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Incentives in Hedge Funds

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Hedge Fund as Delegated Portfolio Management

Investor (Unsophisticated) **1 Unit of Fund, No Withdrawal**

Manager **M Units of Personal Fund: Manage Investor's and Personal Funds**

'Separate Management' or 'Equity Stake'

Weak Regulation, Low Transparency

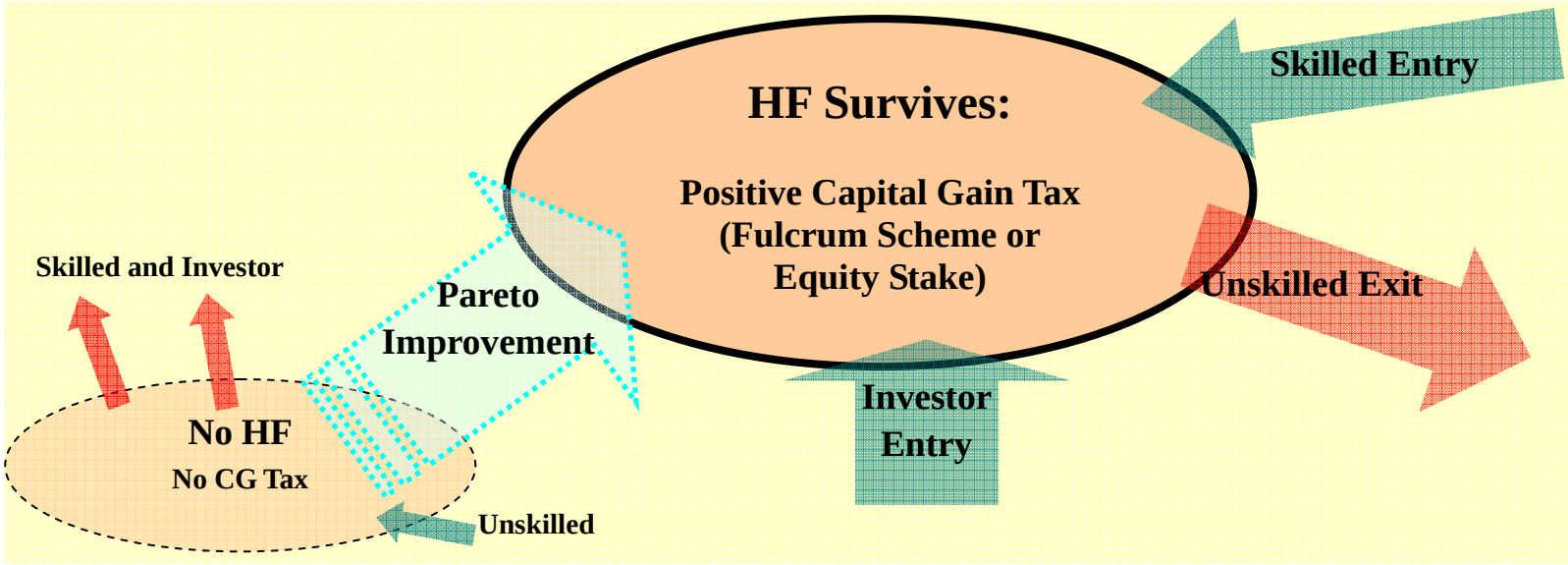
Generate Alpha

Manager
- **Skilled Type** — **Select Alpha (Action)** $a \in [0,\infty)$ **with Non-Pecuniary Cost** $C(a)$
- **Unskilled Type** — **Alpha 0**

Incentive Problem

Hidden Type Investor Cannot Identify whether Manager is skilled or not
Hidden Activity Investor Cannot Observe Manager's Activity

Q: Can We Solve Incentive Problem?

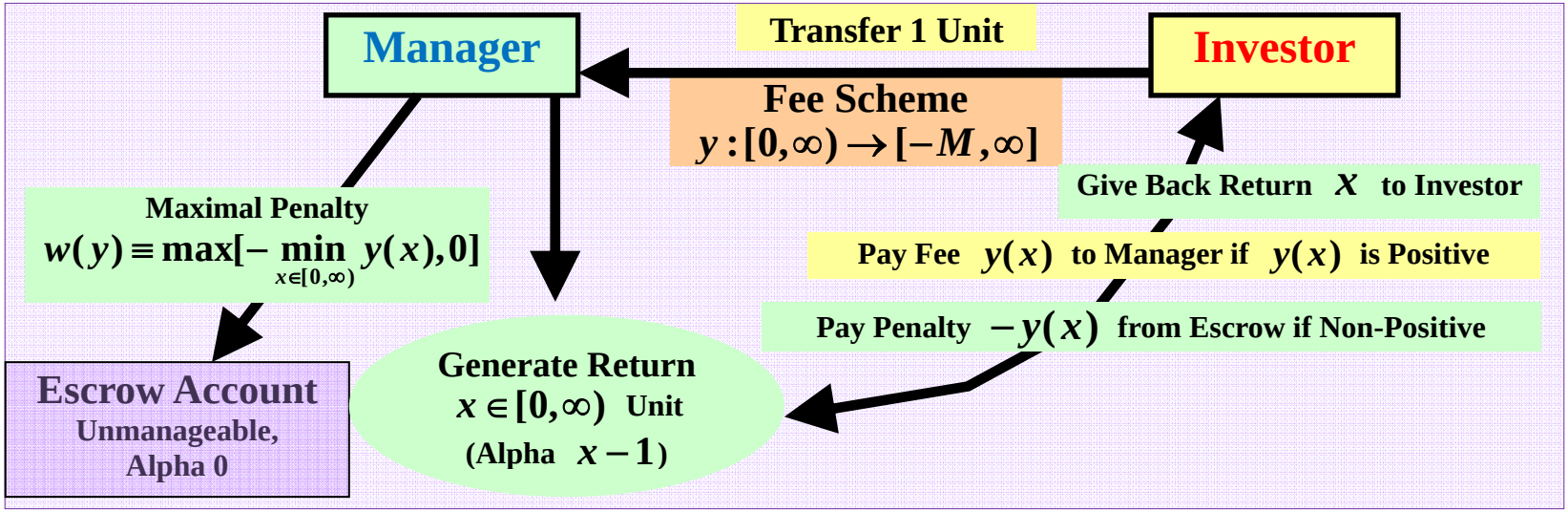


A: Yes, but We Need Capital Gain Tax!

Manager's Incentive Fee Scheme

$$y:[0,\infty)\to[-M,\infty],\quad y(x)\in[-M,\infty)$$

Return-Contingency, Penalty, Escrow for Solvency



Real Fee Scheme

‘2:20’ Scheme

Asymmetry, No Penalty, Convexity, High-Powered

$$y(x) = 0.2x + 0.02$$

Criticisms (Warren Buffet): **‘2:20’ Makes Manager More Risk-Taking by Side Contracting with Third Party. We Should Change ‘2:20’ Scheme to**

‘Fulcrum’ Scheme

Symmetric, Positive Penalty, Linear, Low-Powered

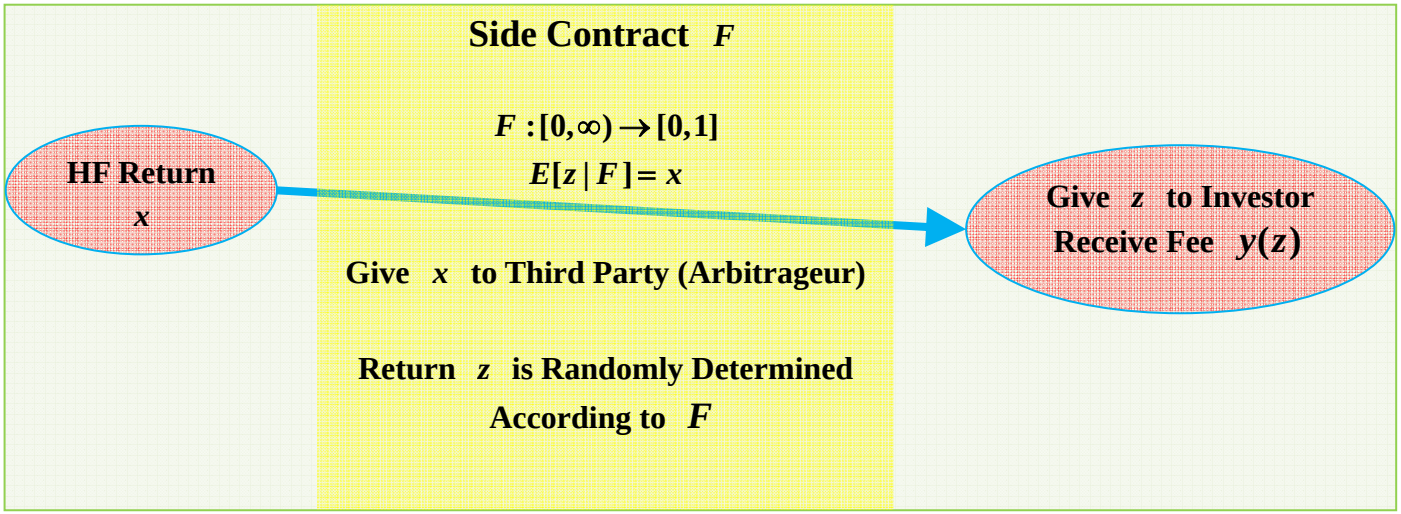
$$y(x) = k(x - 1)$$

Side Contracting: Performance Mimicry

Randomize Return

Cumulative Distribution $F:[0,\infty)\to[0,1]$

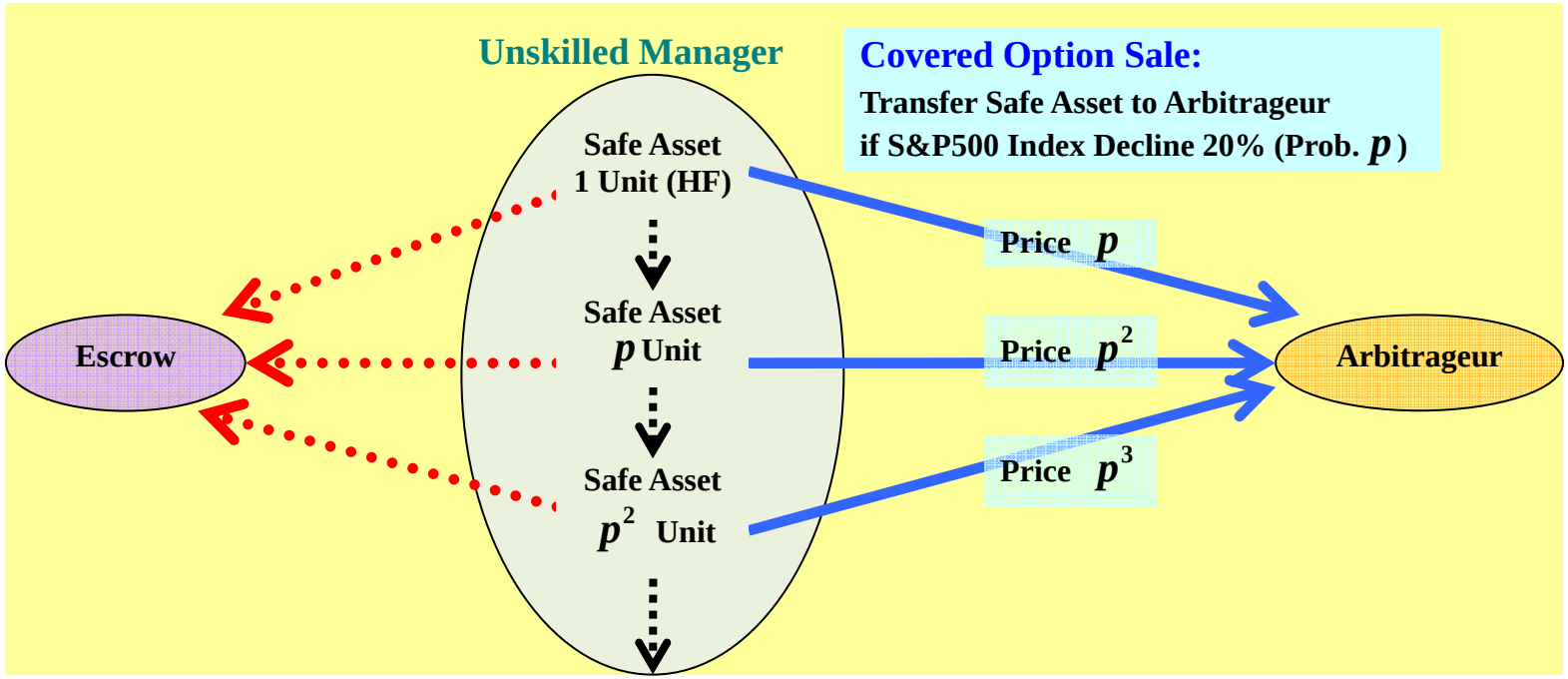
$$E[z\mid F]=x$$



Example (Lo (2001))

Capital Decimation Partners (CDP)

Unskilled Can Generate Alpha $\frac{p}{1-p} > 0$ **with Prob.** $1-p$



Previous Works: Hedge Fund Never Survives

Foster + Young (08/09) — **With No CG Tax, No Scheme Can Solve Incentive Problem**

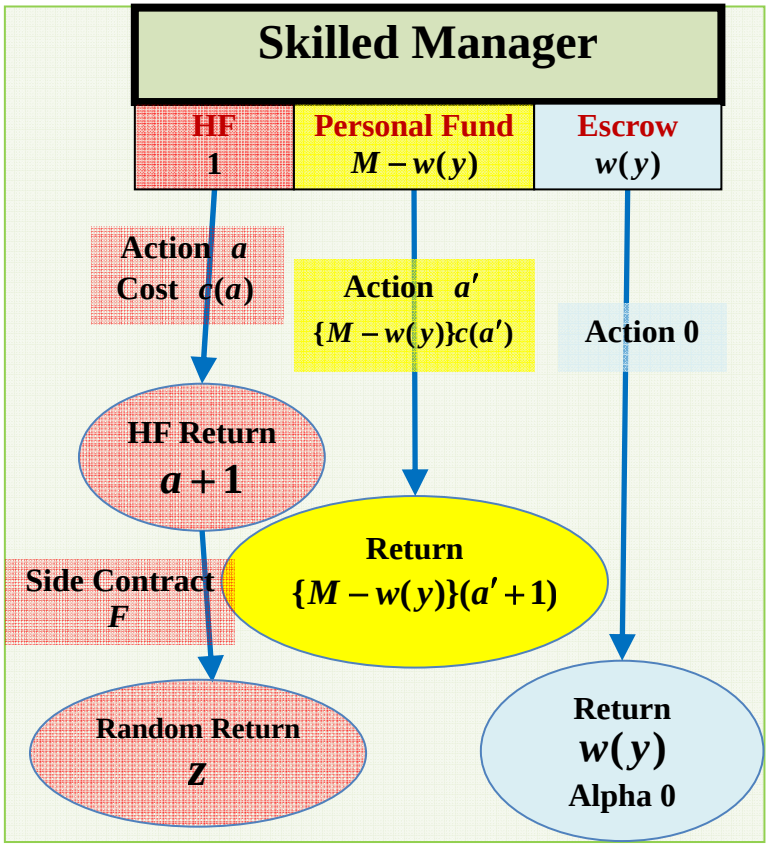
Medias: **FT (18/3/08), NYT (3/8/08)**

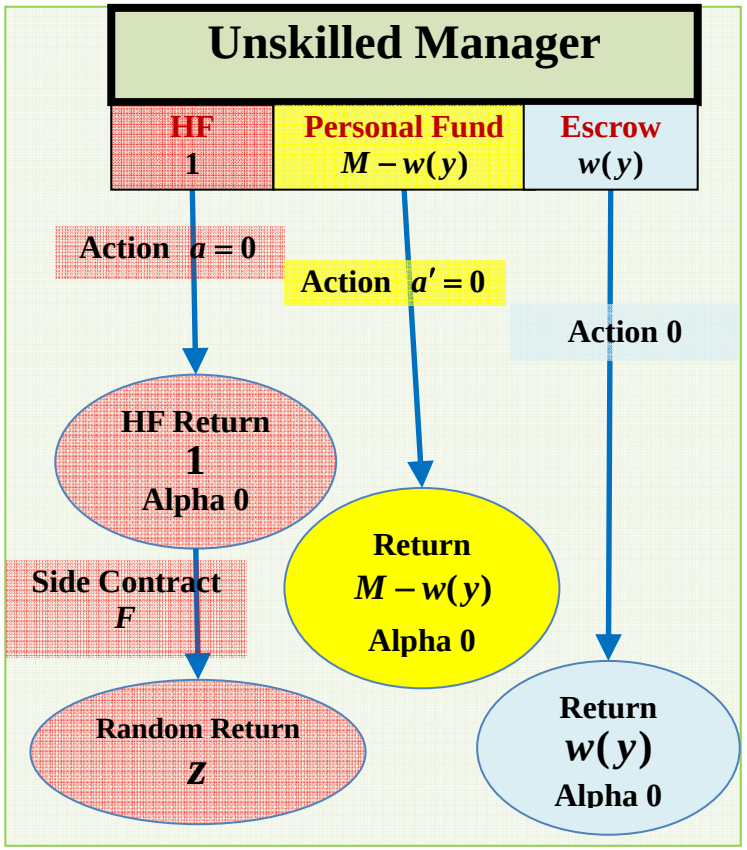
“HF Never Survives. We Need More Transparency!”

Results of This Paper

- **CG Tax Functions**
  - **With No CG Tax, We Cannot Solve Incentive Problem (a la Foster + Young)**
  - **With Positive CGT Rate $t > 0$, We Can Solve Incentive Problem**
- **Constrained Optimal Scheme**
  - **Fulcrum After Taxation: Low-Powered**
- **Income Tax on Fee Functions**
  - **Income Tax Rate Should be Greater than CG Tax Rate, $\tau > t$**
  - **Manager Selects Constrained Optimal Scheme Voluntarily**
- **Equity Stake Functions**
  - **We Can Solve Incentive Problem without Fulcrum**

Assumption: Separate Management





Incentive Problem: Five Constraints

① **Skilled Entry**

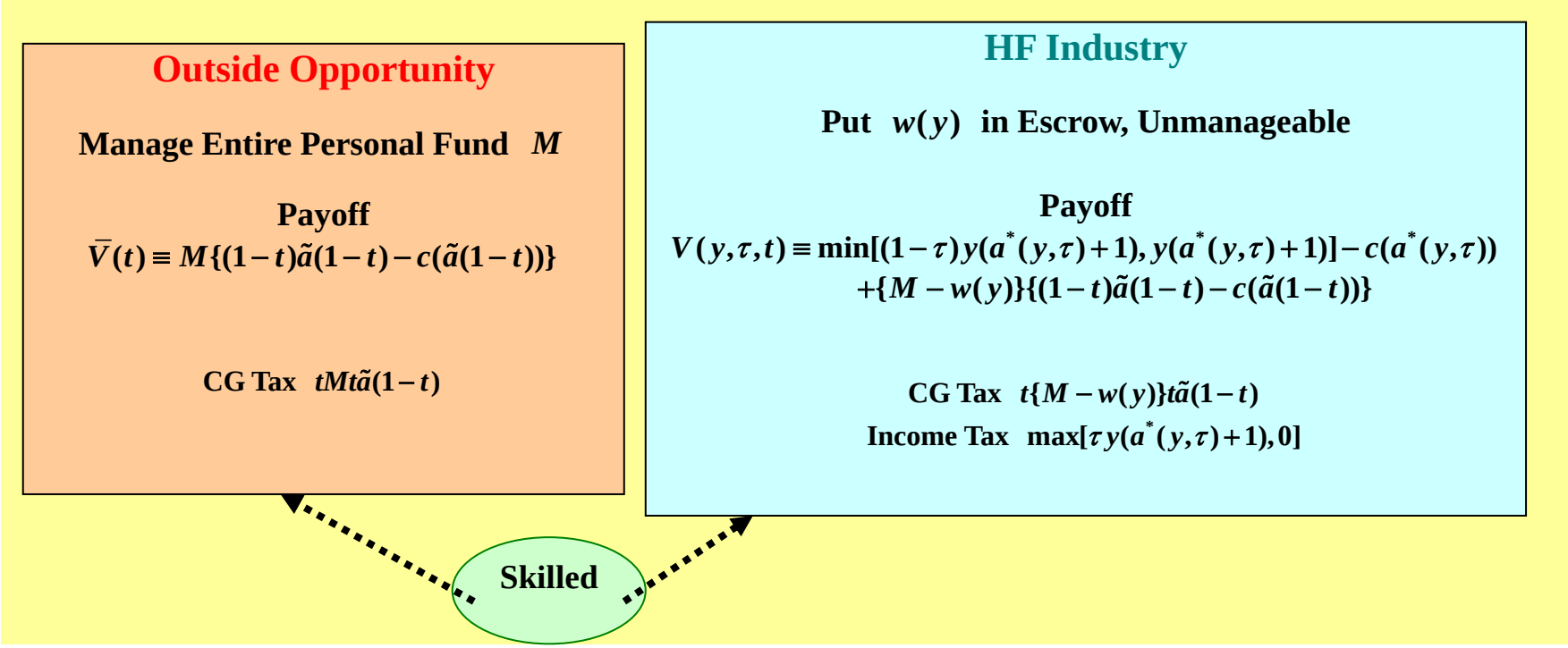
② **Unskilled Exit**

③ **Investor Entry**

④ **Welfare Improvement**

⑤ **Skilled Non-mimicry: Skilled Needs No Third-Party Side Contract**

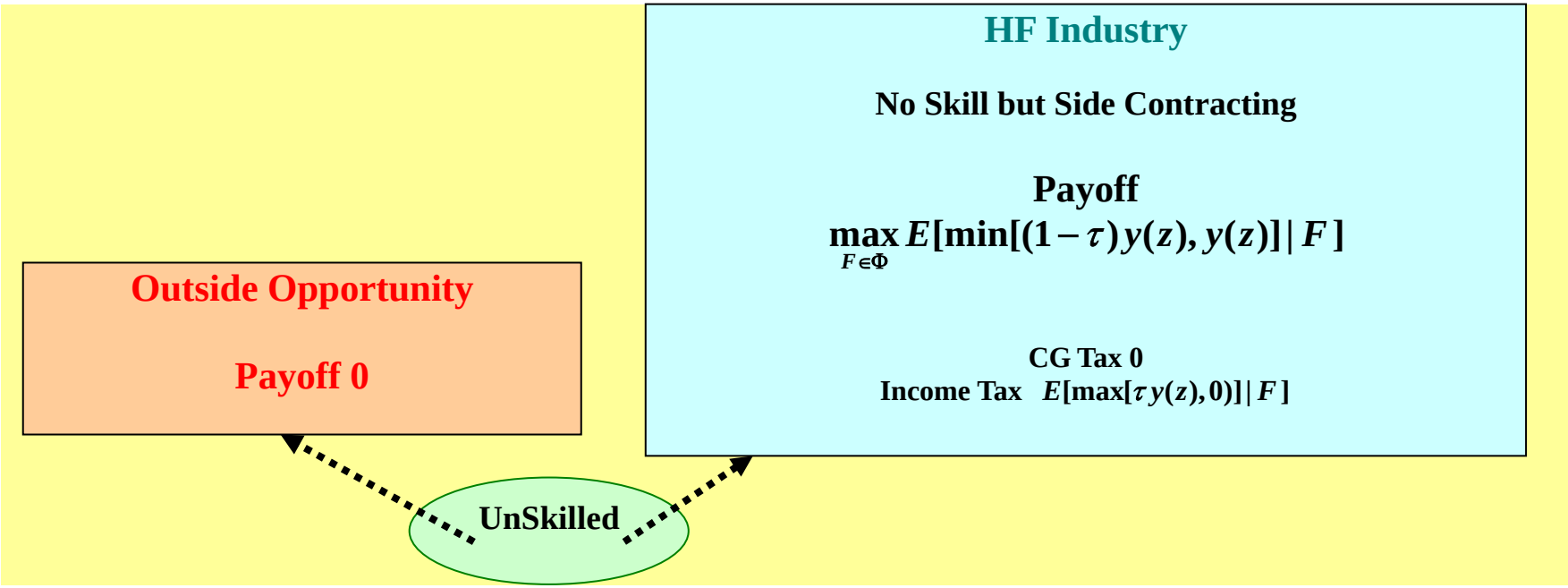
Skilled Entry: $V(y,t,\tau) \geq \bar{V}(t)$



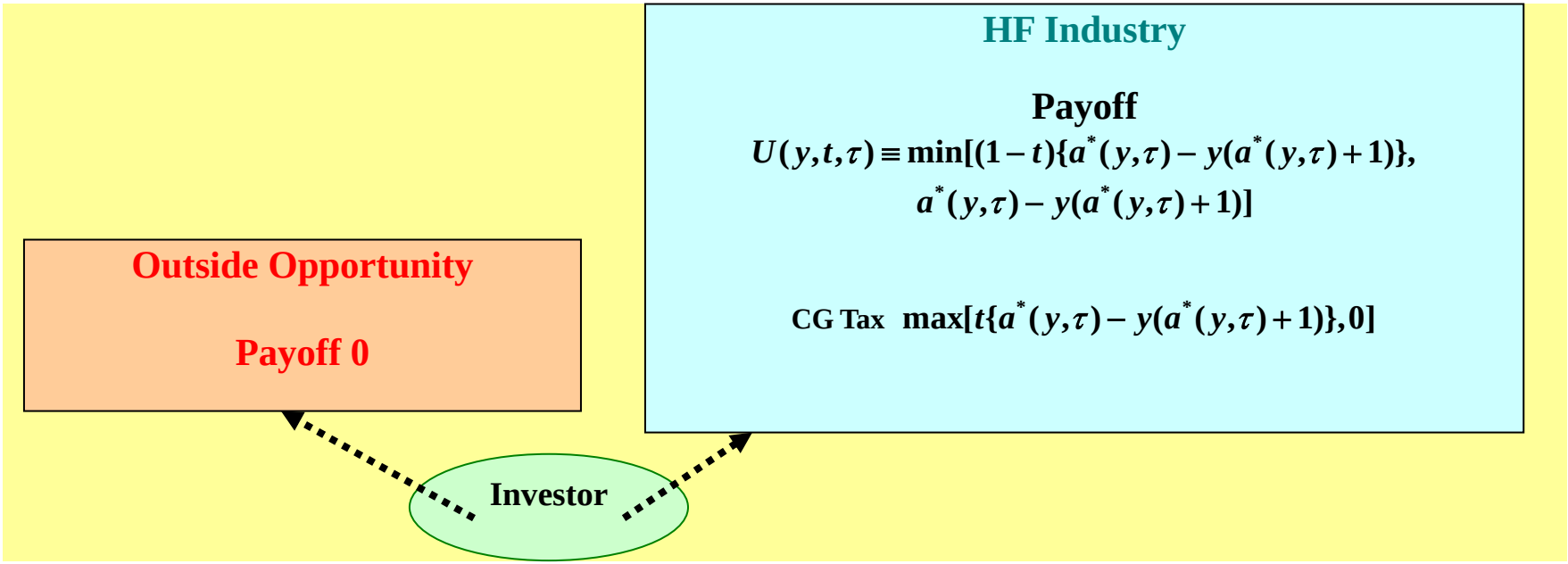
$\tilde{a}(1-t)$ Maximize $(1-t)a - c(a)$

$a^*(y,\tau)$ Maximize $(1-\tau)y(a+1) - c(a)$

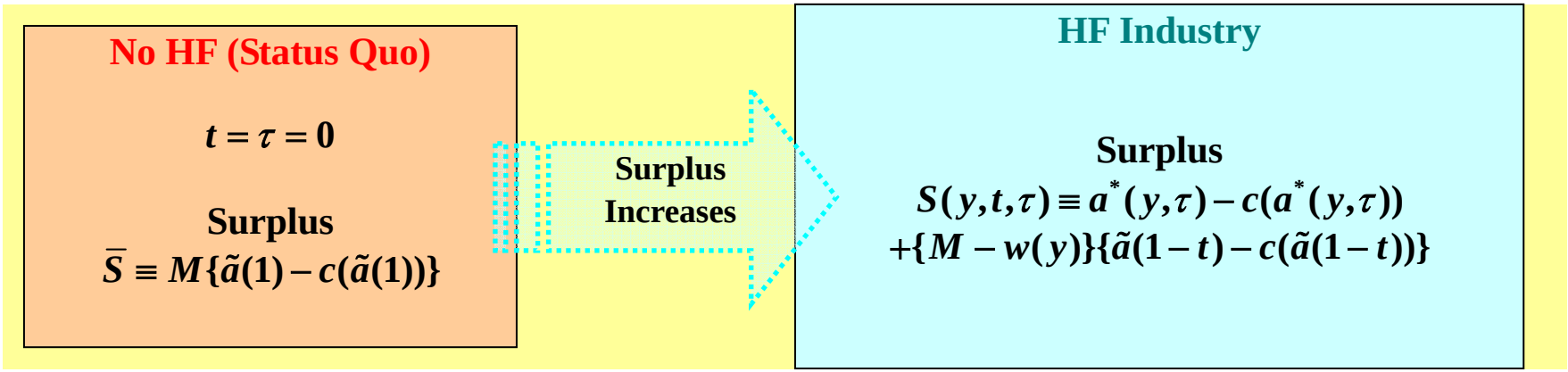
Unskilled Exit: $\max_{F \in \Phi} E[\min[(1-\tau)y(z), y(z)] \mid F] \leq 0$



Investor Entry: $U(y,t,\tau) \geq 0$, i.e., $a^*(y,\tau) \geq y(a^*(y,\tau)+1)$



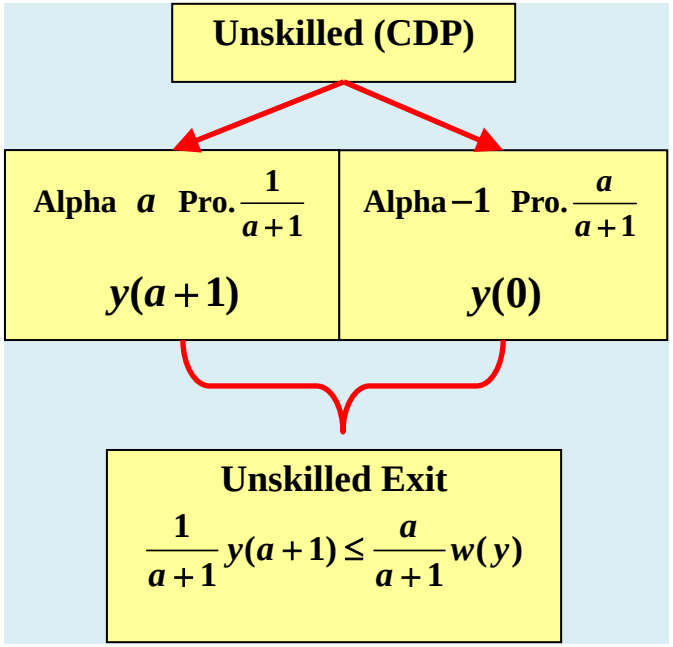
Welfare Improvement: $S(y,t,\tau) > \bar{S}$

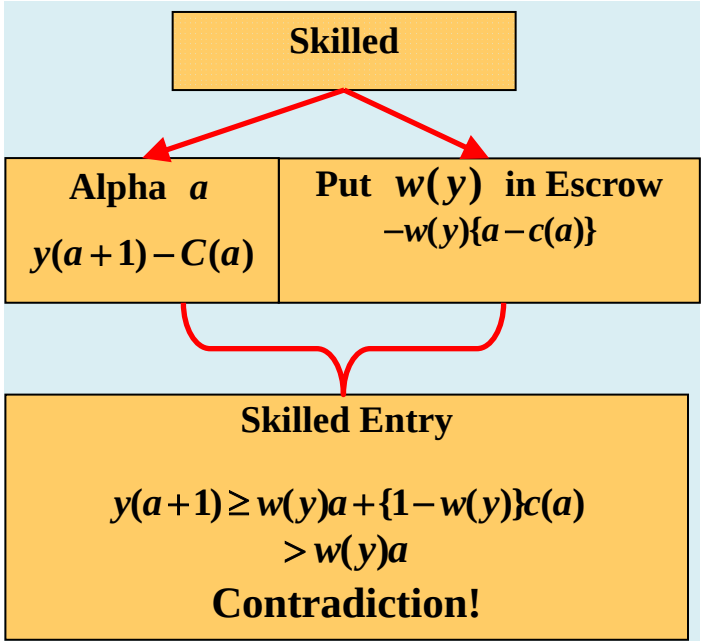


No Capital Gain Tax: Impossibility

Theorem: ***Suppose CGT Rate*** $t = 0$. ***Then, There Exists No Fee Scheme that Satisfies Skilled Entry, Unskilled Exit, and Welfare Improvement.***

Outline of Proof: Assume $a > 0$ is only available, $y(0) = -w(y)$

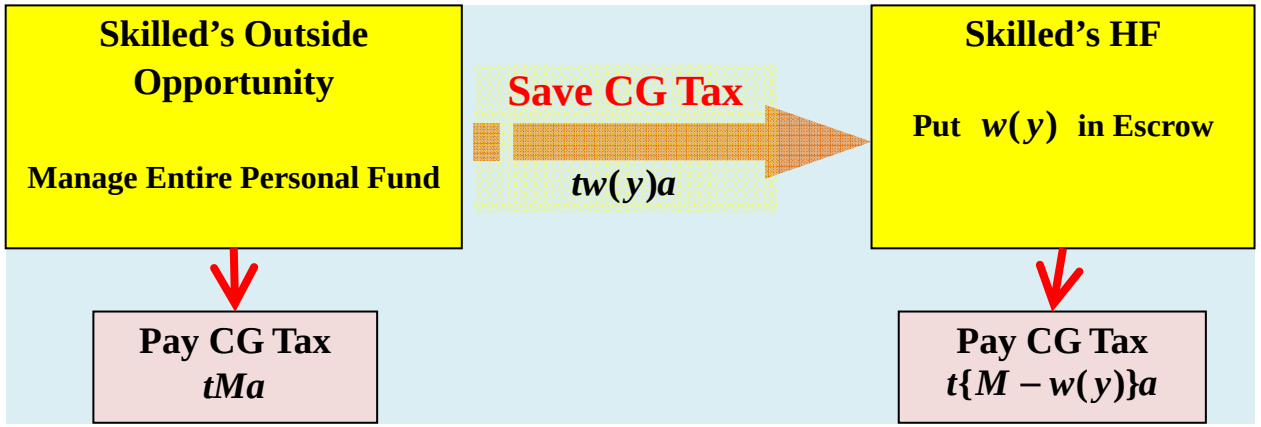




Positive Capital Gain Tax: Possibility

Theorem: ***There exist Tax Rates*** $(t,\tau)\in[0,1]^2$ ***and Fee Scheme*** $y\in Y^*(\tau)$ ***that satisfy All Constraints.***

Outline of Proof: Assume $a>0$ is only available



Constrained Optimization: (y^*, t^*, τ^*)

(1) Fulcrum Scheme after Taxation

$$y(x) = x - 1 \quad \text{for all} \quad x \in [1, \infty)$$

$$y(x) = (1 - \tau)(x - 1) \quad \text{for all} \quad x \in [0, 1)$$

(2) Skilled Entry Binding

$$V(y, t, \tau) = \bar{V}(t)$$

We Specify $(y, t, \tau) = (y^*, t^*, \tau^*)$ As Maximizing Surplus $S(y, t, \tau)$ Subject to (1) and (2)

Theorem: (y^*, t^*, τ^*) *Satisfies All Constraints. There exists No* (y, t, τ) *that Satisfies All Constraints and* $S(y, t, \tau) > S(y^*, t^*, \tau^*)$.

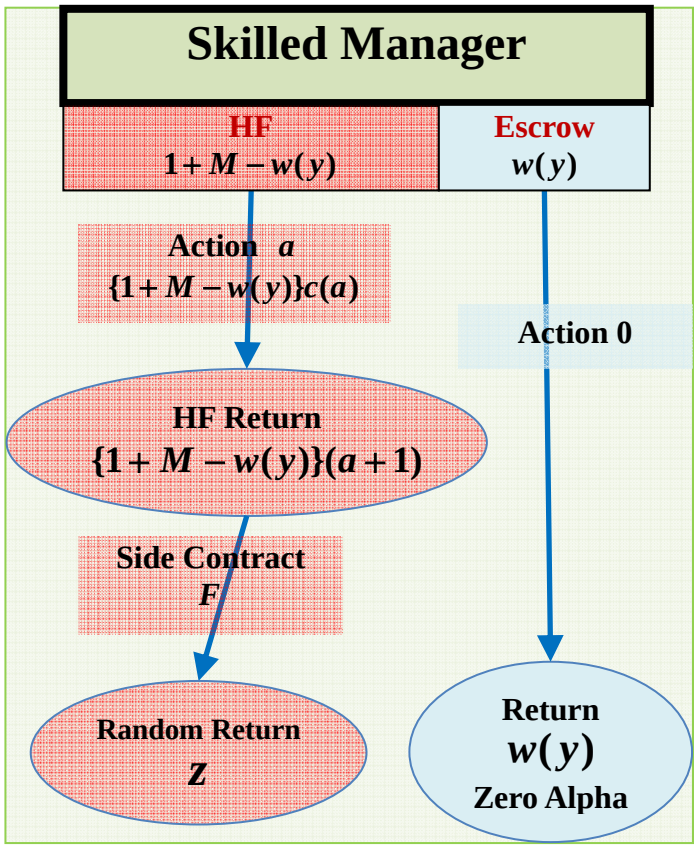
Constrained Optimization: Properties

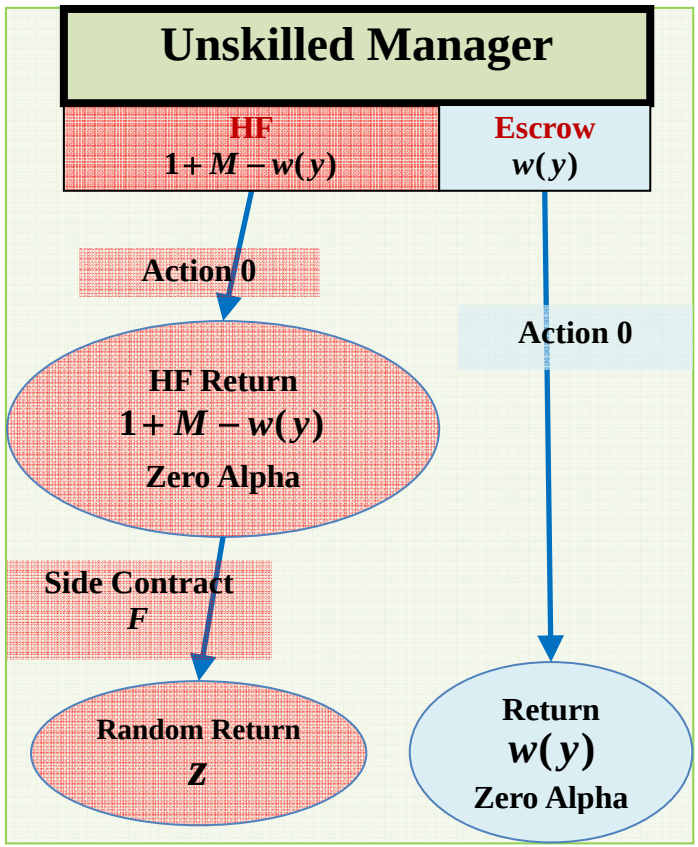
- **Manager is Willing to Select y^* Voluntarily: y^* is the Only Scheme that Satisfies Skilled Entry, Unskilled Exit, Investor Entry, and Skilled Non-mimicry.**

- **Manager Prefers to Put Personal Fund in Escrow as Large as Possible, Distorting Welfare.**

- **Income Tax Rate τ^* is Greater than CG Tax Rate t^*: High Income Tax Rate**

Another Assumption: Equity Stake





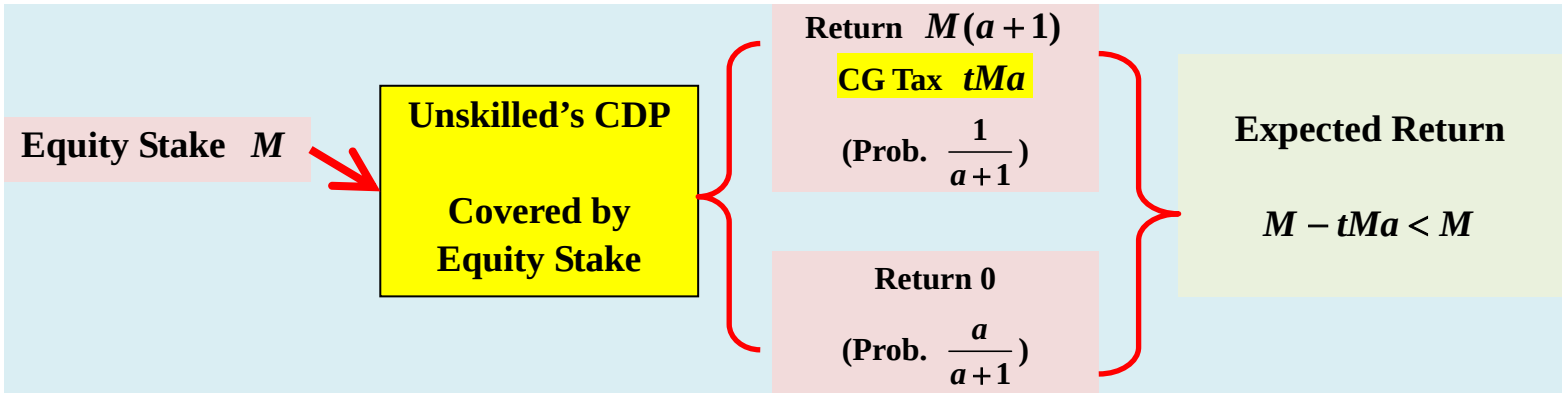
We Don't Need Penalty, But CG Tax and Big Stake

Theorem: ***Suppose CGT Rate*** $t = 0$. ***Then, There Exists No Fee Scheme that Satisfies Skilled Entry, Unskilled Exit, and Welfare Improvement.***

Additional Assumption: $a > 0$ **is only available,** $\tau = 0$

Theorem: ***There exist*** (t, y) ***that Levy No Penalty but Satisfy All Constraints.***

Outline of Proof: **CDP Must be Covered by Not only Investor's Fund But also Personal Fund**



Further Comments

Investor's Optimization

- **Investor Prefers higher-Powered and More Penalty than Constrained Optimal Scheme.**

- **By Transferring Total Tax Revenue to Investor, Government Can Incentivize Investor to Select Constrained Optimal Scheme Voluntarily.**

- **Investor's Payoff May be Greater than Manager's Payoff per Unit: Manager May Fold HF Business.**

Entry Cost

Entry Cost Functions, if, and Only if, It is Non-Pecuniary!