixed.

Heyes [12] argues that lowering wage offer helps to select highly motivated workers who are happy to work for lower wage than unmotivated workers, improves efficency, and concludes "a badly paid nurse (is) a good nurse." See also Nelson and Folbre [14] for criticisms to it. Delfgaauw and Dur [5] consider that a worker having intrinsic motivation to the work as his own private information can choose the effort level which is unobservable for the employer, while Heyes [12] assumes an exogenous correlation between motivation and quality of service. They find that there is a trade-off that raising the wage offer increases the probability of acceptance by allowing unmotivated workers to join but lowers the probability of success conditional on the acceptance.

We would say that the these arguments depend critically on the assumption that how much the work is worthwhile is known to the worker, because otherwise an employer who is simply unwilling to pay could also say that the work is worthwhile for the worker and offer lower wage. Also, what matters for the beneficiary overall is the probability of success prior to acceptance/rejection, not the conditional one, and in our model it is shown to be increasing in wage offer.

2 The simplest model

First we consider the simplest model in which only the value for the beneficiary is private information, and the worker's effort level is exogenous and the degree of how much the worker shares the beneficiary's value is known.

2.1 Setting

There is a project which benefits a beneficiary if it is accomplished by a worker. The beneficiary is informed of the value of the project, denoted θ. The value θ is drawn from $[0, \overline{\theta}]$ according to distribution Φ with density ϕ, where $\overline{\theta}$ can be infinite. Let μ denote the mean of distribution Φ. For the worker, the shared value of the project is $\alpha\theta$, where $\alpha > 0$ is a fixed parameter of degree of value sharing which is known to both parties, but the worker is uninformed of θ. The service requires the worker to pay cost c, which is known to both parties here. The worker has an outside option and its value is given by $\underline{u}$, which is known to both parties. Hence the only private information is θ in this simplest model.

Both parties are assumed to be risk-neutral and the project is assumed to obey no income effect. Denote the wage payment by w. Then the beneficiary's ex-post utility is

$$\theta - w$$

when the worker signs and provides the service, and 0 othiswise. The worker's ex-post utility is

$$\alpha\theta + w - c$$

when he signs and provides the service, and $\underline{u}$ otherwise.

We assume

$$(1+\alpha)\overline{\theta} - c \geq \underline{u},$$

otherwise the project should never be accomplihed.

Consider the following two-stage game.

Stage 1: The beneficiary offers wage w.

Stage 2: The worker accepts or reject the offer. If rejected, the outside option is obtained.

We adopt perfect Bayesian equilibrium as the equilibrium concept.

2.2 The complete information benchmark

Efficiency says that the project should be undertaken if and only if

$$(1+\alpha)\theta - c \geq \underline{u},$$

that is, when

$$\theta \geq \frac{\underline{u} + c}{1 + \alpha}.$$

For any θ satisfying the above condition, in order to implement the project it is optimal for the beneficiary to offer wage $w(\theta)$ so that the participation constraint for the worker is met with equality

$$\alpha\theta + w(\theta) - c = \underline{u},$$

that is,

$$w(\theta) = \underline{u} + c - \alpha\theta.$$

Then the beneficiary extracts the full net surplus $(1 + \alpha)\theta - c - \underline{u}$. Note that $w(\theta)$ is *decreasing* in θ, because when the value of the project is highis the worker is more willing to give up wage payment.

2.3 The incomplete information case

Impossibility of implementing the complete information solution

The complete information wage policy

$$w(\theta) = \underline{u} + c - \alpha\theta$$

cannot be implemented under incomplete information, because there a low-θ beneficiary can always mimic to be a high-θ beneficiary by offering lower wage. Here the fully revealing wage schedule is supposed to convey a message saying "my offering lower wage means the work is more rewarding (so you should be happy to take it)," but a beneficiary who is simply unwilling to pay can say the same thing. Hence the beneficiary cannot commit to the above wage policy under incomplete information, and full-revelation with full surplus extraction must be abondoned when agents are rational.

In order that a rational worker is not deceived in equilibrium like above, the function $w(\cdot)$ must be non-decreasing. It cannot be in equilibrium that $w(\cdot)$ has a strictly increasing component in which the worker accepts the offer, however, bacause it is simply a waste of money for the beneficiary. The story might change when the worker can choose effort level for realizing the value, and we will come to this issue in a later section.

Let us be a little more formal. Let $\mu(w)$ be the expected value of θ conditional on wage offer w, which is believed by the worker and is to be determined in equilibrium. Then the worker accepts the offer when

$$\alpha\mu(w) + w - c \geq \underline{u}.$$

Let $w(\theta)$ denote the wage offer by the beneficiary with type θ.

Proposition 1 In any perfect Bayesian equilibrium there extist no θ and θ' with $\theta < \theta'$ such that

$$\alpha\mu(w(\theta)) + w(\theta) - c, \alpha\mu(w(\theta')) + w(\theta') - c \geq \underline{u}$$

and

$$w(\theta) \neq w(\theta')$$

Proof. Suppose θ and θ' with $\theta < \theta'$ are such that $\alpha\mu(w(\theta)) + w(\theta) - c, \alpha\mu(w(\theta')) + w(\theta') - c \geq \underline{u}$ and $w(\theta) \neq w(\theta')$.

Suppose $w(\theta) > w(\theta')$. Then the beneficiary with type θ can make a profitable deviation to offer $w(\theta')$, which is accepted by the worker as $\alpha\mu(w(\theta')) + w(\theta') - c \geq \underline{u}$, since

$$\theta - w(\theta) < \theta - w(\theta').$$

Suppose $w(\theta) < w(\theta')$. Then the beneficiary with type θ' can make a profitable deviation to offer $w(\theta)$, which is still accepted by the worker as $\alpha\mu(w(\theta)) + w(\theta) - c \geq \underline{u}$, since

$$\theta' - w(\theta') < \theta' - w(\theta).$$

■

Unrevealing equilibrium

The above shows that as far as a wage offer is accepted it cannot reveal type at all, that is, the graph $w(\cdot)$ must be flat over the region in which the offer is accepted. The threshold for such region is the point at which the worker is indifferent between rejecting the offer and accepting the offer knowing only that the value of the project is greater than it.

Proposition 2 Assume that the worker always accepts a wage offer when he is indifferent. (i) When $\alpha\mu - c < \underline{u}$, there is a unique $\theta^* \in (0, \overline{\theta}]$ such that

$$\alpha E[\theta'|\theta' \geq \theta^*] + \theta^* - c = \underline{u}.$$

Then the following is a perfect Bayesian equilibrium:

$$w(\theta)\begin{cases} <\theta^* & \text{if} \quad \theta<\theta^* \\ =\theta^* & \text{if} \quad \theta\geq\theta^* \end{cases}$$

and

$$\mu(w)\begin{cases} <E[\theta'|\theta'\geq\theta^*] & \text{if} \quad w<\theta^* \\ =E[\theta'|\theta'\geq\theta^*] & \text{if} \quad w=\theta^* \\ =\text{free} & \text{if} \quad w>\theta^* \end{cases},$$

and the worker accepts the offer if and only if $\alpha\mu(w)+w-c\geq\underline{u}$.

(ii) When $\alpha\mu-c\geq\underline{u}$, the following is a perfect Bayesian equilibrium:

$$w(\theta)=0$$

for all $\theta\in[0,\overline{\theta}]$ and

$$\mu(w)\begin{cases} =\mu & \text{if} \quad w=0 \\ =\text{free} & \text{if} \quad w>0 \end{cases},$$

and the worker accepts the offer if and only if $\alpha\mu(w)+w-c\geq\underline{u}$.

Proof. (i) It is clear that $\alpha E[\theta'|\theta'\geq\theta]+\theta$ is monotone-increasing, and also that $\alpha E[\theta'|\theta'\geq\theta]+\theta\geq(1+\alpha)\theta$. Since $(1+\alpha)\overline{\theta}\geq\underline{u}+c$, when $\alpha\mu<\underline{u}+c$, there is a unique $\theta^*\in(0,\overline{\theta}]$ such that $\alpha E[\theta'|\theta'\geq\theta^*]+\theta^*-c=\underline{u}$.

The worker's sequential rationality of accepting offer if and only if $\alpha\mu(w)+w-c\geq\underline{u}$ is straightfoward, once we impose the refinement condition.

To check sequential rationality for the beneficiary, note that he always get zero payoff by offering $w<\theta^*$. Now suppose $\theta<\theta^*$. Then, if he offers $w\geq\theta^*$ it is accepted and he gets $\theta-w<\theta^*-\theta^*=0$. Hence it is optimal to offer $w<\theta^*$. Suppose $\theta\geq\theta^*$. Then if he offers $w=\theta^*$ it is accepted and he gets $\theta-\theta^*\geq0$. If he offers $w>\theta^*$, either it is accepted and he gets $\theta-w\leq\theta-\theta^*$, or it is rejected and he gets zero. Hence it is optimal to offer $w=\theta^*$.

To guarantee the consitency of beliefs, since $Prob(\theta'<\theta^*|w<\theta^*)=1$, we see that

$$\mu(w)<\theta^*<E[\theta'|\theta'\geq\theta^*]$$

hold for all $w<\theta^*$.

(ii) The same argument as above follows by considering that θ^* tends to zero. ■

Manipulation and inefficiency

The project should not be undertaken when $\theta < \frac{\underline{u}+c}{1+\alpha}$. Since $\theta^* < \frac{\underline{u}+c}{1+\alpha}$, there is a range of values, $\left[\theta^*, \frac{\underline{u}+c}{1+\alpha}\right)$, in which the project is undertaken despite it should not be. There the worker is manipulated to believe that in expectation the project is worth undertaking, although he is not deceived in the straightforwarded manner as explained in the previous section.

3 Endogenous efforts

Now consider that the worker's effort level e is endogenous, and its cost is given by $c(e)$, whise $c : \mathbb{R}_+ \to \mathbb{R}_+$ is the cost function. Effort level affects the probability of success, which is denoted by $f(e)$, whise $f : \mathbb{R}_+ \to [0, 1]$.

We impose the following standard assumptions.

1. c is twice continuously differentiable over $\mathbb{R}_{++}$, and $c' > 0$, $c'' > 0$. Also, $\lim_{e\to 0} c'(e) = 0$ and $\lim_{e\to\infty} c'(e) = \infty$.

2. f is twice continuously differentiable over $\mathbb{R}_{++}$, and $f' > 0$, $f'' < 0$. Also, $\lim_{e\to 0} f'(e) = \infty$ and $\lim_{e\to\infty} f'(e) = 0$.

Denote the wage payment by w. Then the beneficiary's utility is

$$\theta f(e) - w$$

when the worker signs and provides effort e, and 0 otherwise. The worker's utility when the value is known is

$$\alpha\theta f(e) + w - c(e)$$

when he signs and provides effort e, and $\underline{u}$ otherwise.

Consider the following two-stage game.

Stage 1: The beneficiary offers wage w.

Stage 2: The worker accepts or reject the offer. If rejected, the outside option is obtained. If accepted, the worker chooses effort level e.

We continue to adopt perfect Bayesian equilibrium as the equilibrium concept.

Like in the moral hazard problem we could think of outcome-dependent wage payment, but in order to focus on the signalling role here we focus on outcome-independent wage.

3.1 The complete information benchmark

First consider that the value θ is observable for the worker and the worker's effort is observable for the beneficiary.

Efficiency says that the the following maximization problem must be solved:

$$\max_e \max\{(1+\alpha)\theta f(e) - c(e), \underline{u}\}$$

Let

$$e(\theta) = \arg\max(1+\alpha)\theta f(e) - c(e)$$

Then it is easy to show that $e(\theta)$ and $(1+\alpha)\theta f(e(\theta)) - c(e(\theta))$ are increasing in θ.

Let $\widehat{\theta}$ be such that

$$(1+\alpha)\widehat{\theta} f(e(\widehat{\theta})) - c(e(\widehat{\theta})) = \underline{u}$$

Then the project should be implemented if and only if $\theta \geq \widehat{\theta}$.

When $\theta \geq \widehat{\theta}$, in order to implement the project it is optimal for the beneficiary to offer wage $w(\theta)$ so that the participation constraint for the worker is met with equality,

$$\alpha\theta f(e(\theta)) + w(\theta) - c(e(\theta)) = \underline{u}$$

that is,

$$w(\theta) = \underline{u} + c(e(\theta)) - \alpha\theta f(e(\theta))$$

From the first-order condition it is easy to show that

$$w'(\theta) = -\alpha f(e(\theta)) < 0$$

Hence $w(\theta)$ is again decreasing in θ, and we face the same problem as before. That is, a low-θ beneficiary can always mimic to be a high-θ beneficiary by offering lower wage.

3.2 Incomplete information

We find there are two prominent equilibria. First, the unrevealing equilibrium as seen in the previous section persists even under endogenous efforts. This is contrary to a crude intuition that the beneficiary will have an incentive reveal the value when it is higher, as he may want to motivate the worker to pay more effort by convincing that the value is higher. In the second equilibrium we find this intuition is partially met, in the sense that revelation starts only when the value exceeds some threshold.

Unrevealing equilibrium

Again, let $\mu(w)$ be the expected value of θ conditional on wage offer w, which is believed by the worker and is to be determined in equilibrium. Then the worker accepts the offer when

$$\max_e \alpha\mu(w)f(e) + w - c(e) \geq \underline{u}$$

and rejects otherwise.

In the current specification, only the expectation of value believed by the worker determines effort choice after accepting an offer. Thus, letting v the expectation of value believed by the worker, define

$$\widetilde{e}(v) = \arg\max_e \alpha v f(e) - c(e).$$

Proposition 3 Assume that the worker always accepts a wage offer when he is indifferent. (i) When $\alpha\mu f(\widetilde{e}(0)) - c < \underline{u}$, there is a unique $\theta^* \in (0, \overline{\theta}]$ such that

$$\alpha v^* f(\widetilde{e}(v^*)) + \theta^* f(\widetilde{e}(v^*)) - c(\widetilde{e}(v^*)) = \underline{u},$$

whise

$$v^* = E[\theta'|\theta' \geq \theta^*]$$

Then the following is a perfect Bayesian equilibrium:

$$w(\theta) \begin{cases} < \theta^* f(\widetilde{e}(v^*)) & \text{if} \quad \theta < \theta^* \\ = \theta^* f(\widetilde{e}(v^*)) & \text{if} \quad \theta \geq \theta^* \end{cases}$$

and

$$\mu(w) \begin{cases} < v^* & \text{if} \quad w < \theta^* f(\widetilde{e}(v^*)) \\ = v^* & \text{if} \quad w \geq \theta^* f(\widetilde{e}(v^*)) \end{cases},$$

and the worker accepts the offer if and only if $w \geq \theta^* f(\widetilde{e}(\theta^*))$, and when accepts he provides effort $\widetilde{e}(\theta^*)$.
(ii) When $\alpha\mu f(\widetilde{e}(0)) - c \geq \underline{u}$, the following is a perfect Bayesian equilibrium:

$$w(\theta) = 0$$

for all $\theta \in [0, \overline{\theta}]$ and

$$\mu(w) = \mu$$

for all $w \geq 0$, and the worker allways accepts the offer and provides effort $\widetilde{e}(\mu)$.

Proof. (i) Let

$$v(\theta) = E[\theta'|\theta' \geq \theta]$$

Then it is clear that $\alpha v(\theta) f(\widetilde{e}(v(\theta))) + \theta f(\widetilde{e}(v(\theta)))$ is monotone-increasing, and also that $\alpha v(\theta) f(\widetilde{e}(v(\theta))) + \theta f(\widetilde{e}(v(\theta))) \geq (1+\alpha)\theta f(\widetilde{e}(v(\theta)))$. Since $(1+\alpha)\overline{\theta} f(\widetilde{e}(v(\overline{\theta}))) = (1+\alpha)\overline{\theta} f(\widetilde{e}(\overline{\theta})) \geq \underline{u} + c$, when $\alpha \mu f(\widetilde{e}(0)) < \underline{u} + c$, there is a unique $\theta^* \in (0, \overline{\theta}]$ such that $\alpha v(\theta^*) f(\widetilde{e}(v(\theta^*))) + \theta^* f(\widetilde{e}(v(\theta^*))) - c = \underline{u}$.

Now let

$$v^* = E[\theta'|\theta' \geq \theta^*]$$

To check sequential rationality for the worker given his belief, suppose $w \geq \theta^* f(\widetilde{e}(v^*))$. Then, since $\mu(w) = v^*$ and $\alpha v^* f(\widetilde{e}(v^*)) + \theta^* f(\widetilde{e}(v^*)) - c = \underline{u}$, it is optimal for him to accept the offer and provided effort $\widetilde{e}(v^*)$. Suppose suppose $w < \theta^* f(\widetilde{e}(v^*))$. Then, since $\mu(w) < v^*$ and $\alpha v^* f(\widetilde{e}(v^*)) + \theta^* f(\widetilde{e}(v^*)) - c < \underline{u}$, it is optimal for him to rejct the offer.

To check sequential rationality for the beneficiary, note that he always get zero payoff by offering $w < \theta^* f(\widetilde{e}(v^*))$. Now suppose $\theta < \theta^*$. Then, if he offers $w \geq \theta^* f(\widetilde{e}(v^*))$ it is accepted and he gets $\theta f(\widetilde{e}(v^*)) - w < \theta^* f(\widetilde{e}(v^*)) - \theta^* f(\widetilde{e}(v^*)) = 0$. Hence it is optimal to offer $w < \theta^* f(\widetilde{e}(v^*))$. Suppose $\theta \geq \theta^*$. Then if he offers $w = \theta^* f(\widetilde{e}(v^*))$ it is accepted and he gets $\theta f(\widetilde{e}(v^*)) - \theta^* f(\widetilde{e}(v^*))$. If he offers $w > \theta^* f(\widetilde{e}(v^*))$, it is accepted and he gets $\theta f(\widetilde{e}(v^*)) - w < \theta - \theta^* f(\widetilde{e}(v^*))$. Hence it is optimal to offer $w = f(\widetilde{e}(v^*))$.

Consitency of beliefs is straightforward then.

(ii) The same argument as above follows by considering that θ^* tends to zero. ■

Manipulation and inefficiency

Since $\theta^* < \widehat{\theta}$, where $\widehat{\theta}$ is the point at which the maximal social surplus is zero, there is a range of values $[\theta^*, \widehat{\theta})$ in which the worker is manipulated to believe that the project is worth doing despite it should not be.

Partially revealing equilibrium

We can construct an equilibrium in which the beneficiary with sufficiently high θ reveals the value.

To get an idea, for moments ignore the participation constraint and assume that the worker accepts any offer. Assume that $\widetilde{w}$ is differntiable and monotone-increasing. To show that following $\widetilde{w}$ is incentive-compatible for the beneficiary, consider the maximization problem

$$\max_s \theta f(\widetilde{e}(s)) - \widetilde{w}(s)$$

whise he can pretend to have any type s instead of true θ. Note that hise the beneficiary knows that if he reports s the worker will choose $\widetilde{e}(s)$.

Then the first-order condition says

$$\theta f(\widetilde{e}(s))\widetilde{e}'(s) - \widetilde{w}'(s) = 0$$

In equilibrium it holds $s = \theta$, hence we obtain

$$\widetilde{w}'(\theta) = \theta f(\widetilde{e}(\theta))\widetilde{e}'(\theta)$$

By using integration by part, we obtain

$$\widetilde{w}(\theta) - \widetilde{w}(0) = \theta f(\widetilde{e}(\theta)) - \int_0^\theta f(\widetilde{e}(s))ds.$$

Now can this be a part of solution when the participation constraint for the worker is considered? A natural way of applying the participation constraint is to set threshold $\theta^\star$ defined by

$$\alpha\theta^\star f(\widetilde{e}(\theta^\star)) + \widetilde{w}(\theta^\star) - c(\widetilde{e}(\theta^\star)) = \underline{u}.$$

Then the beneficiary with value lower than $\theta^\star$ can get accepted only by offering wage greater than or equal to $\widetilde{w}(\theta^\star)$. But this does not discrourage such beneficiary to pretend that his value is higher than $\theta^\star$ rather than to get rejected. This still leaves an incentive for the beneficiary to manipulate.

In order that the beneficiary follows an incentive-compatible wage policy, we need two thresholds, one such that the participation constraint for the worker is met with equality there and the beneficiary chooses to get rejected if his value is low than that, the other such that the participation constraint is non-binding and the beneficiary is indifferent between unrevealing and revealing.

Proposition 4 Let

$$\widetilde{w}(\theta) = \theta f(\widetilde{e}(\theta)) - \int_0^\theta f(\widetilde{e}(s))ds$$

Let θ_1, θ_2 with $\theta_1 < \theta_2$ be such that

$$\theta_2 f(\widetilde{e}(v_{12})) - \theta_1 f(\widetilde{e}(v_{12})) = \theta_2 f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2)$$

and

$$\alpha v_{12} f(\widetilde{e}(v_{12})) + \theta_1 f(\widetilde{e}(v_{12})) - c(\widetilde{e}(v_{12})) = \underline{u},$$

whise $v_{12} = E[\theta' | \theta_1 \leq \theta' < \theta_2]$.

Then the following is a perfect Bayesian equilibrium:

$$w(\theta) \begin{cases} < \theta_1 f(\widetilde{e}(v_{12})) & \text{if} \quad \theta < \theta_1 \\ = \theta_1 f(\widetilde{e}(v_{12})) & \text{if} \quad \theta_1 \leq \theta < \theta_2 \\ = \widetilde{w}(\theta) & \text{if} \quad \theta_2 \leq \theta \end{cases}$$

and

$$\mu(w) \begin{cases} < v_{12} & \text{if} \quad w < \theta_1 f(\widetilde{e}(v_{12})) \\ = v_{12} & \text{if} \quad \theta_1 f(\widetilde{e}(v_{12})) \leq w < \widetilde{w}(\theta_2) \\ = \widetilde{w}^{-1}(w) & \text{if} \quad \widetilde{w}(\theta_2) \leq w \end{cases},$$

and the worker accepts the offer if and only if $w \geq \theta_1 f(\widetilde{e}(v_{12}))$, and when accepts he provides effort $\widetilde{e}(\mu(w))$.

Proof. Sequential rationality for the worker is straightforward.

To check sequential rationality for the beneficiary, first suppose $\theta \geq \theta_2$. Then by offering $\widetilde{w}(\theta)$ he gets accepted and gets $\theta f(\widetilde{e}(\theta)) - \widetilde{w}(\theta)$. From the construction of $\widetilde{w}$ there is no profitable deviation in pretending any other $t > \theta_2$.

By offering $w < \theta_1 f(\widetilde{e}(v_{12}))$ he gets rejected and gets zero. By offering $\theta_1 f(\widetilde{e}(v_{12})) \leq w < \widetilde{w}(\theta_2)$ he gets accepted and gets $(\theta - \theta_1) f(\widetilde{e}(v_{12}))$.

Since

$$\begin{aligned} (\theta - \theta_1) f(\widetilde{e}(v_{12})) &= (\theta_2 - \theta_1) f(\widetilde{e}(v_{12})) + (\theta - \theta_2) f(\widetilde{e}(v_{12})) \\ &= \theta_2 f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) + (\theta - \theta_2) f(\widetilde{e}(v_{12})) \\ &< \theta_2 f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) + (\theta - \theta_2) f(\widetilde{e}(\theta_2)) \\ &= \theta f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) \\ &\leq \theta f(\widetilde{e}(\theta)) - \widetilde{w}(\theta) \end{aligned}$$

it is optimal to offer $\widetilde{w}(\theta)$.

Suppose $\theta_1 \leq \theta < \theta_2$. By offering $w < \theta_1 f(\widetilde{e}(v_{12}))$, he gets regected and gets zero. By offering $\theta_1 f(\widetilde{e}(v_{12})) \leq w < \widetilde{w}(\theta_2)$ he gets accepted and gets $(\theta - \theta_1) f(\widetilde{e}(v_{12}))$.

By offering $\widetilde{w}(\theta_2)$, that is, by pretending to be θ_2, he gets accepted and gets

$$\begin{aligned}
\theta f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) &= \theta f(\widetilde{e}(\theta_2)) + \theta_2 f(\widetilde{e}(v_{12})) - \theta_1 f(\widetilde{e}(v_{12})) - \theta_2 f(\widetilde{e}(\theta_2)) \\
&= -(\theta_2 - \theta) f(\widetilde{e}(\theta_2)) + \theta_2 f(\widetilde{e}(v_{12})) - \theta_1 f(\widetilde{e}(v_{12})) \\
&\leq -(\theta_2 - \theta) f(\widetilde{e}(v_{12})) + \theta_2 f(\widetilde{e}(v_{12})) - \theta_1 f(\widetilde{e}(v_{12})) \\
&= \theta f(\widetilde{e}(v_{12})) - \theta_1 f(\widetilde{e}(v_{12}))
\end{aligned}$$

Further, by offering $w > \widetilde{w}(\theta_2)$, that is, by pretending to be $t > \theta_2$, he gets accepted and gets

$$\begin{aligned}
\theta f(\widetilde{e}(t)) - \widetilde{w}(t) &= \theta f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) + \theta[f(\widetilde{e}(t)) - f(\widetilde{e}(\theta_2))] - \widetilde{w}(t) + \widetilde{w}(\theta_2) \\
&\leq \theta f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) + \theta_2[f(\widetilde{e}(t)) - f(\widetilde{e}(\theta_2))] - \widetilde{w}(t) + \widetilde{w}(\theta_2) \\
&= \theta f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) + \theta_2 f(\widetilde{e}(t)) - \widetilde{w}(t) - [\theta_2 f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2)] \\
&\leq \theta f(\widetilde{e}(\theta_2)) - \widetilde{w}(\theta_2) \\
&\leq \theta f(\widetilde{e}(v_{12})) - \theta_1 f(\widetilde{e}(v_{12}))
\end{aligned}$$

Thus it is optimal to offer $\theta_1 f(\widetilde{e}(v_{12}))$.

Suppose $\theta < \theta_1$. Then by offering any $w < \theta_1 f(\widetilde{e}(v_{12}))$ the employer gets rejected and gets zero.

By offering $\theta_1 f(\widetilde{e}(v_{12})) \leq w < \widetilde{w}(\theta_2)$ he gets accepted and gets $(\theta - \theta_1) f(\widetilde{e}(v_{12})) < 0$.

By offering $w > \widetilde{w}(\theta_2)$, he gets accepted and by the same argument as above we obtain

$$\theta f(\widetilde{e}(t)) - \widetilde{w}(t) \leq \theta f(\widetilde{e}(v_{12})) - \theta_1 f(\widetilde{e}(v_{12}))$$

where the right-hand-side is less than zero.

Thus it is optimal to offer $w < \theta_1 f(\widetilde{e}(v_{12}))$.

Now consistency of belief is straightforward. ■

Inefficiency

Since $v_{12} > \theta_1$, it holds

$$\begin{aligned}
\underline{u} &= \alpha v_{12} f(\widetilde{e}(v_{12})) + \theta_1 f(\widetilde{e}(v_{12})) - c(\widetilde{e}(v_{12})) \\
&> \alpha \theta_1 f(\widetilde{e}(v_{12})) + \theta_1 f(\widetilde{e}(v_{12})) - c(\widetilde{e}(v_{12})) \\
&\geq \alpha \theta_1 f(\widetilde{e}(\theta_1)) + \theta_1 f(\widetilde{e}(\theta_1)) - c(\widetilde{e}(\theta_1))
\end{aligned}$$

implying that $\theta_1 < \widehat{\theta}$. Thisefore, there is still a range $[\theta_1, \widehat{\theta})$ in which the project is undertaken despite it should not be. Since we can show $\theta^* \leq \theta_1$ on the other hand, compared to the unrevealing equilibrium the probability of such inefficiency is lower in the partially revealing equilibrium.

4 Degree of value sharing as private information

Finally, we consider that the degree of value sharing is private information of the worker. Assume that degree of value sharing α is drawn from $[0, \overline{\alpha}]$ according to distribution Ψ with density ψ.

4.1 The case of exogenous effort

Given wage offer w and the believed value $\mu(w)$ conditional on w, the worker accepts w if

$$\alpha\mu(w) + w - c \geq \underline{u}.$$

Note that there is no adverse selection here, as the fact that the worker accepts a wage offer simply reveals that he shares value of the beneficiary more and it is a harmless thing for the beneficiary.

Let

$$P_\mu(w) = Prob(\alpha\mu(w) + w - c \geq \underline{u})$$

Note that $P_\mu(\cdot)$ is strictly increasing as far as $\mu(\cdot)$ is positive valued and non-decreasing. Then beneficiary's expected utility of offering w is

$$P_\mu(w)(\theta - w)$$

To simplify the argument we assume

$$\overline{\alpha} = \infty, \quad \overline{\theta} < c + \underline{u}$$

This guarantees that there is always a very small but positive probability that any wage offer is accepted. On can imagine for example the exponential distribution with very thin tail. Also we assume that the project can be undertaken only when the worker has positive empathy, otherwise a beneficary might be able to offer sufficiently high wage so that he does not need to count on the worker's empathy.

Under the assumptions we look for an equilibrium with $\mu(\cdot)$ being non-decreasing and $\mu(w) > 0$ for all $w > 0$. Then, since $w(\theta) \leq \theta \leq \overline{\theta} < c + \underline{u}$, the beneficiary never makes a wage offer greater than $c + \underline{u}$. Also, the assumption implies that $P_\mu(w)$ is always positive while it will be very small when w is small.

Proposition 5 When we restrict attention to a perfect Bayesian equilibrium with $\mu(\cdot)$ being non-decreasing and $\mu(w) > 0$ for all $w > 0$, it must be fully revealing.

Proof. Given any nondcreasing $\mu(\cdot)$, define

$$w_\mu(\theta) = \arg\max_w P_\mu(w)(\theta - w)$$

Let $\theta' > \theta$ and $w' > w$. Then the increasing difference property

$$\begin{aligned} & P_\mu(w')(\theta' - w') - P_\mu(w)(\theta' - w) \\ > \quad & P_\mu(w')(\theta - w') - P_\mu(w)(\theta - w) \end{aligned}$$

follows from

$$(P_\mu(w') - P_\mu(w))(\theta' - \theta) > 0.$$

From the increasing difference property we conclude that $w_\mu(\cdot)$ is strictly increasing.

In equilibrium, it must hold

$$\mu(w) = E\left[\theta' | w_\mu(\theta') = w\right] = w_\mu^{-1}(w)$$

for all $w \in w_\mu([0, \overline{\theta}])$, but since w_μ is strictly increasing μ is strictly increasing. ■

Inefficiency

Since equilibrium wage offer is fully revealing, the worker accepts it when

$$\alpha\theta + w(\theta) - c \geq \underline{u}$$

Recall that the project should be undertaken when

$$(1 + \alpha)\theta - c \geq \underline{u}.$$

Since $w(\theta) \leq \theta$, the first inequality implies the second inequality. That is, it cannot happen that the project is undertaken despite it should not be. This contrasts to what we had for the unrevealing equilibrium when the degree of value sharing is known to the beneficiary.

Inefficiency due to the project not being taken despite it should be arises here, though.

4.2 The case of endogenous effort

Finally we consider the case of endogenous effort when the degree of value sharing private information of the worker. Thise, since an incease of wage attracts a worker with lower degree of value sharing, the probability of success *conditional on* acceptance by the worker will become lower. This sounds like discouraging the beneficiary from raising wage offer based on his value.[1] However, since the beneficiary cares for the probability of acceptance as well, and he gets nothing if the wage offer is rejected, the corresponding increase of acceptance probability exactly kills the above negative effect. In fact, it is shown that the benficiary cares only for pairs of the probability of success prior to accepting/rejecting and the probability of acceptance.

Let $\mu(w)$ be the expected value of θ conditional on wage offer w, which is believed by the worker and is to be determined in equilibrium. Then the worker with type α accepts the offer when

$$\max_e \alpha\mu(w)f(e) + w - c(e) \geq \underline{u}$$

and rejects othiswise.

In the current specification, only the expectation of value believed by the worker determines effort choice after accepting an offer. Thus, letting v the expectation of value believed by the worker, define

$$\widetilde{e}(\alpha, v) = \arg\max_e \alpha v f(e) - c(e).$$

Given wage offer w and the believed value $\mu(w)$ conditional on w, the worker accepts w if

$$\alpha\mu(w)f(\widetilde{e}(\alpha, \mu(w))) + w - c(\widetilde{e}(\alpha, \mu(w))) \geq \underline{u}.$$

Let

$$P_\mu(w) = Prob(\alpha\mu(w)f(\widetilde{e}(\alpha, \mu(w))) + w - c(\widetilde{e}(\alpha, \mu(w))) \geq \underline{u})$$

denote the probability that the worker with belief μ accepts w, and

$$f_\mu(w) = E[f(\widetilde{e}(\alpha, \mu(w)))|\alpha\mu(w)f(\widetilde{e}(\alpha, \mu(w))) + w - c(\widetilde{e}(\alpha, \mu(w))) \geq \underline{u}]$$

[1] Delfgaauw and Dur [5] consider a screening problem in which workers have intrinsic motation, which is private information, and the employer faces a trade-off that increasing wage attracts unmotivated workers while it increase the probability of acceptance.

denote the probability of success conditional on accepting w. Note that $P_\mu(\cdot)$ is strictly increasing as far as $\mu(\cdot)$ is positive valued and non-decreasing.

The beneficiary's expected utility of offering w is

$$\theta P_\mu(w) f_\mu(w) - w P_\mu(w)$$

Note that what matters hise is $P_\mu(w) f_\mu(w)$, which is the probability of success prior to accepting/rejecting, and $P_\mu(w)$, which is the probability of accepting.

Proposition 6 When we restrict attention a perfect Bayesian equilibrium with $\mu(\cdot)$ being non-decreasing $\mu(w) > 0$ for all $w > 0$, it must be fully revealing.

Proof. First, we see that $P_\mu(w) f_\mu(w)$ is strictly increasing in w when $\mu(\cdot)$ is non-decreasing and $\mu(w) > 0$ for all $w > 0$. This is because

$$P_\mu(w) f_\mu(w) = \int_{\frac{c+\underline{u}-w}{\mu(w)}}^{\infty} f(\widetilde{e}(\alpha, \mu(w)))\psi(\alpha) d\alpha$$

and $\frac{c+\underline{u}-w}{\mu(w)}$ is strictly deccreasing in w, and $f(\widetilde{e}(\alpha, \mu(w)))$ is at least weakly increasing in w.

Given any nondcreasing $\mu(\cdot)$, define

$$w_\mu(\theta) = \arg\max_w \{\theta P_\mu(w) f_\mu(w) - w P_\mu(w)\}$$

Let $\theta' > \theta$ and $w' > w$. Then the increasing difference property

$$\begin{aligned} & \{\theta' P_\mu(w') f_\mu(w') - w' P_\mu(w')\} - \{\theta' P_\mu(w) f_\mu(w) - w P_\mu(w)\} \\ > \ & \{\theta P_\mu(w') f_\mu(w') - w' P_\mu(w')\} - \{\theta P_\mu(w) f_\mu(w) - w P_\mu(w)\} \end{aligned}$$

follows from

$$(P_\mu(w') f_\mu(w') - P_\mu(w) f_\mu(w))(\theta' - \theta) > 0$$

as $P_\mu(w) f_\mu(w)$ is strictly increasing in w.

From the increasing difference property we conclude that $w_\mu(\cdot)$ is strictly increasing.

In equilibrium, it must hold

$$\mu(w) = E\left[\theta' | w_\mu(\theta') = w\right] = w_\mu^{-1}(w)$$

for all $w \in w_\mu([0, \overline{\theta}])$, but since w_μ is strictly increasing μ is strictly increasing. ■

Inefficiency

Since equilibrium wage offer is fully revealing, the worker accepts it when

$$\alpha\theta f(\widetilde{e}(\alpha,\theta)) + w(\theta) - c(\widetilde{e}(\alpha,\theta)) \geq \underline{u}.$$

Note that the project should be undertaken when

$$(1+\alpha)\theta f(\widehat{e}(\alpha,\theta)) - c(\widehat{e}(\alpha,\theta)) \geq \underline{u}$$

where $\widehat{e}(\alpha,\theta) = \arg\max_e (1+\alpha)\theta f(e) - c(e)$.

There is not a clear relationship between the above two inequalities. If $w(\theta)$ is always smaller than equal to $\theta f(\widetilde{e}(\alpha,\theta))$ for all α we could draw the similar relathionship as in the exogenous effort case, but $w(\theta)$ may be larger than $\theta f(\widetilde{e}(\alpha,\theta))$ depending on particular realization of α. Thus, in general, there remain two kinds of inefficiency, one that the project is undertaken despite it should not be, the other that the project is not undertaken despite it should be. The evaluation is left to a numerical one.

5 Concluding remarks

We conclude by listing the remaining issues.

(i) When the degree of value sharing α is private information of the worker, it will be resonable to ask if he can signal it in certain way, such as wage proposal from the worker side, and how it affects the economic performance. (ii) In the paper we assumed one beneficiary/employer and one worker, but it will be reasonable to ask what will happen if there is competition between workers or that between beneficiaries/employers. (iii) We considered outcome-independent wage for the purpose of focusing on the signaling role, but it will be reasonable to ask how outcome-dependent wage offer works, in which there will be an interplay between the signaling role and the effort-inducing role as in the standard moral hazard problem. (iv) In the paper we assumed that the degree of value sharing α is exogenously randomly generated, but it may be reasonable to say that it is endogensouly determined in certain way. (v) Finally, in order to predict the evolution of how intrinsic motivation is exploited in the workplace over time, a dynamic yet tractable mode is desired.

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