Aggregating the single crossing property:

theory and applications to comparative statics and Bayesian games

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Extended Abstract: Let $(S, \geq)$ be a partially ordered set and f a real-valued function with $(S, \geq)$ as its domain. The function f is said to obey the *single crossing property* (alternatively, to be a *single crossing function*) if the following holds: $f(s') \geq (>)\, 0 \Longrightarrow f(s'') \geq (>)\, 0$, whenever $s'' > s'$. Let f and g be single crossing functions defined on $(S, \geq)$: under what conditions can we guarantee that $f + g$ is also a single crossing function? This paper provides a thorough examination of this issue and various other issues relating to the aggregative properties of single crossing functions.

The relevance of our results for economic theory arises from the fact that the single crossing property plays a crucial role in the study of comparative statics. In particular, some version of the property is sufficient and, in a sense, necessary to guarantee that the solution to an optimization problem is increasing with respect to certain parameters (see Milgrom and Shannon (1994)). However, it is not always the case that the property can be directly assumed or easily derived from appropriate primitive assumptions. Using our aggregation results, we provide, in a number of important settings, sufficient conditions under which the single crossing property holds.

For example, consider a Bayesian game where one may be interested in guaranteeing that a player's best response is increasing in the signal he receives. This holds if the agent's expected utility function, conditional on his signal, obeys a version of the single crossing property; in this paper, we identify necessary and sufficient primitive conditions for this to be true. An important class of Bayesian games are auctions; we identify primitive conditions under which an agent's bid is increasing in his signal. Our results extend the work of Reny and Zamir (2004) and Athey (2001) on the existence of monotone equilibria in first-price auctions.