Communication Enhancement through Information Acquisition by Uninformed Player

Hitoshi Sadakane and Yasuyuki Miyahara
KIER, Kyoto University

April 13, 2017

1 Introduction

We study strategic information transmission between an informed sender (S) and an uninformed decision maker (D) in a situation where the decision maker can privately acquire information about the states of nature. (Crawford and Sobel (1982) + D's information acquisition.)

- Period 1 : S learns the state of nature.
- Period 2 : D privately acquires information.
- Period 3 : S sends a message to D.
- Period 4 : D makes a decision and this game ends.

(Result)

- Information acquisition by D can enhance communication.
- Information structure is endogenized, namely, it is determined which information D acquires in equilibrium.
- The D's information acquisition and the S's information transmission are mutually dependent.

2 Related Literature

2.1 The cheap-talk game with an informed decision maker

1. S learns the state of the world θ.
2. D privately observes a signal ξ drawn from $\Pr(\xi|\theta)$.
3. S sends a message m to R.
4. D makes a decision depending on m and ξ: $p(m, \xi)$.

- Moreno de Barreda (2013) and Ishida and Shimizu (2017)
  - The welfare can be improved when D has a private information in comparison to the case of no private information.
- Moreno de Barreda (2013) analyzes costly information acquisition by the decision maker.
  - D can acquire private information by herself with cost $C > 0$.
- Our technology of information acquisition: ξ is drawn from $\Pr(\xi|\theta, a)$ with cost $c(a)$.
  - ξ and a are multi-dimensional.

2.2 Papers analyzing information acquisition

- Moreno de Barreda (2013)
- Argenziano et al. (2016)
  - D acquires costly information by herself.
    (vs)
    S acquires costly information about the state, and then, he sends a message to D.
- Austen-Smith (1994), Pei (2015), Venturini (2014)
  - S acquires costly information about the state, and then, he sends a message to D.

3 Motivating Example

- A Ph.D. student (D) is deciding her major field of research.
- There are three possibly promising fields (states): Empirical IO, Theoretical IO, and Game Theory.
- D does not know which is the most promising field.
- She can ask her supervisor (S) for advice to choose a field.
  – S knows which field is most promising.
  – The S's preference biased towards more mathematical research.

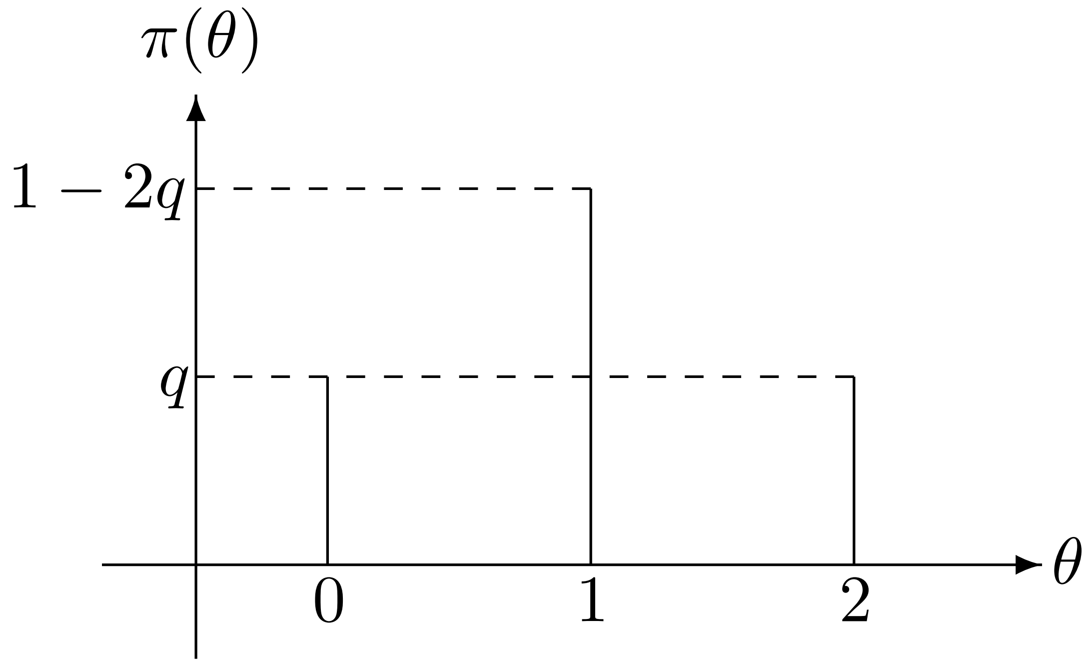
- Additionally, D can privately gather information by reading the academic journals.
  – It is time consuming and costly.
  – $\Rightarrow$ It is a concern which type of journals the student reads.
  – Concentrating the information acquisition on a particular state enhances communication.

4 Model

- Decision maker (D) and Sender (S).
- Project : $Y = \{0, 1, 2\}$.
- State : $\Theta = \{0, 1, 2\}$
  - Empirical IO, Theoretical IO, and Game Theory.
    * Prior probability : $\pi(\theta)$

Utility from the project :

- D : $u^D(y, \theta) = -(y - \theta)^2$
- S : $u^S(y, \theta, b) = -(y - \theta - b)^2$
  - Bias : $b \in (1/2, 1)$.


$\pi(\theta)$
$1-2q$
q
θ
0
1
2

4.1 Investigation

- D maker can acquire noisy signals by her own investigation regarding the possible states.
- For each state, she can investigate whether it is realized or not. : Let a_s be time D spends on investigating whether state s is realized or not.
- D maker decides on the allocation of time, $a = (a_0, a_1, a_2)$.
  - We assume that time is limited, and $a_0 + a_1 + a_2 \leq 1$.
  - Cost : $c \sum_{s=0}^{2} a_s$
- D's payoff : $U^D(y, a, \theta) = -(y - \theta)^2 - c \sum_{s=0}^{2} a_s$

- D maker privately observes three signals : ξ_0, ξ_1, ξ_2
- $\xi_s \in \Xi_s \equiv \{t, f\}$

$$\Pr(\xi_s = t | a_s, \theta) = \begin{cases} \frac{1}{2} + \eta a_s & \text{if } s = \theta, \\ \frac{1}{2} - \eta a_s & \text{if } s \neq \theta. \end{cases}$$

- The sensitivity of signals : $\eta \in (0, 1/2)$.

- The conditional probability of the signal vector : Fix a and θ.

$$\Pr(\xi | a, \theta) = \Pr(\xi_0 | a_0, \theta) \Pr(\xi_1 | a_1, \theta) \Pr(\xi_2 | a_2, \theta).$$

- $\Xi \equiv \Pi_{i=0}^{2} \Xi_i$

4.2 Timing

1. S learns the state of the world θ.
2. D chooses $a = (a_0, a_1, a_2)$ and privately observes $\xi = (\xi_0, \xi_1, \xi_2)$.
3. S sends a message $m \in M$ to D.
4. D chooses a project y.

S's strategy $\mu(\theta)$

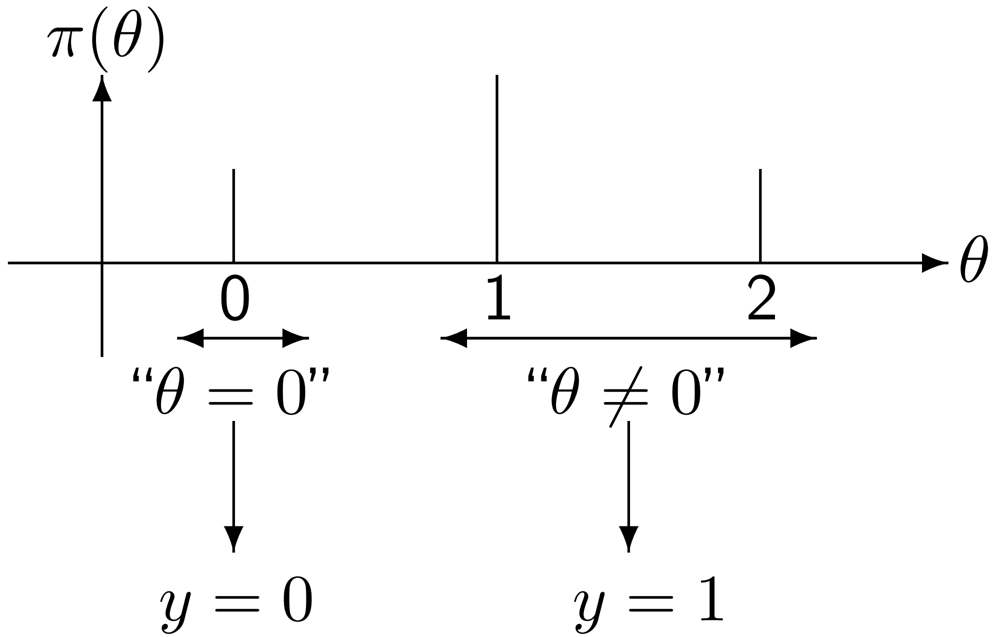
- $\mu : \Theta \to \Delta M$.

D's strategy (a, ρ)

- $a : \{\phi\} \to A \equiv \{(a_0, a_1, a_2) \in [0,1]^3 : a_i \geq 0$ and $a_0 + a_1 + a_2 \leq 1\}$
- $\rho : A \times \Xi \times M \to Y$

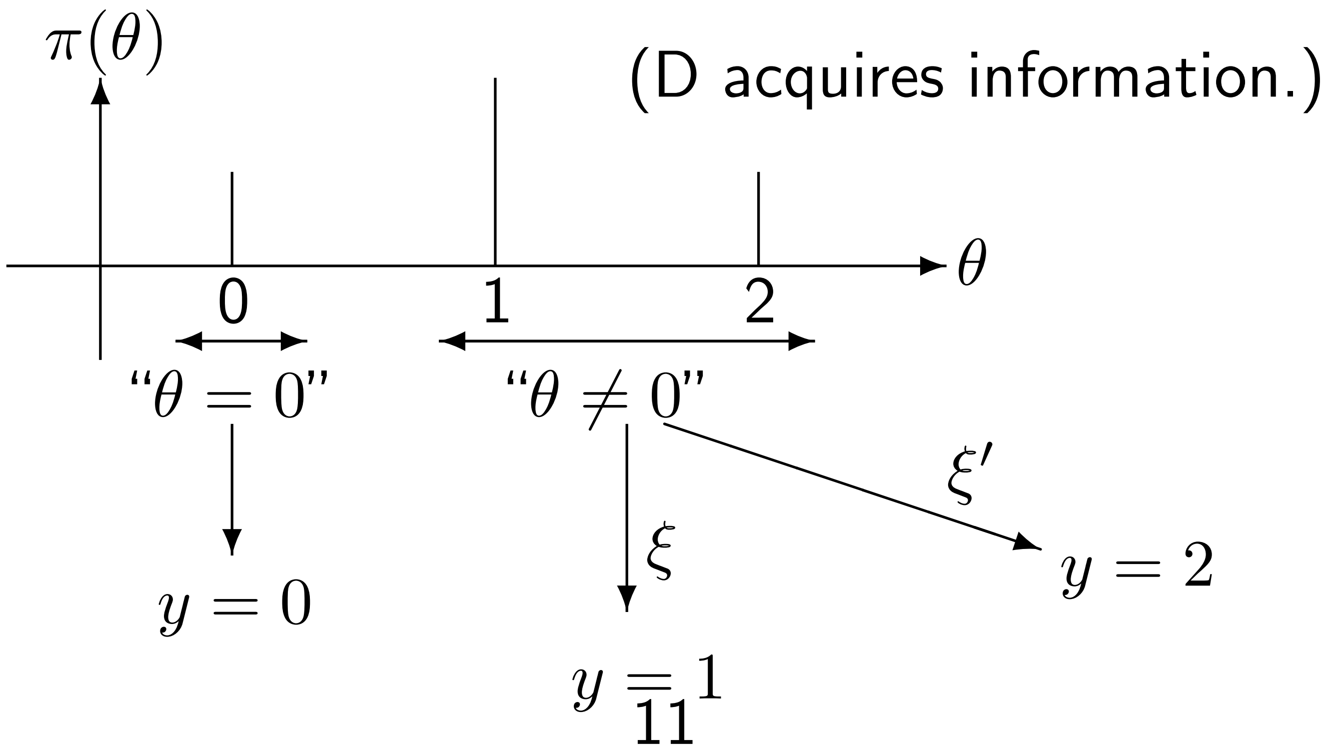
4.3 CS model

- If D can not gather information. There exists no informative equilibrium.
- For example: if $\theta = 0$, S conveys that "$\theta = 0$". Otherwise, he conveys that "$\theta \neq 0$".
- $\Rightarrow$ S of type "$\theta = 0$" has an incentive to deviate from the above message strategy.

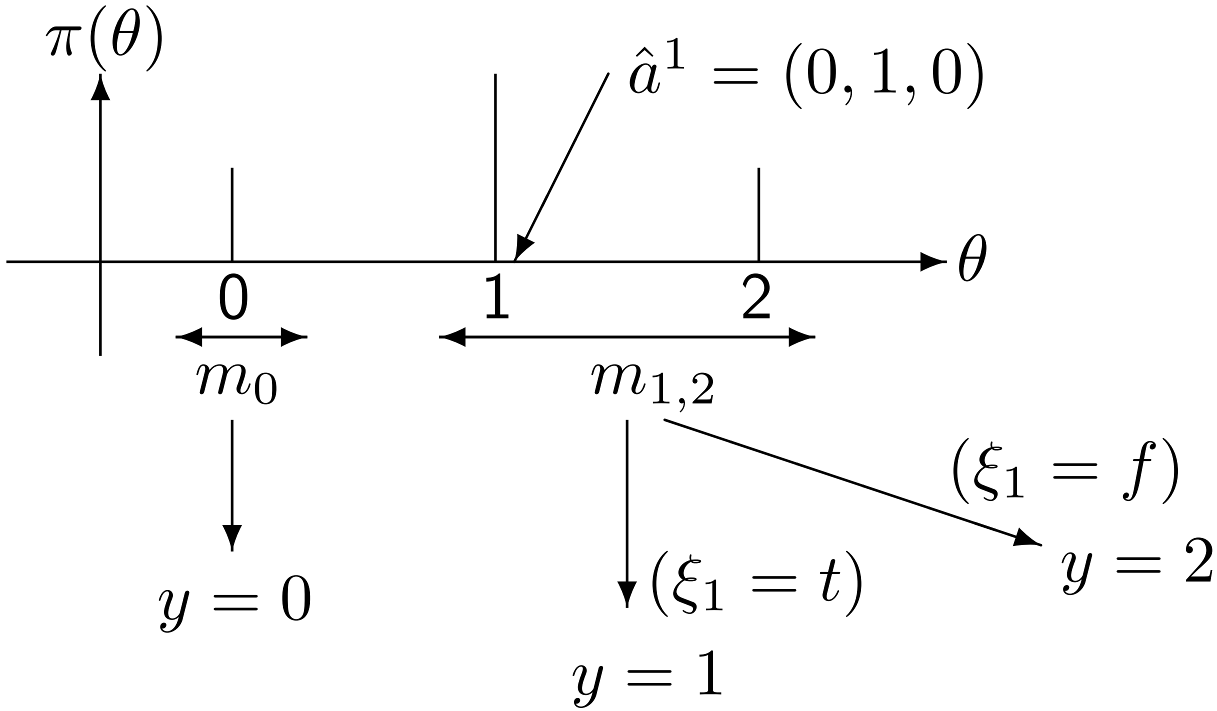


4.4 Idea

- If D has private information, she can choose a project depending on S's message and her private information.
- S of type 0 prefers $y = 0$ to $y = 2$. If $\Pr(\xi' | \theta = 0, a)$ is high enough, S of type 0 conveys "$\theta = 0$" to reduce risk.
- In other words, by choosing a suitably, D can elicit information from S.
- Note that information acquisition is costly $\Rightarrow$ choosing a must be optimal for D.



- In equilibrium, D should acquire information about state 1.

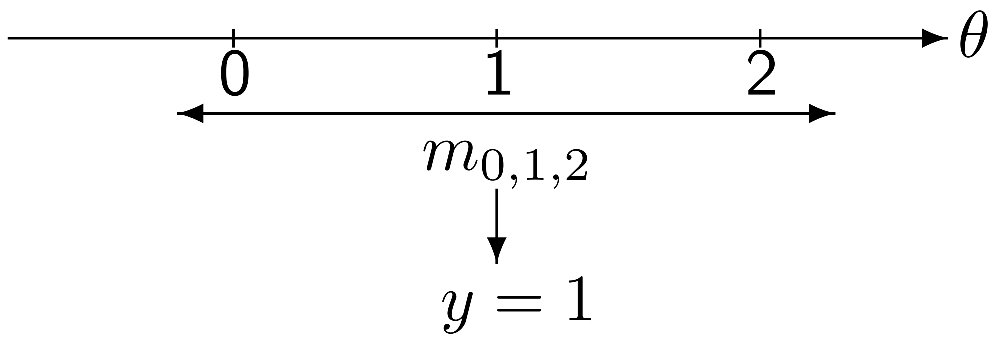


5 Analisis

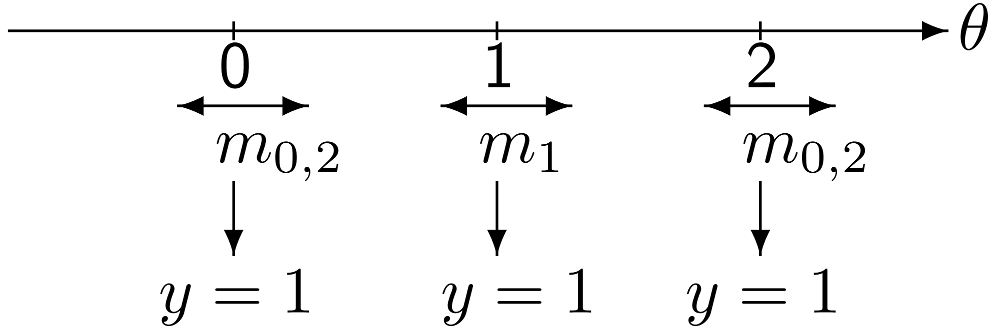
5.1 Benchmark: No Information Acquisition

Proposition 1 Suppose that D cannot acquire information about the states. Then, in any equilibrium, the decision maker chooses project 1 irrelevant to the messages sent by the sender.

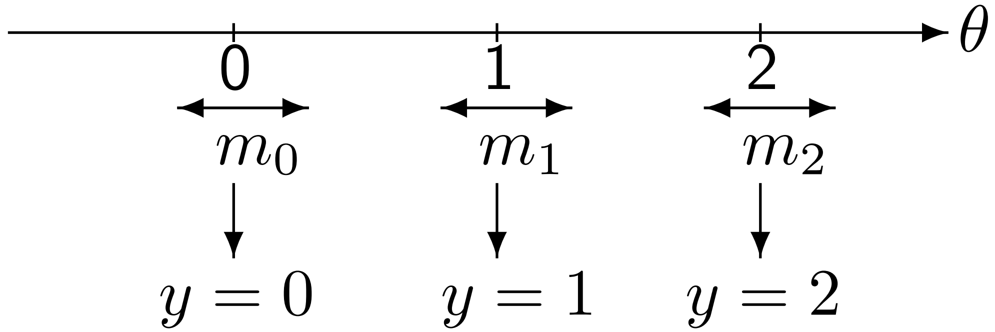
Babbling equilibrium **(○) :** $\{\{0, 1, 2\}\}$



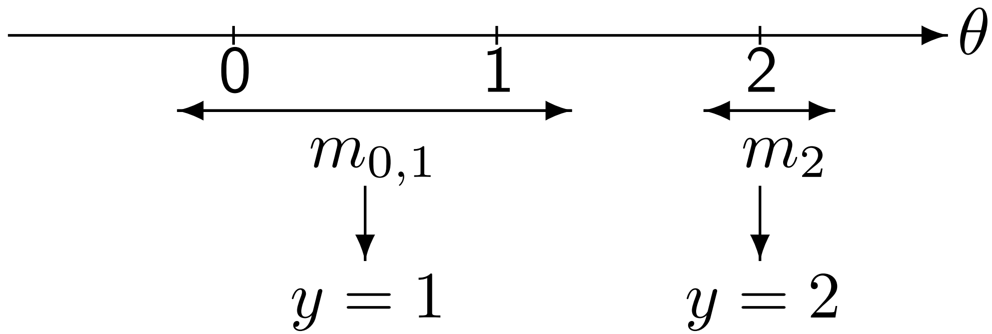
Partially separating equilibrium **(○) :** $\{\{0,2\},\{1\}\}$



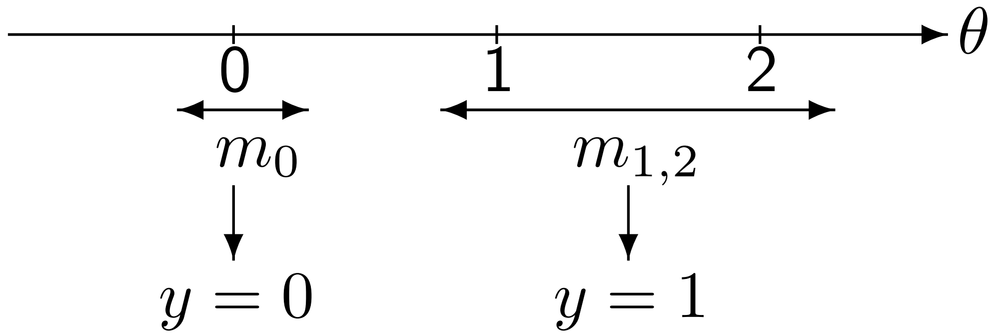
Fully revealing **(**$\theta=0$ and $\theta=1$, $\times$**) :** $\{\{0\},\{1\},\{2\}\}$



Partially separating **(**$\theta = 1,\ \times$**) :** $\{\{0, 1\}, \{2\}\}$



Partially separating **(**$\theta = 0,\ \times$**) :** $\{\{0\}, \{1, 2\}\}$



5.2 Communication Enhancement by Information Acquisition

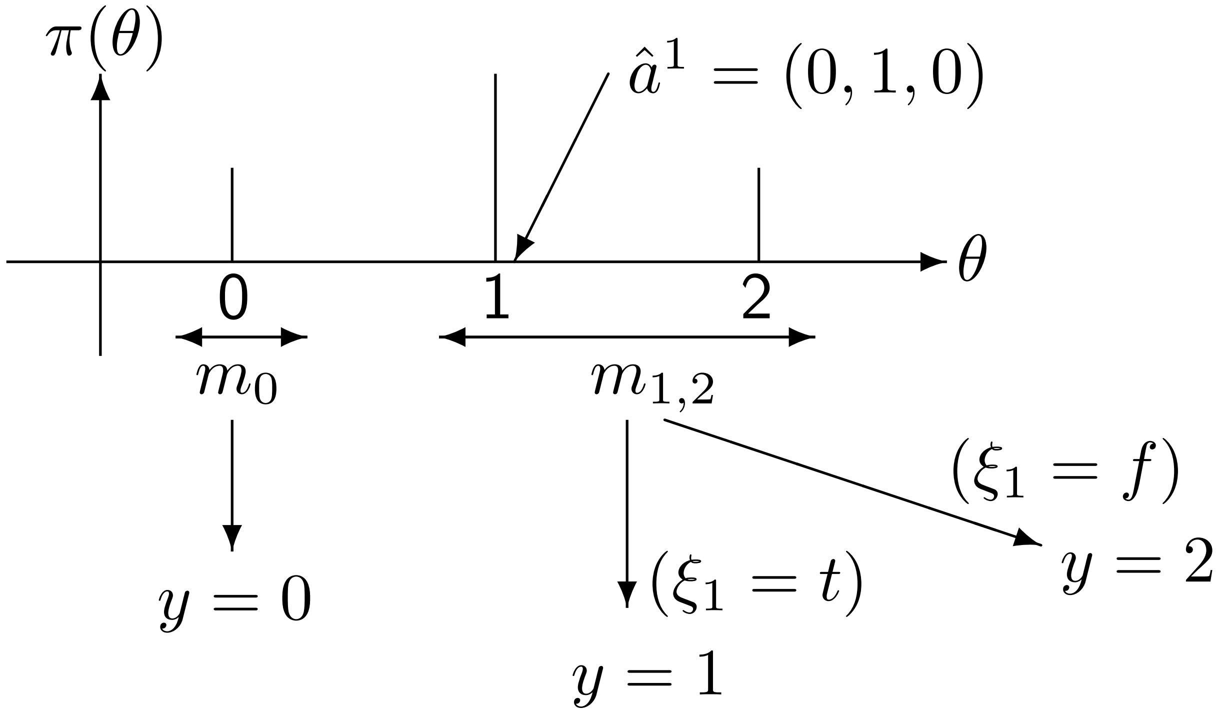
- In order to avoid confusion, $\hat{\mu}^0$ is a pure strategy where S sends m^0 if $\theta = 0$ and he sends $m^{1,2}$ if $\theta = \{1, 2\}$.
- We show that there is an equilibrium where the sender follows $\hat{\mu}^0$. In this equilibrium, D spends all her time on investigating whether state 1 is realized or not, that is, $\hat{a}^1 = (0, 1, 0)$.

Proposition 2 There exists a partially separating equilibrium, $(\hat{\mu}^0, (\hat{a}^1, \hat{\rho}), \hat{\beta})$, if and only if

$$q > c, \text{ and} \tag{1}$$

$$\eta \geq \max\left\{\frac{1}{2} - \frac{q-c}{1-q}, \frac{6b-5}{2(3-2b)}\right\}. \tag{2}$$

- Equilibrium



5.3 Proof. Step 1: Optimal project for D at stage 4.

Lemma 1 Given $\hat{\mu}^0$ and $(a, \xi, m^{1,2})$, the optimal project for the decision maker satisfies that

$$\text{if} \quad \frac{\Pr(\xi_1|a_1,1)\Pr(\xi_2|a_2,1)}{\Pr(\xi_1|a_1,2)\Pr(\xi_2|a_2,2)} \geq \frac{\pi(2)}{\pi(1)}, \quad \text{then } y = 1,$$

$$\text{if} \quad \frac{\Pr(\xi_1|a_1,1)\Pr(\xi_2|a_2,1)}{\Pr(\xi_1|a_1,2)\Pr(\xi_2|a_2,2)} \leq \frac{\pi(2)}{\pi(1)}, \quad \text{then } y = 2.$$

- After receiving $m_0 \Rightarrow y = 0$
- After choosing $\hat{a} = (0, 1, 0)$ and receiving $m_{1,2}$, (On the equilibrium path)
  - if $\eta \geq 1/2 - \pi(2)/(\pi(1) + \pi(2))$, then

$$
\begin{aligned}
y &= 1 \quad \text{if } \xi_1 = t \\
y &= 2 \quad \text{if } \xi_1 = f.
\end{aligned}
$$

  - Otherwise,

$$
\begin{aligned}
y &= 1 \quad \text{if } \xi_1 = t \\
y &= 1 \quad \text{if } \xi_1 = f.
\end{aligned}
$$

Now, we suppose that
$\eta \geq 1/2 - (q - c)/(1 - q) = 1/2 - (\pi(2) - c)/(\pi(1) + \pi(2))$.
It is satisfied that $\eta \geq 1/2 - \pi(2)/(\pi(1) + \pi(2))$.

5.4 Proof. Step 2: S's IC at stage 3

- S has no incentive to deviate when $\theta = 1$ or 2.
- S's type 0 has no incentive to deviate if and only if

$$E[u^S(\hat{\rho}(\hat{a}, \xi, m^0), \theta, b)|\theta = 0] \geq E[u^S(\hat{\rho}(\hat{a}, \xi, m^{1,2}), \theta, b)|\theta = 0]$$

- $\Leftrightarrow$

$$-b^2 \geq \alpha[-(1-b)^2] + (1-\alpha)[-(2-b)^2]$$

  where $\alpha \equiv \Pr(\xi_1 = t|\hat{a}, \theta = 0) = 1/2 - \eta$ and
  $1 - \alpha \equiv \Pr(\xi_1 = f|\hat{a}, \theta = 0) = 1/2 + \eta$.

-

$$-b^2 \geq \alpha[-(1-b)^2] + (1-\alpha)[-(2-b)^2] \quad \text{iff} \quad \eta \geq \frac{6b-5}{2(3-2b)}.$$

- Note that if $b < 5/6$, then $\eta > 0 > \frac{6b-5}{2(3-2b)}$.

5.5 Proof. Step 3: Incentive of Investigation

$$\hat{a} \in \arg\max_{a \in A} \sum_{\theta \in \Theta} \pi(\theta) \left[\sum_{(\xi, m) \in \Xi \times M} \Pr(\xi | a, \theta) \hat{\mu}(m|\theta) U^D(\hat{\rho}(a, \xi, m), a, \theta) \right].$$

$$\Rightarrow \hat{a} \in \arg\max_{a \in A} \pi(0) \times 0$$

$$- \sum_{\theta \in \{1,2\}} \pi(\theta) \left[\sum_{(\xi_1, \xi_2)} \Pr(\xi_1, \xi_2 | (a_1, a_2), \theta) \{\hat{\rho}(a, \xi, m) - \theta\}^2 \right] - c \sum_{i}^{3} a_i$$

- If $\eta \geq 1/2 - (q - c)/(1 - q)$, then $\hat{a} = (0, 1, 0)$.
- If $\eta < 1/2 - (q - c)/(1 - q)$, then $\hat{a} = (0, 0, 0)$.

- $\eta < 1/2 - \pi(2)/(\pi(1 + \pi(2))) \Rightarrow$ (expected) MR is negative wrt a_1 since D always chooses $y = 1$ after receiving $m_{1,2}$.

- $\eta \geq 1/2 - \pi(2)/(\pi(1 + \pi(2))) \Rightarrow$ MR is positive wrt a_1.

- $\eta \geq 1/2 - \pi(2)/(\pi(1 + \pi(2))) + c/(\pi(1) + \pi(2)) \Rightarrow \mathrm{MR} - MC$ is positive wrt a_1.

- Now, we suppose that $\eta \geq 1/2 - \pi(2)/(\pi(1 + \pi(2))) + c/(\pi(1) + \pi(2))$.

- $\hat{a} = (0, 1, 0)$ is optimal for D.

5.6 Discussion: Incentive of Investigation

Under single person decision (In the case where S sends babbling message)

- $(a_0, a_1, a_2) = (0, 1, 0)$ is never optimal for D under single person decision problem.
- $\xi_1 = t \Rightarrow$ D believes that "Probably, $\theta = 1$." $\Rightarrow$ D chooses 1.
- $\xi_1 = f \Rightarrow$ D believes that "Probably, $\theta \neq 1$." $\Rightarrow$ D chooses 1.
- No matter how small c is (no matter how large η is), D never chooses $(0, 1, 0)$.
- When c is small enough, the optimal investigation for decision maker is $(1, 0, 0)$ or $(0, 0, 1)$.

In our model

- If S follows above message strategy (he reveals "$\theta = 0$" or "$\theta \neq 0$"), the investigation vector $(a_0, a_1, a_2) = (0, 1, 0)$ is optimal for D.

5.7 Discussion : Equilibrium where $\hat{a}^2 = (0, 0, 1)$

Proposition 3 There exists a partially separating equilibrium, $(\hat{\mu}^0, (\hat{a}^2, \hat{\rho}), \hat{\beta})$, if and only if

$$\frac{2q+1-3c}{2(1-c)} > b \text{ and } \eta \in \left[\frac{1}{2} - \frac{q-c}{1-q}, \frac{5-6b}{2(3-2b)}\right]. \tag{3}$$

- S's type 0 has no incentive to deviate if and only if

$$-b^2 \geq \gamma[-(1-b)^2] + (1-\gamma)[-(2-b)^2] \Leftrightarrow \eta < \frac{5-6b}{2(3-2b)}$$

  where $1 - \gamma \equiv \Pr(\xi_2 = t | \hat{a}, \theta = 0) = 1/2 - \eta$.

- Equilibrium in Prop 2

$$-b^2 \geq \alpha[-(1-b)^2] + (1-\alpha)[-(2-b)^2]$$

  where $1 - \alpha \equiv \Pr(\xi_1 = f | \hat{a}, \theta = 0) = 1/2 + \eta$.

5.8 Discussion : $q \approx 1/3$, $c \approx 0$, and $b > 5/6$

- $$\eta \geq \max\left\{\frac{1}{2} - \frac{q-c}{1-q}, \frac{6b-5}{2(3-2b)}\right\} = \frac{1}{2} - \frac{q-c}{1-q} \approx 0.$$

- If η is small enough, then the D's equilibrium payoff in Prop 2 is almost equal to $-1/3$.

- When D cannot acquire information, the D's equilibrium payoff is almost equal to $-2/3$.

- The information acquisition provides little additional information about the state for the decision maker. Nevertheless, the decision makers ex ante expected payoff under the equilibrium where she acquire additional information differs vastly from the ex ante expected payoff when she cannot acquire additional information:
$-1/3 - (-2/3) = 1/3$.

5.9 Concluding Remarks

- The present paper analyzed a situation where the decision maker can acquire costly information about the states, and showed that the information acquisition enhances communication.
- It played an essential role that the information acquisition is multi-dimensional.
- Concentrating the information acquisition on a particular state enhanced communication.