THE TAXATION OF SAVING RETURNS IN OVERLAPPING GENERATIONS ECONOMIES WITH STOCHASTIC ASSET BUBBLES

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Abstract. In the standard simple overlapping generations model with production (Diamond (1965)), the steady state competitive equilibrium level of capital under laissez-faire does not typically maximize the net production if the agents can save only in terms of capital. In particular, whenever the steady state competitive equilibrium level of capital is too high, net production and consumption can be increased withdrawing resources from the production process by means of a mechanism —such as fiat money as in Tirole (1985), or a rolled-over public debt as in Diamond (1965)— allowing to transfer savings from the young to the contemporaneous old in order to be consumed. Any such mechanism is essentially based on the agents' certainty that their future claims obtained in exchange of their participation in the mechanism will be honored (by the next generation, by the government,...), i.e. that such claims will have some value. Nevertheless, the actual decentralization of the best possible steady state as a competitive equilibrium by means of, say, fiat money is undermined by the fact that, at that steady state, the agents' demand for money is indeterminate when this same money is known to be valued for sure next period, since then the return to this alternative channel of savings has to be equal to the return to capital at the steady state. Nevertheless, this indeterminacy disappears as soon as there is some chance (no matter how small) that the money (or debt) being offered may be repudiated next period. Weil (1987) showed that competitive equilibria in which money may lose its value (stochastic bubbles) do exist as long as the probability of this happening is small enough. In this paper I show (i) firstly, that the best steady state attainable by means of such "risky" money is not a competitive outcome under laissez-faire; and (ii) secondly, that this best steady state with "risky" money is nonetheless attainable as a competitive outcome if returns to capital savings (but not returns to monetary savings) are taxed adequately and the amount raised returned to the same agents as a lump-sum transfer.

1. Introduction

One of the two main paradigms to address the problem of the intertemporal allocation of resources is the representative agent overlapping generations economy with production considered in Diamond (1965).[1] In that economy the agents' only endowment is their ability to work when young, and output can be reproduced each period using the labor they supply and the share of previously produced output that has not yet been consumed, i.e. the aggregate level of capital. In such a set-up the best possible steady state —i.e. the steady state that maximizes the utility of the representative agent[2]— requires, first, that the aggregate level of capital be such that the net output is maximized and, second, that this net production is split between young and old agents in such a way that the marginal rate of substitution

[1] The other being the neoclassical growth model with endogenous savings of Ramsey (1928).
[2] Also known as the golden rule.

between consumptions when young and old equals the rate at which consumption can be redistributed from young to old at any given period. These two conditions amount to make both the marginal rate of substitution between present and future consumption and the marginal productivity of capital equal to the factor by which the population grows each period. Typically,[3] this requires not to remunerate the factors of production by their marginal productivities or, alternatively, to make intergenerational redistributions of income, should the factors be remunerated this way.

Any of the two ways mentioned above to implement the best steady state is clearly at odds with what characterizes a laissez-faire competitive equilibrium, since the latter both remunerates the factors by their marginal productivities and does not allow for any kind of redistribution of income. It has been shown nonetheless that if the agents are allowed to save part of their labor income in terms of an intrinsically worthless asset (a bubble, in Tirole (1985) terms)[4] that every agent holds for certain will not lose value completely next period, then there is a specific portfolio of money and capital that —*if chosen* by the agents for their savings— allows to attain the first-best steady state as a competitive equilibrium. It turns out, nonetheless, that this last *if* is a too big if.

In effect, whenever too much capital is saved at the competitive equilibrium steady state with only capital to save, the first-best steady state would require to hold strictly positive amounts of both money (or debt) and capital. Therefore, in the absence of uncertainty both assets must have the same return at the steady state, so that the agents must necessarily be completely indifferent about the composition of their savings portfolio. In other words, the agents' choice of the *composition* of their savings portfolio is completely indetermined at the first-best steady state. Thus, although there exists indeed a way to support the best steady state using money to place some of the agents' savings, nothing in the model explains why the agents would actually *choose* to place their savings precisely the way that allows to do so.[5] Note that this indeterminacy is not of the same nature than, say, that of the production plan at equilibrium of a firm with a constant returns to scale technology. In effect, in that case it is widely assumed that production just adjusts to a demand

[3]That is to say, except for the knife-edge case in which the net output maximizing steady state level of capital has a marginal productivity equal to the ratio of consumption when old over capital.
[4]E.g. fiat money or public debt that is rolled-over every period (as with the internal debt in Diamond (1965)).
[5]That the modeler knows this to be the right thing to do does not seem to be a very compelling argument.

that is well determined by prices. Nevertheless, in the case of the savings portfolio choice at the first-best steady state, on the two sides of the money market sits the same representative agent, and *both* face then the same indeterminacy. As a result, there is no well-determined other side of the market here that is able to anchor an indeterminate side. This points to the existence of an element, missing from the model, that would explain why the agents would choose to save exactly the right amounts of capital and money that allow to put the economy on the best possible steady state. Let us then take one step back to think about how money[6] is supposed to support the best steady state in the model of Diamond (1965).

In a competitive equilibrium without money or any other equivalent mechanism of intergenerational transfers, the agents may end up dumping with their saving decisions too much output in the form of capital into the production process, compared to the level that maximizes net production. In order to convince them to withdraw part of these resources from the production process and devote them instead to increase the consumption of their parents,[7] they need to be reassured that they would be treated in the same way by the next generation. That is to say, they must believe that some mechanism in place that allows today to make intergenerational transfers of resources will still be there tomorrow when their turn comes to receive from it, instead of contributing to it.[8] Be this mechanism fiat money, rolled-over public debt or a pay-as-you-go pensions system (promises, promises...), the essential element to make any such social contrivance to be accepted is trust. Now, whenever you trust a promise you run the risk of being abused. Indeed, although fortunately not everyday (but no doubt every now and then yes), states do dissolve, wars are waged, revolutions topple governments, and as a result public debts of previous governments are repudiated, money issued by former regimes becomes worthless, and pension claims are not honoured. Financial crises in which banking and credit institutions disappear do happen and claimants lose their savings as a result. And, nevertheless, *some trust is put recurrently on similar promises*, institutions or social contracts almost immediately after such crises take place.

Thus it seems to be inherent to intergenerational financial arrangements based on

[6]Or public debt, or a pay as you go pensions system.

[7]Which, incidentally, increases the marginal productivity of capital and hence the return to their own savings to an extent that offsets their lower level of savings.

[8]If no intergenerational transfers mechanism is in place, the competitive equilibrium steady state is also inefficient with respect to the best allocation that could be attained using the existing markets of capital and labor. An adequate policy of taxes and transfers allows to implement nevertheless this second best (see Dávila (2008)).

trust that there is some probability, no matter how small, that they collapse, only to be restarted little after. Interestingly enough, it is taking into account the possibility of such a collapse (i.e. the possibility of money being repudiated by the next generation, for instance) what suffices to pin down the agents' choice of portfolio at any equilibrium. In particular, Weil (1989) established conditions for the existence of competitive equilibria in a Diamond (1965) economy in which money risks losing value at any time, and the result was that existence obtained as long as this risk was small enough.[9]

As when the money is assumed to be valued for sure, one can consider which is the best steady state that can be implemented thanks to the possibility of saving in such a "risky" money, on top of in terms of capital. Of course, it will depend on the specific probability of money losing value, and as first approach I will consider that probability to be exogenously determined, as in Weil (1989). For instance, if the probability of money losing value is 1 the economy ends up in the usual Diamond (1965) nonmonetary steady state. If, on the contrary, this probability is 0 then the golden rule steady state is the outcome —if the indeterminacy of the representative agent's savings porfolio *choice* (as opossed to its composition) appearing in that case is, as usual, overlooked. Thus I characterize below the best steady state that "risky" money can buy for an arbitrary probability of money losing value. That steady state turns out, unfortunately, not to be a competitive outcome *under laissez-faire*. In other words, free markets are unable to reach the best steady state allocation of resources implementable through an intergenerational transfers mechanism whenever (quite realistically) the latter may collapse at some point, no matter how small is this risk. That is bad news. The good news is that that same best steady state can nonetheless be attained as a competitive outcome *under a well-defined policy of taxes and transfers*. In the case in which the laissez-faire steady state competitive equilibrium over-accumulates capital with respect to the best steady state, this fiscal policy consists of (i) taxing linearly the returns to capital, (ii) making second period lump-sum transfers, and (iii) not taxing returns on monetary savings.

In case it seems awkward that the implementation of the best steady state when money risks losing value may require the taxation of productive savings and not of unproductive ones, let us recall that the inefficiency comes, to begin with, from the

[9]In Weil (1989) the economy was supposed to revert to a non-monetary equilibrium once the bubble had burst, which eventually happened with probability 1. As a mater of fact, this is inessential since the equilibrium conditions remain the same if new money is supposed to be issued at no cost right after the dismissal of the worthless money.

dumping of too many resources into the productive process in the form of capital, and hence the need to disincentive such savings. At the same time, unproductive monetary savings work instead in the direction of unclogging the production process in this case, from which the need not to disincentive follows. This result may challenge the widespread view that values only productive investments above speculative ones, and hence provide some food for thought about what is the real role of each kind of investments.

The rest of the paper is structured as follows. Section 2 provides, mainly to fix notation and for the sake of completeness, the well-known characterization of the first-best steady state of the Diamond (1965) overlapping generations economy with production. It is also shown there that the first-best steady state is not a competitive equilibrium outcome under laissez-faire in the absence of any kind of intergenerational transfer mechanism. Section 3 introduces such a mechanism, namely the extreme case of a fiat money that is accepted by all generations with probability 1. I argue there that there is indeed an amount of monetary savings that would allow to implement the first-best steady state if chosen by the agents, but that the agents have no reasons for choosing it in a decentralized way. In Section 4 I depart from the assumption of money being always valued for sure allowing for the probability of it losing value completely to be positive at any time. In Section 5 I show that the newly obtained competitive steady state under laissez-faire is not the (second) best steady state that can be attained by means of this "risky" money. Finally, Section 6 establishes that this best steady state can be made into a competitive outcome with the adequate policy of taxes and transfers, which I characterize there.

2. The first-best steady state of the Diamond (1965) OG economy

In the standard overlapping generations economy with production in Diamond (1965), each of the of 2-period-lived identical members of a population of overlapping generations (growing at a rate $n > -1$) is endowed with l units of labor when young. They can produce consumption good —from which they derive a utility $u(c)$, discounted by $0 < \beta < 1$ if consumed when old— out of labor and previously produced and not consumed good used as capital (that does not depreciate) by means of a constant returns technology $F(K, L)$. As it is well known, the highest utility all the agents can get at a steady state[10] follows from solving

$$\max u(c_0) + \beta u(c_1)$$
$$c_0 + \frac{c_1}{1+n} + k = \frac{k}{1+n} + F(\frac{k}{1+n}, l) \tag{1}$$

whose solution (c_0^*, c_1^*, k^*) is characterized by the equations

$$\frac{1}{\beta}\frac{u'(c_0^*)}{u'(c_1^*)} = 1 + n = 1 + F_K(\frac{k^*}{1+n}, l) \tag{2}$$

$$c_0^* + \frac{c_1^*}{1+n} + k^* = \frac{k^*}{1+n} + F(\frac{k^*}{1+n}, l). \tag{3}$$

In effect, such an aggregate level of capital maximizes net output and the latter is distributed between young and old so that their marginal rate of substitution between consumption when young and consumption when old equals the rate at which they can be transformed into each other, i.e. the growth factor of the population.

Unfortunately, this first-best steady state cannot be attained in the absence of fiat money or another intergenerational transfers mechanism. In effect, if the agents can only save in terms of physical capital, they face the problem of deciding how much of the real income $w_t l$ (received from supplying inelastically his labour l at a real wage w_t) to consume immediately, c_t^t, and how much to lend as capital (in exchange of a rental rate r_{t+1}) for production next period, k^t, in order to consume the return $(1 + r_{t+1})k^t$, i.e.

$$\max u(c_t^t) + \beta u(c_{t+1}^t)$$
$$c_t^t + k^t = w_t l \tag{4}$$
$$c_{t+1}^t = (1 + r_{t+1})k^t.$$

given the real wage w_t and the return r_{t+1} to savings in terms of capital. Under standard assumptions about the utility function u, the representative agent's optimal choice is characterized by the equalization of the marginal rate of substitution between first and second period consumptions to the returns to savings and his budget constraints, i.e.

$$\frac{1}{\beta}\frac{u'(w_t l - k^t)}{u'((1 + r_{t+1})k^t)} = 1 + r_{t+1}. \tag{5}$$

[10] Such a steady state is known as the *golden rule*.

Since at a competitive equilibrium the factors must be remunerated by their marginal productivities, i.e.

$$\begin{aligned} r_{t+1} &= F_K(\frac{k^t}{1+n}, l) \\ w_t &= F_L(\frac{k^{t-1}}{1+n}, l) \end{aligned} \tag{6}$$

then the competitive equilibrium saving dynamics is

$$\frac{1}{\beta} \frac{u'(F_L(\frac{k^{t-1}}{1+n}, l)l - k^t)}{u'(F_K(\frac{k^t}{1+n}, l)k^t)} = 1 + F_K(\frac{k^t}{1+n}, l) \tag{7}$$

and the steady state (c_0^c, c_1^c, k^c) is characterized by the conditions

$$\frac{1}{\beta} \frac{u'(c_0^c)}{u'(c_1^c)} = 1 + F_K(\frac{k^c}{1+n}, l) \tag{8}$$

$$\begin{aligned} c_0^c + k^c &= F_L(\frac{k^c}{1+n}, l)l \\ c_1^c &= (1 + F_K(\frac{k^c}{1+n}, l))k^c. \end{aligned} \tag{9}$$

Considering the conditions (2,3) and (8,9) it becomes clear that the competitive equilibrium steady state and the first-best steady state need not coincide. In effect, (2) implies (8) but not the other way around, and (9) implies (3) but not the other way around. They will coincide if, and only if,

$$F_K(\frac{k^c}{1+n}, l) = n \tag{10}$$

i.e. if capital has a marginal productivity exactly equal to the rate at which the population grows over periods —since then k^c satisfies also (2) and (3)— but this needs not be the case (and will typically not be so) in general. Notwithstanding, as it is well known also, the first-best steady state could be attained by means of the introduction of an intrinsically worthless asset that the agents would trade in exchange for good and in terms of which they would save as well. Quite another thing is whether they would choose to do so. We review this possibility in detail in the next section.

3. Introducing fiat money in the economy

Reconsider the problem faced by the representative agent of the overlapping generations economy with production in Diamond (1965) if he has the possibility of saving in terms of money, on top of in terms of capital. Thus, in order to save, the representative agent can lend as before a fraction k^t of his non-consumed income to be used as capital for production next period, but he can also buy with a fraction of that income an amount M^t of an intrisically worthless money at a real price $\frac{1}{p_t}$ that can be resold at $t+1$ to the next young at a price $\frac{1}{p_{t+1}}$.[11] Capital savings deliver a return $(1+r_{t+1})k_{t+1}$ while monetary savings have a return $\frac{p_t}{p_{t+1}}$. The representative agent solves therefore the problem

$$\begin{aligned} \max u(c_t^t)&+v(c_{t+1}^t) \\ c_t^t + k^t + \frac{1}{p_t}M^t &= w_t l \\ c_{t+1}^t = (1+r_{t+1})k^t &+ \frac{1}{p_{t+1}}M^t. \end{aligned} \tag{11}$$

As we have seen above, if the economy is ever going to be at a competitive equilibrium whose allocation coincides with that of the first-best steady state, the equilibrium must be such that the agents *choose* to hold part of their savings in money (otherwise they would end up at the inefficient steady state competitive equilibrium determined by equations (8,9)). And they would also have to hold a positive amount of capital, specifically the one that follows from conditions (10). But an agent chooses to hold strictly positive amounts of money and capital only if both assets have the same return and this return is equal to the marginal rate of substitution between current and future consumption, that is to say

$$\frac{p_t}{p_{t+1}} = \frac{1}{\beta}\frac{u'(w_t - k^t - \frac{1}{p_t}M^t)}{u'((1+r_{t+1})k^t + \frac{p_t}{p_{t+1}}\frac{1}{p_t}M^t)} = 1 + r_{t+1} \tag{12}$$

Moreover, at a steady state money must have a return rate equal to the rate of growth of the population. In effect, the equilibrium of the market for goods at every period t requires that

$$c_t^t + \frac{c_t^{t-1}}{1+n} + k^t = \frac{k^{t-1}}{1+n} + F(\frac{k^{t-1}}{1+n}, l) \tag{13}$$

[11] Thus p_t is the monetary price of the only good.

that is to say, in terms of the agent's budget constraints,

$$\left[w_t l - k^t - \frac{1}{p_t} M^t\right] + \frac{1}{1+n}\left[(1+r_t)k^{t-1} + \frac{1}{p_t} M^{t-1}\right] + k^t = \frac{k^{t-1}}{1+n} + F(\frac{k^{t-1}}{1+n}, l). \quad (14)$$

But, from the homogeneity of degree 1 of the production function and the feasibility of any equilibrium allocation, the output level in the right-hand side cancels out with the remunerations to labor and capital in the left-hand side, so that necessarily

$$\frac{M^{t-1}}{M^t} = 1 + n. \quad (15)$$

As a consequence, since at a steady state the real balances $\frac{1}{p_t} M^t$ hold by each agent is a constant m, then necessarily

$$\frac{p_t}{p_{t+1}} = 1 + n. \quad (16)$$

Therefore, at a competitive equilibrium steady state with money and capital, the savings in capital and real balances (k^m, m) must satisfy

$$1 + n = \frac{1}{\beta} \frac{u'(F_L(\frac{k^m}{1+n}, l)l - k^m - m)}{u'((1 + F_K(\frac{k^m}{1+n}, l))k^m + (1+n)m)} = 1 + F_K(\frac{k^m}{1+n}, l) \quad (17)$$

or, equivalently,

$$\begin{aligned} \frac{1}{\beta} \frac{u'(c_0^m)}{u'(c_1^m)} &= 1 + n = 1 + F_K(\frac{k^m}{1+n}, l) \\ c_0^m &= F_L(\frac{k^m}{1+n}, l)l - k^m - m \\ c_1^m &= (1 + F_K(\frac{k^m}{1+n}, l))k^m + (1+n)m. \end{aligned} \quad (18)$$

Therefore, the solution of the last system of equations is necessarily among the solutions to the equations characterizing the first-best steady state

$$\begin{aligned} \frac{1}{\beta} \frac{u'(c_0)}{u'(c_1)} &= 1 + n = 1 + F_K(\frac{k}{1+n}, l) \\ c_0 + \frac{c_1}{1+n} + k &= \frac{k}{1+n} + F(\frac{k}{1+n}, l) \end{aligned} \quad (19)$$

so that whenever the solution to (19) is guaranteed to be unique the two steady states coincide.

Note nonetheless that, although in the system of equations (18) the amount of money (in real terms) supporting this steady state is determined along with the corresponding levels of consumption and capital, the agent's *demand* for money is not determined by his savings decision. As a matter of fact, the agents choice of the *composition* of the portfolio of money and capital in which to hold their savings is completely indetermined at the first-best steady state. In effect, whenever the two assets, money and capital, have the same returns, i.e. $1 + r_{t+1} = \frac{p_t}{p_{t+1}}$, the agent's problem becomes

$$\begin{aligned} \max u(c_t^t) + \beta u(c_{t+1}^t) \\ c_t^t + [k^t + p_t M^t] = w_t \\ c_{t+1}^t = (1 + r_{t+1})[k^t + p_t M^t] \end{aligned} \tag{20}$$

which determines the agent's overall level of savings $k^t + p_t M^t$, but not the specific shares of capital and money, k^t and $p_t M^t$, within that level of savings. Although looking at equation (18) it is clear that the amount of capital savings within this portfolio that support the first-best steady state must be such that its marginal productivity equals the rate of growth of the population, this does not follow from the representative agent's decision problem, but from the planner's. Therefore there is no reason for the agents to choose, in a decentralized way, precisely the savings portfolio that would allow them to attain the first-best steady state.

In the next section I consider the consequences of introducing, à la Weil (1989), a probability of money becoming worthless at any period, i.e. the consequences of money being "risky" money. As it will become clear there, this solves the indeterminacy problem, so that stochastic bubbles can be shown to exist (see Weil (1989)). Nevertheless, the steady state supported by a stochastic bubble turns out not to be the best steady state implementable through the use of "risky" money. This (second) best steady state can however be attained as a competitive outcome under a fiscal policy to be detailed in Section 5 further below.

4. Laissez-faire competitive equilibria with "risky" money

Suppose now that in the Diamond (1965) economy money has a risky return: with some probability π the money bought by generation t will still have some value at $t+1$ (i.e. generation $t+1$ will be willing to sell some of their real income in exchange for this money), and with some probability $\tilde{\pi} = 1 - \pi$ this money will not be accepted by the next generation. Thus the representative agent's problem is

$$\begin{aligned} \max u(c_t^t) + \frac{1}{\beta}\left[\pi u(c_{t+1}^t) + \tilde{\pi} u(\tilde{c}_{t+1}^t)\right] \\ c_t^t + k^t + \frac{1}{p_t} M^t = w_t l \\ c_{t+1}^t = (1 + r_{t+1})k^t + \frac{1}{p_{t+1}} M^t \\ \tilde{c}_{t+1}^t = (1 + r_{t+1})k^t. \end{aligned} \tag{21}$$

Once more under standard assumptions on u, the solution to this problem is characterized by the first order conditions

$$\frac{1}{\beta} \frac{u'(w_t - k^t - \frac{1}{p_t} M^t)}{\pi u'((1 + r_{t+1})k^t + \frac{1}{p_{t+1}} M^t) + \tilde{\pi} u'((1 + r_{t+1})k^t)} = 1 + r_{t+1} \tag{22}$$

$$\frac{1}{\beta} \frac{u'(w_t - k^t - \frac{1}{p_t} M^t)}{\pi u'((1 + r_{t+1})k^t + \frac{1}{p_{t+1}} M^t)} = \frac{3_t}{p_{t+1}}. \tag{25}$$

Note that the conditions above determine now not only the overall level of savings $k^t + \frac{1}{p_t} M^t$ chosen by agent t, given w_t, r_{t+1}, and $\frac{p_t}{p_{t+1}}$, but actually the very composition of the savings portfolio, i.e. k^t and $\frac{1}{p_t} M^t$. It follows from the conditions above that any equilibrium return to money (unproductive savings) has to be necessarily larger than that of capital (productive savings), that is to say

$$1 + r_{t+1} \leq \frac{p_t}{p_{t+1}} \tag{24}$$

Moreover, if the population grows at a rate $n > -1$, then the condition that, as long as money does not lose completely its value,

$$\frac{p_t}{p_{t+1}} = 1 + n \tag{25}$$

follows, as before, from the feasibility of the competitive equilibrium steady state allocation.

The higher real return for monetary savings is a consequence of the fact that money is a riskier asset than capital in this setup, and hence it requires to bear a higher return for the agents to be willing to accept it at equilibrium.[12]

From the homogeneity of degree 1 of the production function it follows that, at equilibrium,

$$\begin{aligned} r_{t+1} &= F_K(\tfrac{k^t}{1+n}, l) \\ w_t &= F_L(\tfrac{k^{t-1}}{1+n}, l) \end{aligned} \tag{26}$$

at every period t, so that the competitive equilibrium per capita level of capital k^t and monetary savings $\frac{1}{p_t}M^t$ dynamics of this model with risky money is given by

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k^{t-1}}{1+n}, l) - k^t - \frac{1}{p_t}M^t)}{\pi u'((1 + F_K(\frac{k^t}{1+n}, l))k^t + \frac{1}{p_{t+1}}M^t) + \tilde{\pi} u'((1 + F_K(\frac{k^t}{1+n}, l))k^t)} = 1 + F_K(\tfrac{k^t}{1+n}, l) \tag{27}$$

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k^{t-1}}{1+n}, l) - k^t - \frac{1}{p_t}M^t)}{\pi u'((1 + F_K(\frac{k^t}{1+n}, l))k^t + \frac{1}{p_{t+1}}M^t)} = \frac{p_t}{p_{t+1}}. \tag{328}$$

and letting the steady state aggregate demand for real balances be

$$\frac{1}{p_t}M^t = m \tag{29}$$

then the steady state monetary and capital savings m and k are then characterized by the equations

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k}{1+n}, l) - k - m)}{\pi u'((1 + F_K(\frac{k}{1+n}, l))k + (1+n)m) + \tilde{\pi} u'((1 + F_K(\frac{k}{1+n}, l))k)} = 1 + F_K(\tfrac{k}{1+n}, l) \tag{30}$$

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k}{1+n}, l) - k - m)}{\pi u'((1 + F_K(\frac{k}{1+n}, l))k + (1+n)m)} = 1 + n \tag{31}$$

[12] Some may find it surprising, as the only productive investments here, at least directly, are those made in terms of capital. It is worth stressing, at any rate, that money (or for the same token public debt, pay-as-you-go pension systems, or any other intergenerational transfers mechanism) is an unproductive investment only in a strictly direct technological and physical sense, since by allowing to support higher levels of net output at equilibrium, it cannot be deemed socially unproductive, at least indirectly, if only because it allows to move towards the efficiency frontier. Social arrangements or institutions thus certainly matter.

Proposition 3 in Weil (1989) establishes that, if there is a unique steady state for the economy without bubble, i.e. a unique $\bar{k}$ solving

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k}{1+n},l)-k)}{u'((1+F_K(\frac{k}{1+n},l))k)} = 1 + F_K(\tfrac{k}{1+n},l) \tag{32}$$

then there exist a steady state for the economy with a stochastic bubble, i.e. a (k,m) solution to equations (29,30) above, if, and only if,

$$\pi > \frac{1+F_K(\frac{\bar{k}}{1+n},l)}{1+n} \tag{33}$$

Note that, as a consequence, there cannot be such a steady state if $\bar{k}$ is dynamically efficient, i.e. if

$$1 + F_K(\tfrac{\bar{k}}{1+n},l) \geq 1+n \tag{34}$$

5. The best steady state that "risky" money can buy

If we consider the best steady state that the acceptance of this risky money allows to attain —i.e. the steady state maximizing the utility of the representative agent— it would be characterized as a solution to

$$\begin{aligned} \max u(c_0) + &\frac{1}{\beta}\left[\pi u(c_1) + \tilde{\pi} u(\tilde{c}_1)\right] \\ c_0 + k + m &= F_L(\tfrac{k}{1+n},l)l \\ c_1 &= (1+F_K(\tfrac{k}{1+n},l))k + (1+n)m \\ \tilde{c}_1 &= (1+F_K(\tfrac{k}{1+n},l))k \end{aligned} \tag{35}$$

Therefore, the best steady state attainable through the use of risky money is, whenever $\pi, \tilde{\pi} \in (0,1)$ and $\pi + \tilde{\pi} = 1$,

$$\begin{aligned} &\frac{1}{\beta}\frac{u'(F_L(\frac{k}{1+n},l)l-k-m)}{\pi u'((1+F_K(\frac{k}{1+n},l))k+(1+n)m) + \tilde{\pi} u'((1+F_K(\frac{k}{1+n},l))k)} \\ &\qquad = (1+n)\frac{1+F_K(\frac{k}{1+n},l)+F_{KK}(\frac{k}{1+n},l)\frac{k}{1+n}}{1+n+F_{KK}(\frac{k}{1+n},l)\frac{k}{1+n}} \end{aligned} \tag{36}$$

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k}{1+n},l)l-k-m)}{\pi u'((1+F_K(\frac{k}{1+n},l))k+(1+n)m)}=1+n \tag{37}$$

Note that the impact of aggregate capital in the real wage and the return to capital is taken into account in the first equation through the change in the marginal productivity of capital (and implicitly of labor as well) that and increase in k has.

Now, can the best steady state with risky money be attained as a competitive steady state? Comparing equations (29,30) characterizing the competitive equilibrium steady state with risky money, to equations (38) characterizing the best steady state with risky money it becomes clear that they will only have a common solution k if it happens to be such that

$$(1+n)\frac{1+F_K(\frac{k}{1+n},l)+F_{KK}(\frac{k}{1+n},l)\frac{k}{1+n}}{1+n+F_{KK}(\frac{k}{1+n},l)\frac{k}{1+n}}=1+F_K(\frac{k}{1+n},l) \tag{38}$$

or, equivalently,

$$F_K(\frac{k}{1+n},l)=n \tag{39}$$

i.e. if it happens to be the golden rule per capita level of capital. In other words, typically the steady state competitive equilibrium is inefficient under laissez-faire. Is there nonetheless an active fiscal policy allowing to decentralize this steady state as a competitive equilibrium? This issue is addressed in the next section.

6. Implementing the best steady state through taxes and transfers

Assume the government taxes linearly the capital returns of generation t at a rate τ_{t+1} and also distributes to the same generation lump-sum transfer T_{t+1} when old, so that the represerntative agent's problem becomes

$$\begin{aligned} &\max u(c_t^t)+\frac{1}{\beta}\left[\pi u(c_{t+1}^t)+\tilde{\pi}u(\tilde{c}_{t+1}^t)\right]\\ &c_t^t+k^t+p_tM^t=w_tl\\ &c_{t+1}^t=(1+(1-\tau_{t+1})r_{t+1})k^t+T_{t+1}+p_{t+1}M^t\\ &\tilde{c}_{t+1}^t=(1+(1-\tau_{t+1})r_{t+1})k^t+T_{t+1}. \end{aligned} \tag{40}$$

Consider the following policy of taxes and transfers:

(1) tax agent t's returns from capital at a rate

$$\tau_{t+1} = 1 - \frac{1}{F_K(\frac{k^{t-1}}{1+n}, l)} \cdot \left[(1+n) \frac{1 + F_K(\frac{k^{t-1}}{1+n}, l) + F_{KK}(\frac{k^{t-1}}{1+n}, l) k^{t-1}}{1 + n + F_{KK}(\frac{k^{t-1}}{1+n}, l) k^{t-1}} - 1 \right] \tag{41}$$

(2) transfer to agent t when old the amount

$$T_{t+1} = \tau_{t+1} F_K(\tfrac{k^{t-1}}{1+n}, l) k^{t-1} \tag{42}$$

(3) don't tax agent t's returns from monetary savings

The new equilibrium capital and monetary savings dynamics becomes

$$\frac{1}{\beta} \frac{u'(F_L(\frac{k^{t-1}}{1+n}, l) l - k^t - p_t M^t)}{\pi u'((1 + (1-\tau_{t+1}) F_K(\frac{k^t}{1+n}, l)) k^t + T_{t+1} + p_{t+1} M^t) + \tilde{\pi} u'((1 + (1-\tau_{t+1}) F_K(\frac{k^t}{1+n}, l)) k^t + T_{t+1})} = 1 + (1-\tau_{t+1}) F_K(\tfrac{k^t}{1+n}, l) \tag{43}$$

$$\frac{1}{\beta} \frac{u'(F_L(\frac{k^{t-1}}{1+n}, l) l - k^t - p_t M^t)}{\pi u'((1 + (1-\tau_{t+1}) F_K(\frac{k^t}{1+n}, l)) k^t + p_{t+1} M^t)} = \frac{p_{t+1}}{p_t}. \tag{44}$$

Therefore, at the steady state the equations are

$$\frac{1}{\beta} \frac{u'(F_L(\frac{k}{1+n}, l) l - k - m)}{\pi u'((1 + F_K(\frac{k}{1+n}, l)) k + (1+n) m) + \tilde{\pi} u'((1 + F_K(\frac{k}{1+n}, l)) k)} = 1 + (1-\tau) F_K(\tfrac{k}{1+n}, l) \tag{45}$$

$$\frac{1}{\beta} \frac{u'(F_L(\frac{k}{1+n}, l) l - k - m)}{\pi u'((1 + F_K(\frac{k}{1+n}, l)) k + (1+n) m)} = 1 + n. \tag{46}$$

since, as it can be readily checked, at the steady state

$$\tau F_K(\frac{k}{1+n}, l) k = T \tag{47}$$

$$1 + (1-\tau) F_K(\frac{k}{1+n}, l) = (1+n) \frac{1 + F_K(\frac{k}{1+n}, l) + F_{KK}(\frac{k}{1+n}, l) \frac{k}{1+n}}{1 + n + F_{KK}(\frac{k}{1+n}, l) \frac{k}{1+n}} \tag{48}$$

so that the steady state equations under this policy of taxes and transfers are actually

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k}{1+n},l)l-k-m)}{\pi u'((1+F_K(\frac{k}{1+n},l))k+(1+n)m)+\tilde{\pi}u'((1+F_K(\frac{k}{1+n},l))k)} = (1+n)\frac{1+F_K(\frac{k}{1+n},l)+F_{KK}(\frac{k}{1+n},l)\frac{k}{1+n}}{1+n+F_{KK}(\frac{k}{1+n},l)\frac{k}{1+n}} \tag{49}$$

$$\frac{1}{\beta}\frac{u'(F_L(\frac{k}{1+n},l)l-k-m)}{\pi u'((1+F_K(\frac{k}{1+n},l))k+(1+n)m)} = 1+n \tag{50}$$

that is to say, those of the best possible steady state with risky money.

A few remarks are in order at this point. First note that the tax and transfers policy announced at any period t is defined as a function of the capital savings decided by the generation born at $t-1$. Therefore, the policy is defined in terms of information that is known at the time of its announcement, and is not manipulable by the agents to which it applies. Second, by construction the government does not incur in any deficit or superavit at the steady state, since the amount raised by the tax in a distortionary way is given back as a lump sum to the same agents in the same period.

A number of issues remain to be addressed in this setup as, for instance, studying the dynamics out of the steady state, the cost of moving to such a steady state, the endogeneization of the probabilty of breakdown of the intergenerational transfers mechanism. These and other issues are left for further research in the future

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