Core Convergence with Differential Information

Tomoyuki Kamo
Kyoto Sangyo University
kamo@cc.kyoto-su.ac.jp

This version: November 2011

Abstract

We investigate the extension of the core convergence principle in economies with differential information. The paper focuses on the coarse core of Wilson (1978, Econometrica, vol.48, 807-816). We introduce *a generalized constrained market (GM) equilibrium* as a new price equilibrium concept. We establish several limit theorems on the coarse core for the GM equilibrium. We discuss the relationship between our positive results and the negative result reported by Serrano, Vohra, and Volij (2001, Econometrica, vol.69, 1685-1696).

Keywords: Coarse core, Core convergence, Nonstandard analysis
JEL classification: D51, D82, C71

1 Introduction

Since the classic work of Edgeworth [13], there have been many researches on core convergence in perfectly competitive economies without information asymmetries.[1] According to notable contributions in the literature, we may conclude that under some regularity conditions, the core "converges" to the set of competitive allocations when the number of agents becomes larger, which we call *the core convergence principle*:

> The Core Convergence Principle: Any core allocation is approximately competitive for economies with a sufficiently large number of agents.

In recent years, it has been a major focus on the literature in general equilibrium theory whether or not it is possible to extend the core convergence principle to the case that the agents may have private informations.[2]

The present paper investigates the extension of the core convergence principle in economies with differential information. The literarure contains several concepts of core

[1] Anderson [7] provides the comprehensive survey for this topic.

[2] Notable contributions are de Clippel [11], Forges et al. [14], McLean and Postlewaite [23], and Serrano et al. [27]. Forges et al. [16] provide the excellent survey for this topic.

with incomplete information.[3] Among them, this paper focuses on the *coarse core* defined by Wilson [30], which is the notion of the core at the interim stage when no private information is pooled among members of a coalition and incentive constraints are not relevant.[4] No convergence result for the coarse core has been established yet.[5] We provide the two types of limit theorems on the coarse core. The first is the Debreu-Scarf type limit theorem: Any allocation in the coarse core which survives replication is supported as a price equilibrium (Theorem 1). Furthermore, we establish the equivalence result under certain conditons (Theorem 2). The second is the extension of the "Strong Core Theorem" in Anderson [5]: In a large economy with finite types of agents, any coarse core allocation is approximately decentralized by the price, i.e., the core consumptions are near the competitive demands at some price (Theorem 3).

To clarify the position of this paper in the literature, we shall briefly review the background for our research. In his seminal paper [30], Wilson proposed the coarse core as a concept of the core at the interim stage, and the *constrained market (CM) equilibrium* as a corresponding price equilibrium notion. While Wilson himself pointed out that the coarse core contains the CM equilibrium allocations, it has been unsolved for many years whether or not the Debreu-Scarf argument still holds in this framework. Serrano et al.[27] provided the first resolution of this problem. Their answer was negative: They showed, by a simple example, that in the replica sequence of economies, there exists the allocation in the coarse core that cannot be supported as a CM equilibrium.

In spite of their negative result, we claim that the core convergence principle is still valid for the coarse core. A key idea of our results is a price equilibrium notion. We propose another equilibrium notion, a *generalized constrained market (GM) equilibrium*, which is a generalization of the CM equilibrium. In the present paper, we demonstrate that it is possible to recover the core convergence principle if we adopt the GM equilibrium as a price equilibrium concept.[6] A careful examination of the counter-example in [27] suggests the reason for the failure of core convergence: The budget constraints in the CM equilibrium is too stringent for the agent to afford the core allocation at the support price. On the other hand, since budget constraints in the GM equilibrium are more flexible than in the CM equilibrium, the budget-feasibility of the core allocation does not matter for the GM equilibrium. This is an essence of our convergence result.

The difficulty in establishing the convergence in this framework lies in the fact that a standard core theory cannot be applied directly to the coarse core, because a "blocking" concept in the definition of the coarse core is based on a common knowledge argument.

[3] For example, de Clippel [11], Lee and Volij [24], Vohra [28]. Volij [29],Wilson [30], and Yannelis [31] provided the notions of the core at the interim stage, respectively. See also Forges et al. [16].

[4] When incentive constraints are relevant, the corresponding core may be empty. See Forges et al. [15] and Vohra [28].

[5] de Clippel [11] showed that the *subset* of the coarse core, the type-agent core, converges to the constrained market equilibrium allocations under some conditions.

[6] Serrano et.al [27] pointed out that their non-convergence result is also valid for a broader class of price equilibrium concepts including the CM equilibrium. Of course, our GM equilibrium does not come under their category.

We use some tricks to overcome it. We transform an economy with differential information to another economy without information asymmetries, which we call an *auxiliary economy*. Since the auxiliary economy is regarded as the Arrow-Debreu economy with symmetric information, we can naturally define a core and price equilibrium of this economy. It is crucial that the standard core theory works in the auxiliary economy. We establish the limit theorems on the coarse core through the core convergence in the auxiliary economy. The core convergence in the auxiliary economy, however, is not immediate consequences of Anderson [5] or Debreu and Scarf [12], because the preference of the agent may not be continuous. We need a careful treatment about this problem to establish the results.

As the final remak, it is important to point out that our limit theorems have an implication about the case that the incentive compatibility constraints are relevant. Our formulation of a sequence of economies makes the private informations of all agents non-exnclusive[7] in the sense of Postlewaite and Schmeidler [26] when the number of agents are large, which means that the incentive constraints become irrelevant. Hence, our theorems also imply the convergence of the *incentive-compatible coarse core* defined by Vohra [28].

The rest of the paper is organized as follows: The next section is devoted to the description of the economy and the definitions of the fundamental concepts. In Section 3, we introduce our price equilibrium notion. Our convergence results are stated in Section 4 and proved in Section 5. In Section 6, we illustrate our results and discuss the relationship to Serrano et al.[27]. We provide concluding remarks in the last section.

2 Preliminaries

We begin with some notations and definitions. A cardinality of a set S is denoted by $|S|$. Let $x, y \in \mathbb{R}^n$ and $B \subset \mathbb{R}^n$. $x \geq y$ means $x_i \geq y_i$ for all i; $x > y$ means $x \geq y$ and $x \neq y$; $x \gg y$ means $x_i > y_i$ for all i; $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n \mid x \geq 0\}$; $\mathbb{R}^n_\oplus := \{x \in \mathbb{R}^n \mid x > 0\}$; $\mathbb{R}^n_{++} := \{x \in \mathbb{R}^n \mid x \gg 0\}$; $\|x\|_\infty := \max_{1\leq i\leq n}\{x_i\}$; $\|x\| := \sum_{i=1}^n |x_i|$; $\Delta^n := \{x \in \mathbb{R}^n \mid \|x\| = 1\}$; $\Delta^n_+ := \Delta^n \cap \mathbb{R}^n_+$; $\Delta^n_{++} := \Delta^n \cap \mathbb{R}^n_{++}$; $\rho(x, B) := \inf\{\|x - y\| \mid y \in B\}$; $\rho(x, B) = \infty$ if $B = \emptyset$.

Let Ω be a finite set of states where $|\Omega| = m$. An *event* is a subset of Ω. We represent a private information of an agent as a partition $\mathscr{P}$ of Ω. Let $\mathbf{P}(\omega)$ be the element of $\mathscr{P}$ that contains ω. $\mathscr{I}$ is a collection of partitions of Ω. The *meet* of the collection of partitions $\{\mathscr{P}_k\}_{k\in K}$ is the finest partition of Ω that is coarser than $\mathscr{P}_k$ for all $k \in K$, and it is denoted by $\bigwedge_{k\in K} \mathscr{P}_k$.

There are ℓ goods available at each state. A *consumption at state* ω is denoted by $x_\omega = (x_{1,\omega}, \dots, x_{\ell,\omega})$. A *(state-contingent) consumption plan* is a vector of consumptions at every state, which is denoted by x: $x = (x_\omega)_{\omega\in\Omega}$. A set of consumption plans is $X = \mathbb{R}^L_+$ where $L = \ell \times m$. Let us define $X_\oplus := \mathbb{R}^\ell_\oplus \times \cdots \times \mathbb{R}^\ell_\oplus$, i.e., m times direct

[7]Equivalently, the *informational size* of every agent becomes zero in the sense of Mclean and Postlewaite [22].

product of $\mathbb{R}^\ell_\oplus$. $X_\oplus$ is the set of consumption plans such that the consumption at every state is nonzero. We assume that there is a complete set of state-contingent contracts and incentive constraints on contracts are not relevant.The set of prices for state-contingent contracts is Δ^L_+. Given $x \in X$ and $E \subset \Omega$, x_E is a projection of x on E: $x_E = (x_\omega)_{\omega \in E}$. The set of x_E is denoted by X_E.

A *(state-dependent) utility function* is a mapping $u : \mathbb{R}^\ell_+ \times \Omega \to \mathbb{R}$. We assume that $u(\cdot, \omega)$ is continuous and strictly increasing[8] for every $\omega \in \Omega$. A *(subjective) prior probability distribution on* Ω is $\pi = (\pi(\omega))_{\omega \in \Omega}$. We assume $\pi \in \Delta^m_{++}$: $\pi(\omega) > 0$ for all $\omega \in \Omega$. Let $\mathscr{U}$ be a set of the pair (u, π) satisfying the assumptions. Given the pair $(u, \pi) \in \mathscr{U}$ and the event $E \subset \Omega$, an *expected utility of* $x \in X$ *conditional on* E is defined as

$$U(x|E) = \sum_{\omega \in E} \pi(\omega|E) \cdot u(x_\omega, \omega),$$

where $\pi(\omega|E)$ is the probability of ω conditional on E: $\pi(\omega|E) = \pi(\omega) / \sum_{\omega \in E} \pi(\omega)$. We identify a pair $(u, \pi) \in \mathscr{U}$ with a conditional expected utility $U(\cdot|\cdot)$ induced by (u, π).

Definition 1. An *exchange economy* is a map $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ where A is a finite set of agents. We define the informational partition $\mathscr{P}_a$, (expected) utility U_a, and endowment $e(a)$ of agent a by $(\mathscr{P}_a, U_a, e(a)) = \chi(a)$.

Throughout this paper, we assume that any exchange economy χ satisfies:

$$\textbf{(A1)}: \quad \bigwedge_{a \in A} \mathscr{P}_a = \{\Omega\};$$

$$\textbf{(A2)}: \quad \sum_{a \in A} e(a) \gg 0.$$

We assume **(A1)** only for simplicity: In the case that $\bigwedge_{a \in A} \mathscr{P}_a = \{\Omega_1, \ldots, \Omega_n\}$, we can establish the similar results by applying our arguments to every subeconomy where the state space is $\Omega_i \subset \Omega$ $(i = 1, \ldots, n)$. **(A2)** guarantees a positive income for some agent for any price: For any $p \in \Delta^L_+$, there exists $a \in A$ such that $p \cdot e(a) > 0$.

An *allocation* is a map $f : A \to X_\oplus$ such that $\sum_{a \in A} f(a) = \sum_{a \in A} e(a)$. A *coalition* is a nonempty subset of A. An *allocation for a coalition* S is a map $f : S \to X_\oplus$ such that $\sum_{a \in S} f(a) = \sum_{a \in S} e(a)$ and $f_\omega(a) \neq 0$ for all $\omega \in \Omega$ and $a \in S$. An allocation f is *interim efficient* if there exists no allocation g such that for every $a \in A$, $U_a(g(a)|P) \geq U_a(f(a)|P)$ for all $P \in \mathscr{P}_a$ with at least one strict inequality. An allocation f is *interim individually rational* if $U_a(f(a)|P) \geq U_a(e(a)|P)$ for any $a \in A$ and $P \in \mathscr{P}_a$.

Definition 2. Let $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ be an exchange economy. A coalition S has a *coarse objection* to an allocation f if there exist an allocation g for S and an event $E \in \bigwedge_{a \in S} \mathscr{P}_a$ such that

$$U_a(g(a)|P) > U_a(f(a)|P) \text{ for any } a \in S \text{ and } P \in \mathscr{P}_a \text{ satisfying } P \subset E. \tag{1}$$

[8]A real valued function $f : \mathbb{R}^n_+ \to \mathbb{R}$ is *strictly increasing* (or *incereasing*) if $x < y$ (resp. $x \ll y$) implies $f(x) < f(y)$.

The *coarse core* of χ is the set of allocations to which no coalition has a coarse objection, and it is denoted by $\mathscr{CC}(\chi)$.

Remark 1. An event E is *common knowledge for a coalition S at state ω* if $\mathbf{P}_a(\omega) \subset E$ for all $a \in S$. A coalition S has a coarse objection to some allocation f if and only if it is common knowledge among the members of S at some state that f is improved upon by S. This means that the agreement of the coalition members to the coarse objection involves no leakage of private information among them.[9]

To establish the non-emptiness of the coarse core, Wilson [30] considered another economy for which each agent a is split into a number of *subagents*, reindexed as (a, P) for $P \in \mathscr{P}_a$. The subagent (a, P) is assumed to have the utility $U_a(x|P)$ and the initial endowments $e_P(a)$. Wilson proved the non-emptiness of the coarse core by showing that the corresponding NTU game is balanced. A constrained market equilibrium is regarded as a price equilibrium concept associated with this economy.

Definition 3. Let $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ be an exchange economy. A *constrained market (CM) equilibrium of χ* is a pair (p, f) of the price and allocation such that $\sum_{\omega \in P} p_\omega \cdot f_\omega(a) \leq \sum_{\omega \in P} p_\omega \cdot e_\omega(a)$ and $U_a(x|P) > U_a(f(a)|P) \Rightarrow \sum_{\omega \in P} p_\omega \cdot x_\omega > \sum_{\omega \in P} p_\omega \cdot e_\omega(a)$ for any $a \in A$ and any $P \in \mathscr{P}_a$.

It is easy to show that the coarse core contains the CM equilibrium allocations. However, Serrano et al.[27] showed that the coarse core fails to converge to a set of CM equilibrium allocations. In order to recover the core convergence principle, therefore, we have to find another price equilibrium notion.

3 Generalization of CM Equilibrium

In this section, we introduce a generalized version of CM equilibrium. Recall that in the CM equilibrium, each subagent (a, P) is assigned to the income $\beta_a(P) := \sum_{\omega \in P} p_\omega \cdot e_\omega(a)$ from the total income $p \cdot e(a)$ and maximizes her utility $U_a(\cdot|P)$ subject to the budget constraint $\sum_{\omega \in P} p_\omega \cdot x_\omega \leq \beta_a(P)$. Thus, the CM equilibrium can be regarded as a price equilibrium concept under some budget-sharing rule. Here, we define a *budget-sharing rule* as a collection of mappings $\beta := \{\beta_a\}_{a \in A}$ where $\beta_a : \Delta^L_+ \times \mathscr{P}_a \to \mathbb{R}_+$ and $\sum_{E \in \mathscr{P}_a} \beta_a(p, E) = p \cdot e(a)$ for any $p \in \Delta^L_+$.

Given an exchange economy χ and a budget-sharing rule β, for $p \in \Delta^L_+$ and $a \in A$, we define the *demand set* as follows:

$$D(p, a; \beta_a) := \left\{ x \in X \;\middle|\; \forall P \in \mathscr{P}_a : \begin{array}{l} \sum_{\omega \in P} p_\omega \cdot x_\omega \leq \beta_a(p, P) \\ U_a(y|P) > U_a(x|P) \Rightarrow \sum_{\omega \in P} p_\omega \cdot y_\omega > \beta_a(p, P) \end{array} \right\}.$$

Definition 4. Let $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ be an exchange economy. A pair (p, f) of the price and allocation is a *generalized constrained market (GM) equilibrium of χ* if there exists a budget-sharing rule β such that for any $a \in A$,

[9] The common knowledge interpretation of the coarse core is due to Kobayashi [21].

(i) $f(a) \in D(p, a; \beta_a)$; and

(ii) $\beta_a(p, P) \geq \inf\left\{\sum_{\omega \in P} p_\omega \cdot x_\omega \;\middle|\; U_a(x|P) \geq U_a(e(a)|P)\right\}$ for any $P \in \mathscr{P}_a$.

A set of GM equilibrium allocations is denoted by $\mathscr{M}(\chi)$.

Remark 2. Condition (ii) guarantees the interim individual rationality of $f \in \mathscr{M}(\chi)$.

It is important to point out that the first welfare theorem holds under our equilibrium concept: *Any GM equilibrium allocation is interim efficient.* The proof is routine, so we omit it.

4 Core Convergence Theorems

We will state our limit theorems on the coarse core. All the proofs will be provided in the next section.

We begin with the case of the replica sequences. For any exchange economy $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$, the *n-th replica of* χ is the exchange economy $\chi_n : A_n \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ such that $A_n = A \times \{1, \ldots, n\}$, $\mathscr{P}_{(a,i)} = \mathscr{P}_a$, $U_{(a,i)} = U_a$, and $e(a, i) = e(a)$ for any $(a, i) \in A \times \{1, \ldots, n\}$. For any allocation f in χ, f^n is the n-th replication of f: $f^n(a, i) = f(a)$ for $(a, i) \in A_n$. Obviously, f^n is an allocation in χ_n.

Our first convergence result is the Debreu-Scarf type limit theorem. The next theorem asserts that any coarse core allocation which survives replication is a GM equilibrium allocation.

Theorem 1. *Let $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ be an exchange economy, $\chi_n : A_n \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ be the n-th replica of χ, and f be an allocation in χ. If $f^n \in \mathscr{CC}(\chi_n)$ for all $n \geq 1$, then $f \in \mathscr{M}(\chi)$.*

The converse of Theorem 1 is unfortunately not true, because the GM equilibrium allocation does not belong to the coarse core in general. We illustrate this fact in the following example.

Example 1. Let $\Omega = \{\omega_1, \omega_2\}$, $A = \{a, b, c\}$, and $\ell = 2$. The exchange economy χ is defined as follows: The every agent has an identical state-indepent utility function $u(x_1, x_2) = \sqrt[4]{x_1 \cdot x_2}$. The common prior distribution is $\pi(\omega_1) = \pi(\omega_2) = 1/2$. The information structure and initial endowments are given in the table below.

- $\mathscr{P}_a = \mathscr{P}_b = \{\{\omega_1\}, \{\omega_2\}\}$ and $\mathscr{P}_c = \{\Omega\}$;
- $e(a) = ((3, 0), (1, 0))$, $e(b) = ((0, 3), (0, 1))$, and $e(c) = ((0, 0), (2, 2))$.

Notice that $\mathscr{P}_a \wedge \mathscr{P}_b = \{\{\omega_1\}, \{\omega_2\}\} \neq \{\Omega\}$ and $e(a) \neq e(b)$. Consider the allocation f with no risk:

$$f(a) = f(b) = f(c) = ((1, 1), (1, 1)).$$

If we settle the price $p = (1/4, 1/4, 1/4, 1/4)$, then (p, f) is a GM equilibrium under the budget-sharing rule $\beta_a(p, \omega_i) = p \cdot e(a)/2$ and $\beta_b(p, \omega_i) = p \cdot e(b)/2$ for $i = 1, 2$. However,

f does not belong to $\mathscr{CC}(\chi)$, because the coalition $\{a, b\}$ has the coarse objection g at the common knowledge event $\{\omega_1\}$. where $g(a) = ((1.5, 1.5), (1, 0))$ and $g(b) = ((1.5, 1.5), (0, 1))$. □

Example 1 demonstrates that the GM equilibrium allocation is possibly "blocked" by some coalition if the members have "similar" fine informations in the sense that some proper subset of Ω can be common knowledge among them. In Example 1, agents a and b have an identical fine information, so that they take advantage of their information, even without communications, to block the GM allocation through reallocation by themselves. The GM equilibrium is not immune to this kind of objection unless it is a CM equilibrium.

Based on this observation, we provide a sufficient condition for the coarse core to contain all GM equilibrium allocations. In the next theorem, we recover the equivalence result of Debreu and Scarf [12] under some additional conditions.

Theorem 2. *Let $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ be an exchange economy and T be an image of χ where $T = \chi(A) = \{(\mathscr{P}_t, U_t, e(t))\}_{t \in T}$. Suppose that*

(i) $U_t(\cdot|P)$ is concave for all $P \in \mathscr{P}_t$ and $t \in T$,

(ii) $\mathscr{P}_t \wedge \mathscr{P}_{t'} = \{\Omega\}$ for any distinct $t, t' \in T$.

Then, $f \in \mathscr{M}(\chi)$ if and only if $f^n \in \mathscr{CC}(\chi_n)$ for all $n \geq 1$ where χ_n be the n-th replica of χ.

Notice that both (i) and (ii) concern with the image set T, which are independent of the number of agents. That is, if χ satisfies (i) and (ii), then any replica χ_n ($n \geq 2$) also does so. Differently from Brown and Robinson [10] and Hildenbrand [19],[10] our eqivalence result requires the convexity condition (i). Loosely speaking, (ii) means that the private informations of agents with different characteristics are so "different" among each other that only the whole state space Ω can be common knowledge for any coalition. For example, (ii) is satisfied if $\Omega = \{\omega_1, \dots, \omega_m\}$, $T = \{(\mathscr{P}_1, U_1, e(1)), \dots, (\mathscr{P}_m, U_m, e(m))\}$, and $\mathscr{P}_1 = \{\{\omega_1\}, \{\omega_2, \dots, \omega_m\}\}, \dots, \mathscr{P}_m = \{\{\omega_m\}, \{\omega_1, \dots, \omega_{m-1}\}\}$.[11] Although (ii) is indeed stringent, Theorem 2 still covers many examples in the literatures. In particular, we would like to stress that Theorem 2 holds for the counter-example of Serrano et al.[27].[12]

As Serrano et.al.[27] remarked, it is important to notice that the *equal treatment property*, the basic ingredient in the Debreu-Scarf argument, does *no*t hold for the coarse core.[13] In this sense, Theorems 1 and 2 may be insufficient for a characterization of the asymptotic behavior of $\mathscr{CC}(\chi_n)$. This fact motivates a more general convergence result.

[10] See Theorem 4 (p.253) of Brown and Robinson [10] and Corollary 2 (Chapter 3, p.201) of Hildenbrand [19].

[11] In the case that $n = 3$, this corresponds to Example 2 of Wilson [30].

[12] See also Section 6.

[13] We will discuss this issue in Section 6.

We shall establish the strong convergence result for the type sequences of economies, including the replica sequences. A *type sequence of economies* is a sequence of exchange economies $\chi_n : A_n \to T$ where T is a finite subset of $\mathscr{I} \times \mathscr{U} \times \mathbb{R}^L_+$ satisfying **(A1)** and **(A2)**: $\bigwedge_{t\in T} \mathscr{P}_t = \{\Omega\}$ and $\sum_{t\in T} e(t) \gg 0$. The next theorem is the extension of Theorem 3.3 of Anderson [5] to the coarse core:[14] Any coarse core consumptions are approximately GM-demands in the large economy.

Theorem 3. *Let $\chi_n : A_n \to T$ be a type sequence of economies satisfying*

(i) $|A_n| \to \infty$*;*

(ii) $\inf_n \dfrac{|\chi_n^{-1}(t)|}{|A_n|} > 0$ *for any* $t \in T$*.*

Then,

$$\lim_{n\to\infty} \sup_{f\in\mathscr{CC}(\chi_n)} \inf_{\substack{p\in\Delta^L_+ \\ \beta}} \frac{1}{|A_n|} \sum_{a\in A_n} \rho(f(a), D(p, a; \beta_a)) = 0.$$

The analogue of Theorem 2 holds for the type sequence of economies: If T satisfies (i) and (ii) in Theorem 2, then $\mathscr{M}(\chi_n) \subset \mathscr{CC}(\chi_n)$ for all $n \geq 1$. The proof is essentially same as the case of Theorem 2.

5 Proof of Theorems

In this section, we prove our main results, Theorems 1, 2, and 3. We shall provide the overview of our proof. At first, we introduce the concept of *auxiliary economy* associated with an original economy, and define the core and price equilibrium of the auxiliary economy. Secondly, we establish the relationship between the coarse core and GM equilibria of the original economy and the core and equilibria of the auxiliary economy. Finally, we prove the theorems via the results in the auxiliary economy. As we pointed out in the Introduction, it is crucial in our proof that a standard core convergence arguments can be applied to the core and equilibria of the auxiliary economy. We use nonstandard analysis to prove Theorems 1 and 3.

5.1 Auxiliary Economy

Given $(\mathscr{P}, U) \in \mathscr{I} \times \mathscr{U}$, a vector of conditional expected utilities of $x \in X$ is

$$U(x|\mathscr{P}) = (U(x|P))_{P\in\mathscr{P}}.$$

We define a binary relation $\succsim$ on X as follows:

$$\forall x, y \in X : \; x \succsim y \Longleftrightarrow U(x|\mathscr{P}) \geq U(y|\mathscr{P}).$$

Let $\succ$ be an asymmetric part of $\succsim$. A *preference* is a binary relation $\succsim$ on X induced by some $(\mathscr{P}, U) \in \mathscr{I} \times \mathscr{U}$ in this manner. The set of preferences is denoted by $\mathcal{P}$.

[14] See also Proposition 4 (Chapter 3, p.200) of Hildenbrand [19].

It is obvious that $\succsim$ is transitive and strongly monotonic. Note that $\succsim$ is *not* continuous in general: $\{(x, y) \in X \times X \mid x \succ y\}$ may *not* be open in $X \times X$ even if $u(\cdot, \omega)$ is continuous for all $\omega \in \Omega$. Note also that strict concavity of $U(\cdot|P)$ does *not* imply strong convexity of $\succ$,[15] because of incompleteness of $\succsim$.

Definition 5. Let $\chi : A \to \mathscr{I} \times \mathscr{U} \times \mathbb{R}_+^L$ be an exchange economy where $\chi(a) = (\mathscr{P}_a, U_a, e(a))$ for all $a \in A$. The *auxiliary economy* associated with χ is the mapping $\widetilde{\chi} : A \to \mathcal{P} \times \mathbb{R}_+^L$ such that for any $a \in A$, (i) $\succsim_a$ is induced by $(\mathscr{P}_a, U_a)$ and (ii) $\widetilde{e}(a) = e(a)$, where $\widetilde{\chi}(a) = (\succsim_a, \widetilde{e}(a))$ for all $a \in A$.

Since an auxiliary economy is regarded as the Arrow-Debreu economy with symmetric information, we can naturally define the core and price equilibrium of this economy.

Definition 6. Let χ be an exchange economy and $\widetilde{\chi}$ be the associated auxiliary economy. An allocation f is *blocked* by a coalition S in $\widetilde{\chi}$ if there exists the allocation g for S such that $g(a) \succ_a f(a)$ for all $a \in S$. The *core* of $\widetilde{\chi}$ is the set of allocations which cannot be blocked by any coalition, and it is denoted by $\mathscr{C}(\widetilde{\chi})$.

For $p \in \Delta_+^L$ and $(\succsim, e) \in \mathcal{P} \times \mathbb{R}_+^L$, the *demand set* is

$$d(p, \succ, e) = \left\{ x \in X \;\middle|\; p \cdot x \leq p \cdot e \text{ and } y \succ x \Rightarrow p \cdot y > p \cdot e \right\}.$$

By abuse of notation, we let $d(p, a) = d(p, \succ_a, e(a))$ for $a \in A$.

Definition 7. Let χ be an exchange economy and $\widetilde{\chi}$ be the associated auxiliary economy. A *Walrasian equilibrium* of $\widetilde{\chi}$ is a pair (p, f) of the price and allocation such that $f(a) \in d(p, a)$ for any $a \in A$.

Remark 3. If f is a Walrasian alloation in $\widetilde{\chi}$, then $f \in \mathscr{C}(\widetilde{\chi})$. The proof is routine, so we omit it.

5.2 Preliminary Results

We begin with a property of the induced preference relation $\succsim$. As we have already remarked, $\succsim$ is not continuous. It is easy to check that the result in the next lemma holds if $\succsim$ is continuous. The lemma says that the same result still holds even in the absence of the continuity.

Lemma 1. *For any $x, y \in X$, if $x \succ y$, then there exists $z \in \mathbb{R}_+^L \setminus \{0\}$ such that $x - z \succ y$.*

Proof . See Appendix A. $\square$

The next lemma clarifies the relationship between the coarse core of the original economy and the core of the associated auxiliary economy: The coarse core is a subset of the core of the auxiliary economy.

[15] A binary relation $\succ$ on X is *strongly convex* if for any $x, y \in X, x \neq y$, either $\frac{x+y}{2} \succ x$ or $\frac{x+y}{2} \succ y$ holds.

Lemma 2. *$\mathscr{C}\mathscr{C}(\chi) \subset \mathscr{C}(\widetilde{\chi})$ for any exchange economy χ.*

Proof . See Appendix A. $\square$

The next lemma states that the GM equilibrium in the original economy is almost equivalent to the Walrasian equilibrium in the associated auxiliary economy.

Lemma 3. *Let χ be an exchange economy and $p \in \Delta^L_{++}$. For any $a \in A$, $x \in D(p, a; \beta_a)$ for some β_a if and only if $x \in d(p, a)$. In particular, (p, f) is a GM equilibrium of χ if and only if (p, f) is a Walrasian equilibrium of $\widetilde{\chi}$ and f is interim individually rational.*

Proof . See Appendix A. $\square$

By Lemmas 2 and 3, we see that the core convergence in the original economy is derived from that in the auxiliary economy. Therefore, our main task is to establish the core convergence in the auxiliary economy.

For $p \in \Delta^L_+$, $x, e \in \mathbb{R}^L_+$, and $\succsim \in \mathcal{P}$, we define the *demand gap* ϕ as follows:

$$\phi(p, x, \succsim, e) := |p \cdot (x - e)| + |\inf\{p \cdot (y - e) \mid y \succ x\}|.$$

Roughly speaking, if $\phi(p, x, \succsim, e)$ is small, then x is "near" a *quasi-demand* at $(p, \succsim, e)$; $p \cdot x \leq p \cdot e$ and $y \succ x \Rightarrow p \cdot y \geq p \cdot e$. The following lemma on the demand gap is due to Anderson [2]. The lemma claims that there is the upper bound for the total demand gap of the core allocations in the auxiliary economy.

Lemma 4. *Let χ be an exchange economy and $\widetilde{\chi}$ be the associated auxiliary economy. If $f \in \mathscr{C}(\widetilde{\chi})$, then there exists $p \in \Delta^L_+$ such that $\sum_{a \in A} \phi(p, f, a) \leq 4L \max_{a \in A} \|e(a)\|_\infty$ where $\phi(p, f, a) = \phi(p, f(a), \succsim_a, e(a))$.*

Proof . Since the preference $\succsim$ is transitive and monotonic, the proof in Anderson [2] also works in our setting. We provide the formal proof in Appendix A for the reader's convinience. $\square$

The proofs of Theorems 1 and 3 are based on nonstandard analysis,[16] especially Loeb measure theory developed by Anderson [1], [4], and Loeb [25], while Theorem 2 is proved in a standard way. For the readers unfamiliar with nonstandard analysis, we shall sketch the proofs informally. At first, notice that the upper bound $4L \max_{a \in A_n} \|e(a)\|_\infty$ in Lemma 4 is independent of index n for any type sequence of economies χ_n. Hence, the *average* of the demand gaps, $\frac{1}{|A_n|} \sum_{a \in A_n} \phi(p, f, a)$, is "infinitesimal" when $|A_n|$ is "infinite". This means that $\phi(p, f, a)$ is "infinitesimal" for "almost all" $a \in A_n$. Next, if $\phi(p, f, a)$ is "infinitesimal" and $p \gg 0$, then $f(a)$ is "near" the Walrasian demand $d(p, a)$.[17] Since we can show $p \gg 0$, we see that the core allocation f is approximately Walrasian. Combined with Lemmas 2 and 3, we obtain the desired results.

The readers who are not interested in technical arguments may skip the following formal proofs to Section 6.

[16] Brown and Robinson [9], [10] are pioneering works in mathematical economcs. Anderson [6] is the excellent introduction to nonstandard analysis with applications to economics.

[17] We use Lemma 1 to prove this statement.

5.3 Proof of Theorem 1

Let f be an allocation in χ. Suppose that $f^n \in \mathscr{C}\mathscr{C}(\chi_n)$ for all n. Then, Lemma 2 implies that $f^n \in \mathscr{C}(\widetilde{\chi}_n)$ for all n. By the Transfer Principle, $f^n \in {}^*\mathscr{C}(\widetilde{\chi}_n)$ for $n \in {}^*\mathbb{N} \setminus \mathbb{N}$. Also, by Lemma 4 and the Transfer Principle, for $n \in {}^*\mathbb{N} \setminus \mathbb{N}$, there exists $p \in {}^*\Delta^L_+$ such that $(1/|A_n|) \sum_{\alpha \in A_n} {}^*\phi(p, f^n, \alpha) \simeq 0$, because $\max_{\alpha \in A_n} \|e(\alpha)\|_\infty = \max_{a \in A} \|e(a)\|_\infty$ is finite. Since both $f^n(\alpha)$ and $e(\alpha)$ are finite, ${}^*\phi({}^\circ p, f^n, \alpha) \simeq 0$ for $L(\nu)$-almost all $\alpha \in A_n$ where ν is the counting probability measure and $L(\nu)$ is the associated Loeb measure.[18]

We show that ${}^\circ p \gg 0$. Suppose not, i.e., ${}^\circ p_{\omega,i} = 0$ for some ω and i. It follows from **(A2)** that ${}^\circ p \cdot e(a) > 0$ for some $a \in A$. Let us define $B_n := \{a\} \times \{1, \ldots, n\}$, so $\nu(B_n) = 1/|A|$. Without loss of generality, ${}^*\phi({}^\circ p, f^n, \alpha) \simeq 0$ for all $\alpha \in B_n$. If ${}^\circ p \cdot f(a) = 0$, then ${}^\circ p \cdot f(a) < {}^\circ p \cdot e(a)$, which contradicts ${}^*\phi({}^\circ p, f^n, \alpha) \simeq 0$ for $\alpha \in B_n$. Therefore, ${}^\circ p_{\omega',j} > 0$ and $f_{\omega',j}(a) > 0$ for some ω' and j. For notational simplicity, we assume that ${}^\circ p_1 = {}^\circ p_{\omega,i} = 0$ and ${}^\circ p_2 = {}^\circ p_{\omega',j} > 0$ by reindexing the goods. If we set $x := f(a) + (1, 0, \ldots, 0)$, then $x \succ_\alpha f(a)$ and ${}^\circ p \cdot x = {}^\circ p \cdot f(a)$ for all $\alpha = (a, i) \in B_n$. Fix $\alpha \in B_n$ arbitrarily. Since ${}^*\phi({}^\circ p, f^n, \alpha) \simeq 0$, ${}^\circ p \cdot x = {}^\circ p \cdot e(a)$. Since $u_a(\cdot, \omega')$ is continuous and strictly incerasing, if we set $y = x - (0, \varepsilon, 0, \ldots, 0)$ for sufficiently small $\varepsilon > 0$, then $y \succ_\alpha f(a)$ and ${}^\circ p \cdot y < {}^\circ p \cdot e(a)$. That is contradiction.

We show that $({}^\circ p, f)$ is the Walrasian equilibrium in $\widetilde{\chi}$. Suppose not. Then, there exists $a \in A$ and $y \in \mathbb{R}^L_+$ such that $y \succ_a f(a)$ and ${}^\circ p \cdot y \leq {}^\circ p \cdot e(a)$. By Lemma 1, we may assume that ${}^\circ p \cdot y < {}^\circ p \cdot e(a)$. Therefore, there exists $\varepsilon > 0$ such that $\phi({}^\circ p, f(a), a) > \varepsilon$. For any $n \in \mathbb{N}$, there exists $C_n \subset A_n$ such that $C_n = \{a\} \times \{1, \ldots, n\}$ and $\phi({}^\circ p, f^n(\alpha), \alpha) > \varepsilon$ for all $\alpha \in C_n$. By the Transfer Principle, for $n \in {}^*\mathbb{N} \setminus \mathbb{N}$, ${}^*\phi({}^\circ p, f^n(\alpha), \alpha) > \varepsilon$ for $\alpha \in C_n$ and $\nu(C_n) = 1/|A|$. That is contradiction.

Finally, Lemma 3 implies that $({}^\circ p, f)$ is a GM equilibrium in χ, so we obtain the desired result.

5.4 Proof of Theorem 2

The "if" part is a direct consequence of Theorem 1.

For the converse, we will show that $\mathscr{M}(\chi) \subset \mathscr{C}\mathscr{C}(\chi)$ under (i) and (ii). This implies that $\mathscr{M}(\chi_n) \subset \mathscr{C}\mathscr{C}(\chi_n)$ for $n \geq 1$, because χ_n also satisfies (i) and (ii) if χ does so.

Suppose that a coalition S has a coarse objection to an allocation $f \in \mathscr{M}(\chi)$. Then, there exist the event $E \in \bigwedge_{a \in S} \mathscr{P}_a$ and the allocation g for S satisfying (1).

If S contains several types of agents, i.e., there exist $a, b \in S$ such that $\chi(a) \neq \chi(b)$, then (ii) implies $\bigwedge_{a \in S} \mathscr{P}_a = \{\Omega\}$. Since (1) implies $g(a) \succ_a f(a)$ for all $a \in S$, f is blocked by S in $\widetilde{\chi}$. On the other hand, It follows from Lemma 3 and Remark 3 that $f \in \mathscr{C}(\widetilde{\chi})$. That is contradiction.

If S consists of single type of agents, i.e., $S \subset \chi^{-1}(t)$ for some $t \in T$, then $E \in \mathscr{P}_t$ and

$$\sum_{a \in S} g(a) = \sum_{a \in S} e(a) = |S| \cdot e(t) \iff \frac{1}{|S|} \cdot \sum_{a \in S} g(a) = e(t).$$

[18] See Anderson [4], [6], Hurd and Loeb [20], and Loeb [25] for details on the Loeb measure.

Let $a^* \in \arg\min_{a\in S}\{U_t(g(a)|E)\}$. By (1) and (i),

$$U_{a^*}(e(a^*)|E) = U_t(e(t)|E) = U_t\left(\frac{1}{|S|}\cdot\sum_{a\in S} g(a) \;\middle|\; E\right) \geq U_t(g(a^*)|E) > U_{a^*}(f(a^*)|E).$$

Thus, f is not interim individually rational. That is contradiction.

5.5 Proof of Theorem 3

Fix $p \in \Delta^L_{++}$ and $(\succsim, e) \in T$ arbitrarily.[19] We show that

$$\forall\varepsilon > 0\ \exists\delta > 0\ \forall x \in \mathbb{R}^L_+ : \phi(x, p, \succsim, e) < \delta \Longrightarrow \rho(x, d(p, \succsim, e)) < \varepsilon. \tag{2}$$

Suppose not. Then there exist some $\varepsilon > 0$ and a sequence $\{x^{(n)}\} \subset \mathbb{R}^L_+$ such that $\phi(x^{(n)}, p, \succsim, e) \to 0$ and $\rho(x^{(n)}, d(p, \succsim, e)) \geq \varepsilon$. Since $\{x^{(n)}\}$ is bounded, we may assume that $x^{(n)} \to x$ for some $x \in \mathbb{R}^L_+$. Then $\phi(x, p, \succsim, e) = 0$, that is, $|p\cdot(x - e)| = 0$ and $\left|\inf\{p\cdot(y-e) \mid y \succ x\}\right| = 0$. It follows from $p \gg 0$ and Lemma 1 that $y \succ x \Rightarrow p\cdot y > p\cdot e$. Thus $x \in d(p, \succsim, e)$, which contradicts $\rho(x^{(n)}, d(p, \succsim, e)) \geq \varepsilon$ for all n.

By the transfer of (2), given $p \in {}^*\Delta^L_{++}$ and $(\succsim, e) \in {}^*T$, we obtain

$$\forall x \in {}^*\mathbb{R}^L_+ : {}^*\phi(x, p, \succsim, e) \simeq 0 \Longrightarrow {}^*\rho(x, {}^*d(p, \succsim, e)) \simeq 0. \tag{3}$$

Let $f_n \in \mathscr{CC}(\chi_n)$ be any sequence of the core allocations. By Lemma 2, $f_n \in \mathscr{C}(\widetilde{\chi}_n)$. We consider the hyperfinite economy ${}^*\widetilde{\chi}_n : A_n \to {}^*T$ for $n \in {}^*\mathbb{N}\setminus\mathbb{N}$. Note that in ${}^*\widetilde{\chi}_n$, $e(a)$ is finite for all $a \in A_n$ and e is S-integrable, because T is finite. By Lemma 4 and the Transfer Principle, there exists $p \in {}^*\Delta^L_+$ such that $(1/|A_n|)\sum_{a\in A_n} {}^*\phi(p, f_n(a), a) \simeq 0$. Since $(1/|A_n|)\sum_{a\in A_n} f_n(a) = (1/|A_n|)\sum_{a\in A_n} e(a)$, $f_n(a)$ is finite for $L(\nu)$-almost all $a \in A_n$. Thus, ${}^*\phi(p, f_n(a), a) \simeq 0$ for $L(\nu)$-almost all $a \in A_n$. By the similar argument in the proof of Theorem 1, we can show that ${}^\circ p \gg 0$. Therfore, (3) implies ${}^*\rho(f_n(a), d({}^\circ p, a)) \simeq 0$ for $L(\nu)$-almost all $a \in A_n$.

Since $\|x\| \leq \min\{1/{}^\circ p_1, \dots, 1/{}^\circ p_L\}\cdot\|e(a)\|$ for any $x \in d({}^\circ p, a)$, ${}^*\rho(f_n(a), d({}^\circ p, a)) \leq \|f_n(a)\| + \min\{1/{}^\circ p_1, \dots, 1/{}^\circ p_L\}\cdot\|e(a)\|$. Following the argument in the Appendix of Anderson [3], if e is S-integrable, then the core allocation f_n is also S-integrable. It follows from Corollary 5 in Anderson [1] that ${}^*\rho(f_n(a), d({}^\circ p, a))$ is S-integrable. Therefore,

$$\frac{1}{|A_n|}\sum_{a\in A_n} {}^*\rho(f_n(a), d({}^\circ p, a)) = \int_{A_n} {}^\circ({}^*\rho(f_n(a), d({}^\circ p, a)))dL(\nu) \simeq 0,$$

because the integrand is zero for $L(\nu)$-almost all $a \in A_n$.

Hence, given $\varepsilon \in \mathbb{R}_{++}$, for any $n \in {}^*\mathbb{N}\setminus\mathbb{N}$ and $f_n \in \mathscr{CC}(\chi_n)$, there exists $p \in \Delta^L_{++}$ such that

$$\frac{1}{|A_n|}\sum_{a\in A_n} {}^*\rho(f_n(a), d(p, a)) < \varepsilon.$$

By the Overspill Principle, for sufficiently large $n \in \mathbb{N}$,

$$\frac{1}{|A_n|}\sum_{a\in A_n} \rho(f_n(a), d(p, a)) < \varepsilon.$$

[19] Note that if $p \gg 0$, then $d(p, \succ, e) \neq \emptyset$.

By Lemma 3, there exists β such that

$$\frac{1}{|A_n|}\sum_{a\in A_n}\rho(f_n(a), D(p,a;\beta_a)) < \varepsilon.$$

Since ε is arbitrary, we complete the proof.

6 Discussion

Serrano et al.[27] is the most related research to this paper. We need to clarify the relationship between our results and theirs. The aim of this section is to examine the counter-example given by [27] in detail and to illustrate how our theory works well.

Example 2 ([27], p.1689)**.** Let $\Omega = \{\omega_1, \omega_2\}$, $\ell = 2$, and $A = \{a, b\}$. The exchange economy χ^{SVV}, which we call *the SVV economy*, is defined as allows:

- $\mathscr{P}_a = \{\{\omega_1\}, \{\omega_2\}\}$ and $\mathscr{P}_b = \{\{\omega_1, \omega_2\}\}$.
- $u_a(x_1, x_2, \omega) = u_b(x_1, x_2, \omega) = \sqrt[4]{x_1 \cdot x_2}$ for any $\omega \in \Omega$.
- $e_{\omega_1}(a) = e_{\omega_2}(a) = (24, 0)$ and $e_{\omega_1}(b) = e_{\omega_2}(b) = (0, 24)$.
- $\pi_a(\omega) = \pi_b(\omega) = \frac{1}{2}$ for any $\omega \in \Omega$.

Let us define the allocation g in χ and the price q as follows:

$$\begin{aligned} &g(a) = g(b) = ((12, 12), (12, 12)),\\ &q = (q_{1,\omega_1}, q_{2,\omega_1}, q_{1,\omega_2}, q_{2,\omega_2}) = (1/4, 1/4, 1/4, 1/4). \end{aligned}$$

Then, (q, g) is unique CM equilibrium of χ^{SVV}.

Consider the allocation $f^* : A \to \mathbb{R}^2_\oplus \times \mathbb{R}^2_\oplus$ in χ^{SVV} as follows:

$$\begin{aligned} &f^*(a) = (f^*_{\omega_1}(a), f^*_{\omega_2}(a)) = ((15, 15), (8, 8)),\\ &f^*(b) = (f^*_{\omega_1}(b), f^*_{\omega_2}(b)) = ((9, 9), (16, 16)). \end{aligned}$$

Then, the replicated allocation f^{*n} is in $\mathscr{CC}(\chi^{SVV}_n)$ for all n, but f^* is not supported as a CM equilibrium, because $f^* \neq g$. □

Let us begin with identifying $\mathscr{CC}(\chi^{SVV})$ and $\mathscr{M}(\chi^{SVV})$. Consider an allocation $f : A \to \mathbb{R}^2_\oplus \times \mathbb{R}^2_\oplus$ satisfying

$$\textbf{(S):}\ \ f_{1,\omega_i}(a) = f_{2,\omega_i}(a),\ \ f_{1,\omega_i}(b) = f_{2,\omega_i}(b) \quad \text{for } i = 1, 2.$$

For such f, we define $\widehat{f} : A \to \mathbb{R}^2_\oplus$ as follows: $\widehat{f} = (\widehat{f}_{\omega_1}, \widehat{f}_{\omega_2})$ and

$$\widehat{f}_{\omega_i}(a) := f_{1,\omega_i}(a) = f_{2,\omega_i}(a),\ \ \widehat{f}_{\omega_i}(b) := f_{1,\omega_i}(b) = f_{2,\omega_i}(b) \quad \text{for } i = 1, 2.$$

The set of all allocations satisfying **(S)** is denoted by $\widehat{F}$:

$$\widehat{F} = \big\{\widehat{f} : A \to \mathbb{R}^2_\oplus \ \big|\ \widehat{f}_{\omega_i}(a) + \widehat{f}_{\omega_i}(b) = 24 \text{ for } i = 1, 2\big\}.$$

Then, we can show that

(i) $\mathscr{CC}(\chi^{SVV})$ coincides with $\widehat{F}$, and

(ii) $\mathscr{M}(\chi^{SVV})$ is a one-dimensional curve in $\widehat{F}$.

The formal discussion and proofs about these facts will be provided in Appendix B.

The figure illustrates the observations described above. In the figure, $\widehat{F}$ is represented as the "Edgeworth Box". All the points in the square region are elements in $\mathscr{CC}(\chi^{SVV})$ while $\widehat{g}$ is the unique CM equilibrium allocation. $\mathscr{M}(\chi^{SVV})$ is represented as the curve (thick line) in the region. Clearly, the curve passes through the point $\widehat{g}$.

It is important to note that $\widehat{f^*}$ is on the curve, that is, $f^* \in \mathscr{M}(\chi^{SVV})$. Indeed, f^* is the GM equilibrium allocation under the price $p^* = (2/7, 2/7, 3/14, 3/14)$.[20] In the figure, the indifference curve of agent b through $\widehat{f^*}$ is tangent to the budget line under p^* while the demand set of agent a at p^* is the budget line itself. This fact is consistent with our theory: f^* survives replication, because $f^* \in \mathscr{M}(\chi^{SVV})$!

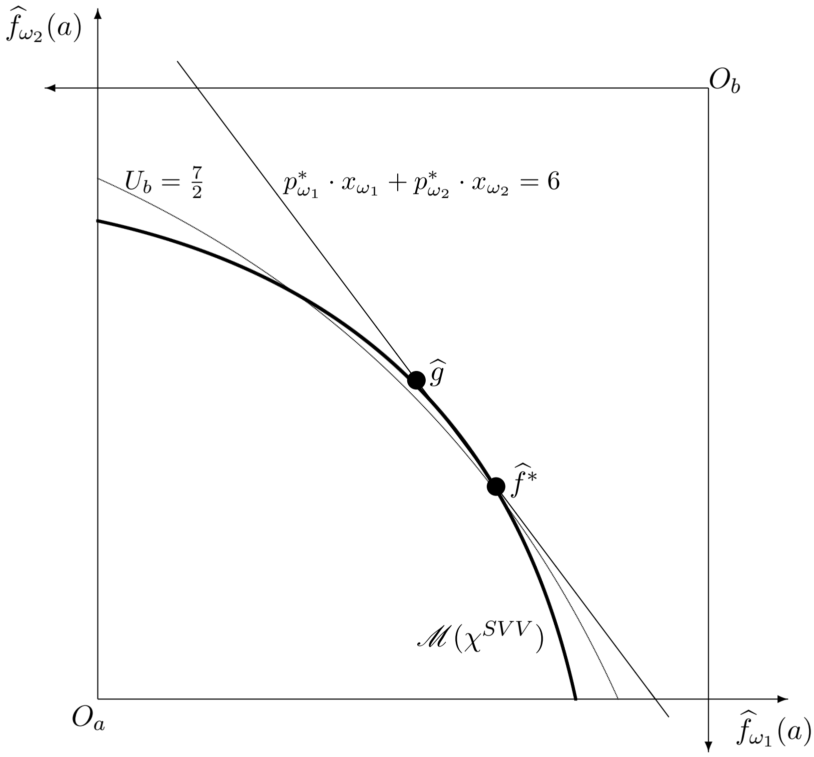

FIGURE: The SVV economy

The main reason for the non-convergence to a CM equilibrium is that the budget constraint in a CM equilibrium is too stringent to afford the core allocation at the supporting price: In particular, agent $(a, \{\omega_1\})$ does not afford the consumption $f^*_{\omega_1}(a)$ at p^*, i.e., $p^*_{\omega_1} \cdot f^*_{\omega_1}(a) = 60/7 > 48/7 = p^*_{\omega_1} \cdot e_{\omega_1}(a)$. This problem does not occur in the GM equilibrium, because there exists a budget-sharing rule under which $f^*_{\omega_1}(a)$ is the demand of agent (a, ω_1) at p^*.

It is important to note that the SVV economy satisfies the assumptions in Theorem 2. This means that the set of coarse core allocations which survive replication coincides

[20]See Appendix B for detail.

with $\mathscr{M}(\chi^{SVV})$. This fact demonstrates the relevance of our equilibrium concept: It seems necessary to consider the GM equilibrium allocations if we examine the limiting behavior of the coarse core.

Serrano et al.[27] also showed that the equal treatment property does not hold in the example (p.1692 of [27]). Let us explain this fact from our point of view. We can show that the coarse core of the SVV economy coincides with the core of the auxiliary economy: $\mathscr{CC}(\chi_n^{SVV}) = \mathscr{C}(\widetilde{\chi}_n^{SVV})$ for $n \geq 1$.[21] Therefore, we can restrict our attention to the auxiliary economy. As we pointed out in Section 5, the auxiliary economy is the Arrow-Debreu economy with symmetric information. Now, it is clear why the Debreu-Scarf argument does not work in $\widetilde{\chi}^{SVV}$: This is because *the preference* $\succsim_a$ *of agent a in* $\widetilde{\chi}^{SVV}$ *is not strongly convex.* As discussed in [12], it is obvious that the equal treatment property cannot be expected when the preference is not strongly convex.

Finally, notice that χ_n^{SVV} trivially satisfies the non-exclusivity condition of Postlewaite and Schmeidler [26] with $n \geq 2$. By Lemma 3.1 of Vohra [28], we can safely say that any allocation in $\mathscr{CC}(\chi_n^{SVV})$ is *incentive compatible*.[22] Our result, therefore, implies the convergence of the *incentive compatible coarse core* of Vohra [28].[23]

7 Concluding Remarks

In this paper, the core convergence principle was extended to economies with differential information. We established the several limit theorems on the coarse core. Furthermore, we provided the core-equivalence result for the replicated economies under certain conditions. As the final remark, we notice that the GM equilibrium allocation may not be in the coarse core. Although we cannot expect the inclusion in the general case, we may extend the core-equivalence reult of Aumann [8] within the framework of Theorem 2. This issue is left to the future work.

References

[1] Anderson, R. M. (1976): "A Non-standard Representation for a Brownian Motion and Ito Integration," *Israel Journal of Mathematics*, 25, 15-46.

[2] Anderson, R. M. (1978): "An Elementary Core Equivalence Theorem," *Econometrica*, 46, 1483-1487.

[3] Anderson, R. M. (1981): "Core Theory with Strongly Convex Preferences," *Econometrica*, 49, 1457-1468.

[4] Anderson, R. M. (1982): "Star-Finite Representations of Measure Spaces," *Transaction of the American Mathematical Society*, 271, 667-687.

[21] See Appendix B for details.

[22] We can transform any coarse core allocation into the incentive-compatible allocation, based solely on the private informations reported by agents, even though the true state is not observable or verifiable.

[23] Notice that the incentive compatible coarse core of χ_n^{SVV} is non-empty with $n \geq 2$.

[5] Anderson, R. M. (1985): "Strong Core Theorems with Nonconvex Preferences," *Econometrica*, 53, 1283-1294.

[6] Anderson, R. M. (1991): "Nonstandard Analysis with Applications to Economics," in *Handbook of Mathematical Economics (Vol. IV)*, ed. by W. Hildenbrand and H. Sonnenschein. Amsterdam; North Holland, Chapter 39.

[7] Anderson, R. M. (1992): "The Core in Perfectly Competitive Economies," in *Handbook of Game Theory with Economic Applications (Vol. I)*, ed. by R. J. Aumann and S. Hart. Amsterdam; North Holland, Chapter 14.

[8] Aumann, R. J. (1964): "Markets with a Continuum of Traders," *Econometrica*, 32, 39-50.

[9] Brown, D.J., and A. Robinson (1974): "Nonstandard Exchange Economies," *Econometrica*, 43, 41-55.

[10] Brown, D.J., and A. Robinson (1974): "The Cores of Large Standard Exchange Economies," *Journal of Economic Theory*, 9, 245-254.

[11] de Clippel, G., (2007): "The Type-Agent Core for Exchange Economies with Asymmetric Information," *Journal of Econmic Theory*, 135, 144-158.

[12] Debreu, G., and H. Scarf (1963): "A Limit Theorem on the Core of an Economy," *International Economic Review*, 4, 235-246.

[13] Edgeworth, F.Y. (1881): *Mathematical Psychic*. London: Kegan Paul

[14] Forges, F., A. Heifetz, and E. Minelli (2001): "Incentive Compatible Core and Competitive Equilibria in Differential Information Economies," *Economic Theory*, 18, 349-365.

[15] Forges, F., J-F. Mertens, and R. Vohra (2002): "Ex-Ante Incentive Compatible Core in the Absence of Wealth Effects," *Econometrica*, 70, 1865-1892.

[16] Forges, F., E. Minelli, and R. Vohra (2002): "Incentives and Core of an Exchange Economy: A Survey," *Journal of Mathematical Economics*, 38, 1-41.

[17] Gale, D., and A. Mas-Colell (1975): "An Equilibrium Existence Theorem for a General Model without Ordered Preferences," *Journal of Mathematical Economics*, 5, 9-15.

[18] Holmström, B., and R. Myerson (1983): "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica*, 51, 1799-1819.

[19] Hildenbrand, W. (1974): *Core and Equilibria of an Large Economy*, Princeton, Princeton University Press.

[20] Hurd, A., and P.A. Loeb (1985): *Introduction of Nonstandard Real Analysis*, New York, Academic Press.

[21] Kobayashi, T. (1980): "Equilibrium Contracts for Syndicates with Differential Information," *Econometrica*, 48, 1635-1665.

[22] McLean, R., and A. Postlewaite (2002):"Informational Size and Incentive Compatibility," *Econometrica*, 70, 2421-2453.

[23] McLean, R., and A. Postlewaite (2005):"Core Convergence with Asymmetric Information," *Games and Economic Behavior*, 50, 58-78.

[24] Lee, D., and O. Volij (2002): "The Core of Economies with Asymmetric Information: Axiomatic Approach," *Journal of Mathematical Economics*, 38, 43-63.

[25] Loeb, P.A. (1975): "Conversion from Nonstandard to Standard Measure Spaces and Applications in Probability Theory," *Transactions of the American Mathematical Society*, 211, 113-122.

[26] Postlewaite, A. and D. Schmeidler (1986):"Implementation in Differential Information Economies," *Journal of Economic Theory*, 39, 14-33.

[27] Serrano, R., R. Vohra, and O. Volij. (2001): "On the Failure of Core Convergence in Economies with Asymmetric Information" *Econometrica*, 69, 1685-1696.

[28] Vohra, R. (1999): "Incomplete Information, Incentive Compatibility, and the Core," *Journal of Economic Theory*, 86, 123-147.

[29] Volij, O. (2000): "Communication, Credible Improvement, and the Core of an Economy with Asymmetric Information," *International Journal of Game Theory*, 29, 63-79.

[30] Wilson, R. (1978): "Information, Efficiency, and the Core of an Economy," *Econometrica*, 46, 807-816.

[31] Yannelis, N. (1991): "The Core of an Economy with Differential Information," *Economic Theory*, 1, 183-198.

Appendix

A Proof of Lemmas

A.1 Proof of Lemma 1

If $x \succ y$, then $U(x|P) \geq U(y|P)$ for all $P \in \mathscr{P}$ with at least one strict inequality. Suppose that $U(x|P^*) > U(y|P^*)$ for some P^*. Since $U(\cdot|P^*)$ is continuous and strictly increasing on X_{P^*}, there exists $w \in X_{P^*}$ such that $w > 0$ and $U(x - w|P^*) > U(y|P^*)$. If we define $z = (z_\omega)_{\omega\in\Omega}$ as

$$z_\omega = \begin{cases} w_\omega & \text{if } \omega \in P^* \\ 0 & \text{otherwise,} \end{cases}$$

then we obtain the desired result.

A.2 Proof of Lemma 2

We show that if an allocation f is blocked by a coalition S, then S has a coarse objection to f. Suppose that f is blocked by S. Then, there exists the allocation g for S such that for any $a \in S$, $U_a(g(a)|P) \geq U_a(f(a)|P)$ for all $P \in \mathscr{P}_a$ with at least one strict inequality. Select a^* from S arbitrarily. Let P^* be the element of $\mathscr{P}_{a^*}$ such that $U_{a^*}(g(a^*)|P^*) > U_{a^*}(f(a^*)|P^*)$. Let E be the element of $\bigwedge_{a\in S} \mathscr{P}_a$ satisfying $P^* \subset E$. It suffices to show that there exists the allocation $\widetilde{g}$ for S such that

$$\forall \omega \in E \ \exists a_\omega \in S : U_{a_\omega}(\widetilde{g}(a_\omega)|\mathbf{P}_{a_\omega}(\omega)) > U_{a_\omega}(f(a_\omega)|\mathbf{P}_{a_\omega}(\omega)).$$

For if so, then all the member of S can be better off on E by the commodity-transfer from a_ω to all the other members at each $\omega \in E$.

We shall construct $\widetilde{g}$ by modifying g in the following manner. At the first step, we consider the event E_0 such that

$$E_0 := \{\omega \in E \mid \forall a \in S : U_a(g(a)|\mathbf{P}_a(\omega)) = U_a(f(a)|\mathbf{P}_a(\omega))\}.$$

Note that $E_0 \subsetneq E$, because $P^* \subset E \setminus E_0$. If $E_0 = \emptyset$, then we have done by setting $\widetilde{g} = g$. Suppose $E_0 \neq \emptyset$. Then, there exists $a_0 \in S$ and $P_0 \in \mathscr{P}_{a_0}$ such that $P_0 \not\subset E_0$ and $P_0 \cap E_0 \neq \emptyset$, because $E \in \bigwedge_{a\in S} \mathscr{P}_a$. By the definition of E_0, for any $\omega_0 \in P_0 \cap (E \setminus E_0)$, there is $b_0 \in S$ such that $U_{b_0}(g(b_0)|\mathbf{P}_{b_0}(\omega_0)) > U_{b_0}(f(b_0)|\mathbf{P}_{b_0}(\omega_0))$. If g^0 is the modification of g obtained by the sufficiently small transfer of goods from b_0 to a_0 at ω_0, then $U_{a_0}(g^0(a_0)|P_0) > U_{a_0}(f(a_0)|P_0)$. At the next step, we consider the event E_1 as follows:

$$E_1 := \{\omega \in E \mid \forall a \in S : U_a(g^0(a)|\mathbf{P}_a(\omega)) = U_a(f(a)|\mathbf{P}_a(\omega))\}.$$

It follows from the construction of g^0 that $E_1 \subsetneq E_0$. If $E_1 = \emptyset$, then we have done by setting $\widetilde{g} = g^0$. If $E_1 \neq \emptyset$, then we obtain the allocation g^1 for S by applying the similar arguments. By repeating this procedure, we obtain the decreasing sequence $E_0 \supsetneq E_1 \supsetneq \cdots$. Since E is finite, $E_n = \emptyset$ for some n. By setting $\widetilde{g} = g^{n-1}$, we obtain the desired result.

A.3 Proof of Lemma 3

Let $p \in \Delta^L_{++}$ and $a \in A$. Suppose $x \in D(p, a; \beta_a)$ for some β_a. It is obvious that $p \cdot x \leq p \cdot e(a)$. Suppose that $y \succ x$. Then, $U_a(y|P) \geq U(x|P)$ for all $P \in \mathscr{P}_a$ with at least one strict inequality. Since $x \in D(p, a; \beta_a)$, this implies that $\sum_{\omega \in P} p_\omega \cdot y_\omega \geq \beta_a(p, P)$ for all $P \in \mathscr{P}_a$ with at least one strict inequality. Therefore,

$$p \cdot y = \sum_{P \in \mathscr{P}_a} \sum_{\omega \in P} p_\omega \cdot y_\omega > \sum_{P \in \mathscr{P}_a} \beta_a(p, P) = p \cdot e(a),$$

which means $x \in d(p, a)$.

Conversely, Suppose $x \in d(p, a)$. Set the budget-sharing rule β as follows:

$$\beta_a(p, P) = \sum_{\omega \in P} p_\omega \cdot x_\omega \quad \text{for all } P \in \mathscr{P}_a.$$

Fix $P \in \mathscr{P}_a$ arbitrarily. Suppose that $U_a(y|P) > U_a(x|P)$ for some $y \in X$. If we consider another $\widetilde{y} \in X$ satisfying that

$$\widetilde{y}_\omega = \begin{cases} y_\omega & \text{if } \omega \in P, \\ x_\omega & \text{otherwise,} \end{cases}$$

then $\widetilde{y} \succ_a x$. Since $x \in d(p, a)$, $p \cdot \widetilde{y} > p \cdot e(a)$, which implies $\sum_{\omega \in P} p_\omega \cdot y_\omega > \beta_a(p, P)$. Therefore, we obtain $x \in D(p, a; \beta_a)$.

A.4 Proof of Lemma 4

We reproduce the proof of the lemma in Anderson [2]. For each $a \in A$, let $\phi(a) := \{x - e(a) \in \mathbb{R}^L \mid x \succ_a f(a)\}$, and define $\Phi := \sum_{a \in A} \phi(a)$. For notatinal simplicity, we write $M := \max_{a \in A} \|e(a)\|_\infty$.

At first, let us show that $\Phi \cap -\mathbb{R}^L_{++} = \emptyset$. Suppose $Z \in \Phi \cap -\mathbb{R}^L_{++}$. Then there exist $z(a) \in \phi(a)$ for every $a \in A$ such that $Z = \sum_{a \in A} z(a)$. Let us define $S := \{a \in A \mid z(a) \neq 0\}$ and

$$g(a) := z(a) + e(a) - \frac{1}{|S|} \cdot Z \text{ for all } a \in S.$$

Then $g(a) > z(a) + e(a)$ and

$$\sum_{a \in S} g(a) = \sum_{a \in S} z(a) + \sum_{a \in S} e(a) - Z = \sum_{a \in S} e(a).$$

Since $z(a) \in \phi(a)$ (and free disposal), $g(a) \succ_a f(a)$ for all $a \in S$. Hence S blocks f, which is contradiction.

Next, we will show that $\mathrm{co}\,\Phi \cap \{z \in \mathbb{R}^L | z_i < M, \forall i = 1, \ldots, L\} = \emptyset$. Suppose $Z \in \mathrm{co}\,\Phi \cap \{z \in \mathbb{R}^L | z_i < M, \forall i = 1, \ldots, L\}$. By Shapley-Folkman Theorem, we can write

$$Z = \sum_{i=1}^{k} z(a_i) + \sum_{a \notin \{a_1, \ldots, a_k\}} z(a)$$

where $z(a) \in \operatorname{co}\phi(a)$, $z(a) \in \phi(a)$ for $a \notin \{a_1 \ldots, a_k\}$, and $k \leq L$. Let us define $z'(a)$ for every $a \in A$ as follows;

$$z'(a) := \begin{cases} z(a) & \text{if } a \notin \{a_1 \ldots, a_k\} \\ 0 & \text{otherwise.} \end{cases}$$

Let $Z' := \sum_{a \in A} z'(a)$. Then $Z' \in \Phi$ and

$$Z' = Z - \sum_{i=1}^{k} z(a_i) \leq Z + \sum_{i=1}^{k} e(a_i) \leq Z + (M, \ldots, M) < 0.$$

That is contradiction.

By Minkowski's separation hyperplane theorem, there exists $p \in \Delta_+$ such that p separates $\operatorname{co}\Phi$, therefore Φ, from $\{z \in \mathbb{R}^L | z_i < M, \forall i = 1, \ldots, L\}$. Thus $\inf p \cdot \Phi \geq \sup\{p \cdot z | z_i < M, \forall i = 1, \ldots, L\} = -M$.

Finally, we will show that $\sum_{a \in A} |p \cdot (f(a) - e(a))| \leq 2M$ and $\sum_{a \in A} |\inf\{p \cdot z | z \in \phi(a)\}| \leq 2M$, which imply $\sum_{a \in A} \phi(p, f, a) < 4M$. Let $B := \{a \in A | p \cdot (f(a) - e(a)) < 0\}$. Then

$$\begin{aligned} &\sum_{a \in B} p \cdot (f(a) - e(a)) \geq \sum_{a \in A} \inf p \cdot \phi(a) \geq -M, \textit{ and} \\ &\sum_{h \in A} p \cdot (f(a) - e(a)) = 0. \end{aligned}$$

Therefore

$$\sum_{a \in A} |p \cdot (f(a) - e(a))| = 2 \sum_{a \in B} |p \cdot (f(a) - e(a))| \leq 2M$$

$$\begin{aligned} \sum_{a \in A} |\inf p \cdot \phi(a)| &\leq -\sum_{a \in A} \inf p \cdot \phi(a) + \sum_{a \notin B} \inf p \cdot (f(a) - e(a)) \\ &\leq M + M = 2M. \end{aligned}$$

This completes the proof.

B Core and Equilibria of the SVV Economy

This section supplements our arguments in Section 6. We provide a complete characterization of the coarse core and GM equilibrium allocations in the SVV economy.

We begin with the characterization of $\mathscr{C}\mathscr{C}(\chi^{SVV})$. Since there are only two agents in χ^{SVV}, the next proposition is straightforward.

Proposition B.1. *$f \in \mathscr{C}\mathscr{C}(\chi^{SVV})$ if and only if f is ex-post efficient and interim individually rational.*

Proof . Suppose that $f \in \mathscr{C}\mathscr{C}(\chi^{SVV})$. Clearly, f must be interim individually rational. If f is not ex-post efficient, then there exist $x : A \to \mathbb{R}^2_{\oplus}$ and $\omega_i \in \Omega$ such that

$u_a(x(a),\omega_i) > u_a(f_{\omega_i}(a),\omega)$, $u_b(x(b),\omega_i) > u_b(f_{\omega_i}(b),\omega_i)$, and $x(a)+x(b) = (24,24)$. Consider the allocation g as follows:

$$g_\omega(a) := \begin{cases} x(a) & \text{if } \omega = \omega_i \\ f_\omega(a) + z & \text{otherwise} \end{cases}, \quad \textit{and} \quad g_\omega(b) := \begin{cases} x(b) & \text{if } \omega = \omega_i \\ f_\omega(b) - z & \text{otherwise} \end{cases}$$

where $z \in \mathbb{R}^2_\oplus$ is a commodity-transfer from b to a. Then, g becomes a coarse objection to f for A when $\|z\|$ is sufficiently small. That is contradiction.

Conversely, if $f \notin \mathscr{C}\mathscr{C}(\chi^{SVV})$, then there exist a coalition $S \subset A$, an allocation g for S, and an event $E \in \bigwedge_{a\in S} \mathscr{P}_a$ satisfying (1). If $S = \{a\}$ or $\{b\}$, then f is not interim individually rational. If $S = A$, then $E = \Omega$. Since $U_b(g(b)|\Omega) > U_b(f(b)|\Omega)$ by (1), there exists $\omega \in \Omega$ such that $u_b(g_\omega(b),\omega) > u_b(f_\omega(b),\omega)$. It follows from this fact and (1) that f is not ex-post efficient at ω. $\square$

By Proposition B.1, we can explicitly calculate all allocations in $\mathscr{C}\mathscr{C}(\chi^{SVV})$. On the one hand, the ex-post efficiency in χ^{SVV} implies the symmetric consumption of goods at every state:

$$\textbf{(S)} \quad \forall \omega \in \Omega : f_{1,\omega}(a) = f_{2,\omega}(a), \quad f_{1,\omega}(b) = f_{2,\omega}(b).$$

On the other hand, the individual rationality gives no restriction on f in χ^{SVV}. Hence, we obtain the following result as a corollary.

Corollary B.1. *$\mathscr{C}\mathscr{C}(\chi^{SVV})$ coincides with a set of all allocations satisfying **(S)**.*

We shall provide a characterization of $\mathscr{M}(\chi^{SVV})$. The next proposition states that $\mathscr{M}(\chi^{SVV})$ is a "one-dimensional curve" in the feasible set.

Proposition B.2. *$f \in \mathscr{M}(\chi^{SVV})$ if and only if there exists $t \in \mathbb{R}$, $0.5 < t < 2$ such that*

$$\begin{aligned} f(a) &= ((24-12t, 24-12t), (24-12/t, 24-12t)) \\ f(b) &= ((12t, 12t), (12/t, 12/t)). \end{aligned} \tag{4}$$

Proof . Since $\mathscr{M}(\chi^{SVV}) \subset \mathscr{C}\mathscr{C}(\chi^{SVV})$ by Theorem 2, we can restrict our attention to the allocation satisfying **(S)**. Therefore, any GM equilibrium price $p \in \Delta^4_{++}$ in χ^{SVV} also satisfies the similar symmetric condition: $p_{1,\omega} = p_{2,\omega}$ for all $\omega \in \Omega$. The set of such prices is denoted by Δ_S. For any $p \in \Delta_S$, let us denote $\widehat{p}_\omega := p_{1,\omega} = p_{2,\omega}$ for $\omega \in \Omega$.

We shall derive the demand set of agent b, $D(p,b)$.[24] Recall that $D(p,b)$ is a solution of the maximization problem

$$\begin{aligned} \max \quad & \frac{1}{2}(\sqrt[4]{x_{1,\omega_1}\cdot x_{2,\omega_1}} + \sqrt[4]{x_{1,\omega_2}\cdot x_{2,\omega_2}}) \\ \text{sub.to} \quad & p_{1,\omega_1}\cdot x_{1,\omega_1} + p_{2,\omega_1}\cdot x_{2,\omega_1} + p_{1,\omega_2}\cdot x_{1,\omega_2} + p_{2,\omega_2}\cdot x_{2,\omega_2} \le p_{2,\omega_1}\cdot 24 + p_{2,\omega_2}\cdot 24, \\ & x_{1,\omega_1} \ge 0,\ x_{2,\omega_1} \ge 0,\ x_{1,\omega_2} \ge 0,\ x_{2,\omega_2} \ge 0. \end{aligned}$$

[24]Note that the budget sharing rule is not relevant for agent b.

For $p \in \Delta_S$, this problem is reduced to the following:

$$\begin{array}{ll} \max & \frac{1}{2}(\sqrt{\widehat{x}_{\omega_1}} + \sqrt{\widehat{x}_{\omega_2}}) \\ \text{sub.to} & \widehat{p}_{\omega_1} \cdot \widehat{x}_{\omega_1} + \widehat{p}_{\omega_2} \cdot \widehat{x}_{\omega_2} \leq 6,\ \widehat{x}_{\omega_1} \geq 0,\ \widehat{x}_{\omega_2} \geq 0, \end{array}$$

where $\widehat{x}_{\omega_i} := x_{1,\omega_i} = x_{2,\omega_i}$ for $i = 1, 2$. Hence, we can explicitly calculate $D(p, b)$ for $p \in \Delta_S$ as follows:

$$\begin{aligned} D_{1,\omega_1}(p, b) = D_{2,\omega_1}(p, b) = 12 \cdot \widehat{p}_{\omega_2}/\widehat{p}_{\omega_1}, \\ D_{1,\omega_2}(p, b) = D_{2,\omega_2}(p, b) = 12 \cdot \widehat{p}_{\omega_1}/\widehat{p}_{\omega_2}. \end{aligned} \tag{5}$$

Therefore, it is necessary for $f \in \mathscr{M}(\chi^{SVV})$ to satisfy (4).

It is easy to calculate the demand of agent a, because it is almost arbitrary: For any $p \in \Delta_S$ and any $(x_{\omega_1}, x_{\omega_2}) \in \mathbb{R}^2_{++}$ satisfying $\widehat{p}_{\omega_1} \cdot x_{\omega_1} + \widehat{p}_{\omega_2} \cdot x_{\omega_2} = 6$, there exists β_a such that $D(p, a; \beta_a) = ((x_{\omega_1}, x_{\omega_1}), (x_{\omega_2}, x_{\omega_2}))$. It is easy to check this fact by setting $\beta_a(p, \omega_i) = 2p_{\omega_i} \cdot x_{\omega_i}$ for $i = 1, 2$. So the proof is left to the reader.

We shall show that if an allocation f satisfies (4), then $f \in \mathscr{M}(\chi^{SVV})$. Let f be an allocation satisfying (4). For any $t \in \mathbb{R}_+$ satisfying $0.5 < t < 2$, there exists $p \in \Delta_S$ such that $t = \widehat{p}_{\omega_2}/\widehat{p}_{\omega_1}$. It follows form (5) that $f(b) = D(p, b)$. Since

$$p_{\omega_1} \cdot \left(24 - 12 \cdot \frac{\widehat{p}_{\omega_2}}{\widehat{p}_{\omega_1}}\right) + \widehat{p}_{\omega_2} \cdot \left(24 - 12 \cdot \frac{\widehat{p}_{\omega_1}}{\widehat{p}_{\omega_2}}\right) = 6,$$

$f(a) = D(p, a; \beta_a)$ for some β_a. □

It follows from Proposition B.2 that $f^* \in \mathscr{M}(\chi^{SVV})$, because f^* satisfies (4) when $t = 3/4$. Since $t = \widehat{p}_{\omega_2}/\widehat{p}_{\omega_1}$ and $\widehat{p}_{\omega_1} + \widehat{p}_{\omega_2} = 1/2$, $p_{\omega_1} = 2/7$ and $p_{\omega_2} = 3/14$. Thus, (p^*, f^*) is the GM equilibrium of χ^{SVV} where $p^* = (2/7, 2/7, 3/14, 3/14)$.

Finally, since χ_n^{SVV} satisfies the conditions in Theorem 2 for all $n \geq 1$, we obtain the following proposition. The proof is essentially same as Theorem 2.

Proposition B.3. *$\mathscr{CC}(\chi_n^{SVV}) = \mathscr{C}(\widetilde{\chi}_n^{SVV})$ for all $n \geq 1$.*

Proof . Suppose that a coalition $S \subset A_n$ has a coarse objection to an allocation f in χ_n^{SVV}. Then, there exists an allocation g for S and an event $E \in \bigwedge_{\alpha \in S} \mathscr{P}_\alpha$ satisfying (1). If S contains both types of agents, then $E = \Omega$, which means f is blocked by S through g. If S consists of single type of agents, then some agent $\alpha^* \in S$ prefers the average $\frac{1}{|S|}\sum_{\alpha \in S} g(\alpha)$ to $g(\alpha^*)$ conditional on E. On the other hand, since $\frac{1}{|S|}\sum_{\alpha \in S} g(\alpha)$ is either $e(a)$ or $e(b)$, $U_{\alpha^*}(f(\alpha^*)|E) < 0$. That is impossible, because f is an allocation, i.e., $f(\alpha) \in \mathbb{R}^4_+$ for all $\alpha \in A_n$. □

All allocations in χ_n^{SVV} are interim individually rational for all $n \geq 1$. By Lemma 3, Remark 3, and Proposition B.3, we see that the core convergence in the original economy is equivalent to that in the auxiliary economy.