The Optimality of Delegation under Imperfect Commitment

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Abstract

Should a principal delegates authority (decision right) to his or her agent if the agent has private information? This paper answers this question under the “imperfect commitment” assumption that compensation schemes are contractable but decisions are not verifiable. Our conclusions are that (i) the allocation of authority is determined by a trade-off between a self-commitment cost due to centralization and an incentive cost due to delegation, (ii) and that the principal should adopt a performance-based compensation scheme under both delegation and centralization; however, the optimal compensation schemes are rather different. Furthermore, we find a new kind of advantage from delegation.

KEYWORDS: Authority, Commitment, Hierarchy, Compensation.
JEL Classification: C72, D24, D82, D86

1 Introduction

A design problem about an allocation of authority in organizations is interrelated closely to their incentive systems. Wulf (2007) finds that the pay for division managers with broader authority is more dependent on performance of their firms. Nagar (2002) examines the banking industry in U.S. and finds a positive correlation between the extent of delegation and

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the incentive intensity of compensation scheme. As a result of the linkage between the two problems, the informativeness of performance plays an important role not only in incentive systems but also in an allocation of authority. Colombo and Delmastro (2004) consider an impact of advanced communication technologies on the allocation of authority among Italian manufacturing plants and their parent companies. They conclude that an introduction of the communication technologies improves the performance measures, which increases a probability of delegation.

The above empirical evidence naturally poses the following theoretical questions. Why does a compensation design based on more informative performance fosters a decentralized decision process? What effect does the allocation of authority make on an optimal compensation scheme? More generally, what combination of the authority allocation and the compensation is optimal and what factors influence the optimal compensation?

Few theoretical studies, however, consider the above questions. While many of the existing studies on an optimal allocation of authority focus on “local information” (i.e., an informational advantage of a division manager over their headquarters), their frameworks inherently involve an analytical difficulty to address the two design problem at one time, i.e. an allocation of authority and an incentive design.

The first approach on an allocation of authority points out a trade-off as follows under an “incomplete contract assumption” in which the headquarters cannot write any contract (Alonso and Matouschek, 2005; Dessein, 2002; and Harris and Raviv, 2005; Holmstrom, 1984; Jensen and Meckling, 1992) but does not consider an optimal compensation scheme. On the one hand, in cases where a headquarters holds authority (centralization), the headquarters who is ignorant about the information his or her subordinates have will invariably make an inappropriate decision. On the other hand, if the headquarters delegates authority to his or her subordinate (decentralization), the subordinate, who is not under the control of the headquarters, abuses the authority. In this approach, therefore, the answer is that if the conflict of interests between a headquarters and his or her subordinate is not serious, authority should be delegated to the subordinate. Although the trade-off explains why a headquarters should delegate authority to his or her subordinate, these papers fail to analyze the optimal compensation scheme because of the definition of the incomplete contract assumption.

By adopting the “complete contract assumption” instead, in which a

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1 The similar observations are obtained in Demers et al. (2002), Foss and Laursen (2005) and DeVaro and Kurtulus (2006).
2 Abernethy et al. (2004) and Moers (2006) also observe the similar facts.
3 While many papers consider utilization of local information as a benefit of delegation, some papers focus on another effect (Aghion and Tirole, 1997; Baker et al., 1999; Bolton and Farrell, 1990; Athey and Roberts, 2001). For example, Aghion and Tirole (1997) argues that a benefit of delegation is to strengthen an incentive of authority holder.
Table 1: The Complete Contract, Imperfect Commitment, and Incomplete Contract Assumptions

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<tr>
<th></th>
<th>Message</th>
<th>Performance</th>
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<tr>
<td>Complete contract</td>
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<td>Imperfect commitment</td>
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<td>Not V</td>
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<tr>
<td>Incomplete contract</td>
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Note that: V denotes “verifiable.”

complete contract can be written (i.e., a compensation and decisions are contingent on message), we find an optimal compensation scheme. However, an optimal compensation scheme under a decentralized process cannot be analyzed as the revelation principle states that it is optimal for the headquarters to always maintain his authority (Melumad and Shibano, 1991 and Harris and Raviv, 1998). In sum, either approach, the incomplete contract or the complete contract, cannot analyze a relationship between authority allocation and incentive systems.

To address the two design problems, this paper considers the third, intermediate situation we call “imperfect commitment assumption.” Except for that assumption, our model is similar to a well-known standard adverse selection model. There are a headquarters and a plant manager who have to decide whether to install a high-performance machine for production or a low-performance machine. While the high-performance machine brings greater profit to the headquarters through improvement in a quality of products, it also entails greater maintenance costs to the plant manager such as the frequent cleaning or adjustment of the machine. The plant manager privately knows the maintenance cost caused by introduction of the machine in the plant while the headquarters does not know it since she does not observe the maintenance ability of the plant. The timing of the model is as follows. After observing the private information, the plant manager sends a message about the private information to the headquarters. Second, the holder of authority makes a decision. We call the decision process centralized if the headquarters makes the decision and the process decentralized or delegation of authority if the plant manager makes the decision.

The important assumptions that distinguishes our paper from the previous existing literature are that (i) the decision itself is not verifiable but division manager’s message is verifiable, (ii) the headquarters can measure a noisy but verifiable performance of the plant manager about the decision (e.g., production volume and yield rate of production). In other words, firms

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4According to the custom in contract theory literature, we use “she” to refer to a pronoun of a principal or a headquarters, and the pronoun “he” to refer to an agent or a plant manager.
cannot design a decision rule contingent on message but can design a compensation scheme contingent on the manager’s message and performance. Our paper calls this situation an “imperfect commitment assumption.”

This is an intermediate assumption between the two assumptions in the existing literature (see table 1). While the incomplete commitment assumption eliminates a contingent compensation (i.e. any variable is not verifiable), the complete contract assumption means that firms can develop a message-contingent decision rule to completely control decision-making activities. Under the imperfect commitment assumption, firms can develop a compensation scheme but their ability to influence decision-making activities is limited.5

By analyzing our model, we find a new trade-off between centralization and delegation. Suppose that the decision process is centralized. In this case, although the headquarters prefers to install the low performance machine in the plant of low ability manager *ex ante*, she *ex post* is tempted to install high-performance machine regardless of the maintenance ability of the plant manager, as she is not concerned about the plant manager’s pain (maintenance cost) for operating the plant normally.6 This commitment problem causes a self-commitment cost because the headquarters has to devise a compensation scheme based on a noisy performance to deter herself from acting in this manner. Under a decentralized decision process, the headquarters bears an incentive cost to lead the plant manager to make a appropriate decision, because the manager prefers to the low-performance machine occurring the low maintenance cost. Therefore, the optimal allocation of authority in organizations is determined by the trade-off between the self-commitment cost and the incentive cost. This is different from the trade-off which is typically considered under the incomplete contract assumption (Dessein, 2002; Holmstrom, 1984; and Jensen and Meckling, 1992).

An analysis of the trade-off gives us four results. First, it is shown that although the headquarters can write a contract, she strictly prefers delegation to centralization if the self-commitment cost outweighs the incentive cost. As a result, we can analyze an optimal allocation of authority and an optimal incentive design at one time. Second, the headquarters should design a compensation scheme contingent on performance under both delegation and centralization, however the optimal schemes are rather different. Under delegation, the headquarters should offer to a high ability agent a compensation

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5The other specification of this intermediate situation is also feasible. Rantakari (2007) considers the situation in which performance is verifiable but message and decision are not verifiable.

6A result in the standard adverse selection model is that an *ex ante* optimal decision of the principal is not optimal *ex post*. Under the imperfect commitment assumption, this time-inconsistency causes the self-commitment problem, i.e., the principal has an incentive to overturn the *ex ante* optimal decision, because decisions are not verifiable by the definition.
more dependent on his performance and to a low ability agent a compensation that is less dependent on his performance, as the headquarters wants the high ability plant manager to install high-performance machine. Under centralization, the principal adopts a performance-based compensation scheme for the low ability plant manager as the principal needs to refrain from installing it excessively. Third, delegation is more likely as (i) decisions are more important for the headquarters; (ii) local information is more important; and (iii) performance measure is more informative. The comparative statics are consistent with the empirical findings.

Finally, delegation offers a unique advantage under the imperfect commitment assumption. Although the self-commitment cost due to centralization is always present, the headquarters sometimes can avoid the incentive cost under the delegation. This is because (i) a high ability plant manager has information rent like in the standard adverse selection model and (ii) under delegation, by redistributing the rent, the headquarters can give the high ability plant manager the incentive to choose a costly machine without any cost. On the other hand, when the decision process is centralized, the headquarters cannot use the information rent for the self-commitment as the low ability plant manager, who has been assigned a low performance machine to, earns no information rent. These factors make delegation more beneficial than centralization.

There are many papers that analyze the allocation of authority. One approach assumes the complete contract circumstance but avoids the revelation principle by introducing new elements of organizations into the model: (i) communication cost (Segal, 2001; Melumad et al., 1992; Bolton and Dewatripont, 1994; Zandt, 1999; Radner, 1993; Melumad et al., 1997), (ii) renegotiation (Poitevin, 2000), and (iii) collusion (Baliga and Sjostrom, 1993; Melumad et al., 1997; Laffont and Martimort, 1998; Faure-Grimaud et al., 2003; Mookherjee and Tsumagari, 2004). Another approach deals with no-information side of delegation under the incomplete contract assumption (Aghion and Tirole, 1997; Baker et al., 1999; Bolton and Farrell, 1990; Athey and Roberts, 2001). Our paper is different from these papers as ours consider the informational side without introducing these elements.

Our model is closely related to Krishna and Morgan (2005), Ottaviani (2000), Prendergast (2002), and Rantakari (2007). They, too, focus on the assumption that the decision is not verifiable and compensation is feasible. However, Krishna and Morgan (2005) and Ottaviani (2000) are different from ours in that (i) the ability of the principal to contract is more limited (the contract is contingent only on the agent’s message) and (ii) they implicitly assume incentive systems in organizations (the agent’s utility function is single-peaked). While Prendergast (2002) and Rantakari (2007) do not allow contracts dependent on a message substantially, the principal designs

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the message-contingent compensation in our model.

The remainder of the paper is organized as follows. Section 2 develops our model. In section 3, we consider the complete contract and the imperfect contract cases as benchmarks. In section 4, a basic trade-off between centralization and delegation under the imperfect commitment assumption is shown. In section 5, we analyze the conditions that delegation is optimal. In section 6, we discuss our results. Section 7 provides some concluding remarks on our model.

2 Framework

Circumstances We consider a principal (e. g., a headquarters) and an agent (e. g., her plant manager) who has to choose a decision $d$ from a set of available decisions $D = \{d_H, d_L\}$ ($\Delta d = d_H - d_L > 0$) such as introducing a high-performance machine or a low performance-machine to a plant managed by the agent. When a decision $d$ is executed, the principal obtains $v_P(d)$ ($\Delta v_P = v_P(d_H) - v_P(d_L) > 0$) and the agent bears cost $\theta x$.

$\theta \in \Theta = \{\theta_0, \theta_1\}$ is the random variable that represents the nature of decisions ($\Delta \theta = \theta_1 - \theta_0 > 0$) and whose density function is $f(\theta)$. We denote $f = f(\theta_0)$ and $1 - f = f(\theta_1)$. For example, while the high-performance machine brings greater profit to the headquarters through improvement in a quality of some product, it also entails greater maintenance costs to the plant manager such as the frequent cleaning or adjustment of the machine.\(^8\) Such maintenance costs depend on an ability of a plant manager $\theta$ as a high ability manager can deal with an accident to the machine very well. Only the agent is informed about $\theta$.

The decision making process has two stages: (i) the agent sends a message, $m \in M$, about private information to the principal and (ii) after the message is sent, either the principal or the agent —depending on who has the decision right— chooses $d$. Authority is delegated to the agent (the decision process is decentralized) if the agent chooses $d$. A decision process is centralized if the principal chooses $d$.

The execution of the decision generates a noisy signal about the decision ($y \in Y = \{G, B\}$). The probability on $y$ when a decision $d$ is executed, is denoted by $g(y; d)$. Let $\Delta g = g(G; d_H) - g(G; d_L) > 0$. We assume that $y$ is independent of $\theta$. Examples of $y$ is internal evaluation of performance in the plant such as production volume or yield rate of production. When $\theta$ and $y$ are generated, the \textit{ex post} payoffs of the principal and the agent are given by

\[
U_P(x, w; \theta, y) = v_P(d) - w, \\
U_A(x, w; \theta, y) = w - \theta d,
\]

\(^8\)This story is consistent with Colombo and Delmastro (2004).
where \( w \) is a compensation described in some contract.

In this circumstance, \( d_H \) is efficient if marginal benefit of \( d \) exceeds its marginal cost (i.e., \( \Delta v_P \geq \theta \Delta d \)). An efficient pair of decisions is given by

\[
(d(\theta_0), d(\theta_1)) = \begin{cases} 
(d_H, d_H) & \text{if } \Delta v_P > \theta_1 \Delta d \\
(d_H, d_L) & \text{if } \theta_1 \Delta d \geq \Delta v_P \geq \theta_0 \Delta d \ \\
(d_L, d_L) & \text{if } \theta_0 \Delta d > \Delta v_P
\end{cases}
\]

The second case is particularly important as the principal needs the agent’s private information to implement the efficient pair of decisions. Our analysis in the subsequent sections begins with the second case, then proceeds to all the cases.

**Contract** The principal can design a contract but her ability to write the contract is limited. We use the following terminology of the principal’s ability to commit. The ability is called “complete” (complete contract assumption) if message \( m \), signals \( y \) and decision \( d \) are verifiable, while “no ability to commit” (incomplete contract assumption) means that message \( m \), signals \( y \), and decision \( d \) are not verifiable. We call the ability imperfect (imperfect commitment assumption) if message \( m \) and signal \( y \) are verifiable, while \( d \) are not verifiable. The imperfect commitment assumption reflects more realistic situations where firms can design the compensation scheme contingent not on decisions but on performance. We thus adopt the imperfect commitment assumption in most cases, while the complete contract assumption and the incomplete contract assumption are treated as benchmarks.\(^9\) Under the imperfect commitment assumption, the principal designs a compensation scheme contingent on a signal \( y \) and a message \( m \), i.e., \( w(y, m) \).

Throughout this paper, we focus on a contract in which the agent’s \( ex \)

\(^9\)Along with the imperfect commitment assumption, we implicitly assume that the decision right \( D \) is transferred contractably. One might question why the decision set \( D \) is contractable while elements in the decision set are not contractable, in particular, one may question why the principal does not overturn the authority when delegating it to the agent. The answer is that if the delegation is \( ex \) ante beneficial for the principal, she can honor the delegation of authority by using the following procedure: (i) the principal does not monitor the agent’s decision making process (e.g., by increasing the physical distance between herself and the agent, or by removing the monitoring institution); (ii) the agent sends message and implements the decision at the same time. Under this procedure, the principal is always concerned about her \( ex \) ante payoff even if the turnover of authority is feasible and thus, keeps the promise.

\(^{10}\)Throughout this paper, we assume that performance \( y \) is most informative and thus the headquarters does not use her profit \( v_P(d) \) on an optimal compensation. The headquarters’ profit \( v_P(d) \) is indeed a random variable and is usually determined not only by her plants but also by her marketing departments, her R&D departments and so on. Thus, we consider \( v_P(d) \) as less informative measure than \( y \).
post payoff is nonnegative, i.e.,

\[ w(\theta, y) - \theta_i d \geq 0 \quad \text{for any } \theta_i, y. \] (ex-PCiy)

The ex post participation constraint (ex-PCiy) means that \( i \)-type agent has no wealth to afford the pecuniary maintenance cost \( \theta_i d \). As described in an analysis of the subsequent two sections, these constraints imply that an indirect control over an authority holder through a noisy signal \( y \) is costly.\textsuperscript{11}

**Timing and Payoffs** The timing of the game is given as follows.

1. The agent privately observes \( \theta \).
2. The principal offers a contract \( \{w(m, y)\} \) and allocates the control right.
3. The agent accepts or rejects the contract.
4. The agent sends a message \( m \in M \).
5. The principal or the agent makes a decision.
6. \( y \) is observed and the contract is executed.

We call “interim” a point in time before observing performance after observing private information, while the ex ante before observing performance and private information. The interim payoffs are denoted by

\[
E_y[U_P(d, w; \theta, y)] = v_P(d) - \sum_{y \in Y} g(y; d)w(\theta, y),
\]

\[
E_y[U_A(d, w; \theta, y)] = \sum_{y \in Y} g(y; d)w(\theta, y) - \theta d.
\]

The ex ante payoffs are denoted by

\[
E_{y,\theta}[U_P(d, w; \theta, y)] = \sum_{\theta_i \in \Theta, y \in Y} (v_P(d) - g(y; d)w(\theta_i, y))f(\theta_i),
\]

\[
E_{y,\theta}[U_A(d, w; \theta, y)] = \sum_{\theta \in \Theta, y \in Y} (g(y; d)w(\theta, y) - \theta d)f(\theta_i).
\]

**3 Benchmarks**

In this section, we establish two benchmark results under the complete contract assumption and the incomplete contract assumption.

\textsuperscript{11}We assume (ex-PCiy) in order to avoid the unessential classification. Of course, the utilization of a noisy signal causes costs together with limited liability constraints, i.e., \( w(\theta, y) \geq 0 \) for any \( \theta, y \). We discuss this point in concluding remarks.
Benchmark under the Complete Commitment Assumption

We begin with the analysis of the centralized decision process under the complete contract assumption that \( d, m, \) and \( y \) are verifiable. Let \( (d(m), w(m, y)) \) be an allocation when \( m \) is sent and \( y \) is observed. The principal’s problem is to choose a contract \( \{(d(m), w(m, y))\} \) to maximize her \textit{ex ante} expected payoff. We assume that \( M = \Theta \) without loss of generality from the revelation principle.

In choosing centralized decision process, the principal faces the following problem,

\[ \max_{\{(d, w)\}} E_{\theta, y} [v_P(d(\theta)) - w(\theta, y)] \]

\[ \text{s.t. } E_{\theta}[w(\theta_i, y)] - \theta_i d(\theta_i) \geq 0 \quad \text{for any } \theta_i, \quad (PC_i) \]

\[ E_{\theta}[w(\theta_i, y)] - \theta_i d(\theta_i) \geq E_{\theta}[w(\theta_j, y)] - \theta_i d(\theta_j) \quad \text{for any } \theta_i, \theta_j, \quad (IC_i) \]

\[ w(\theta_i, y) - \theta_i d \geq 0 \quad \text{for any } \theta_i, y. \quad (ex-PC_{i,y}) \]

The participation constraint \((PC_i)\) implies that an \( i\)-type agent must obtain at least his reservation utility, which we normalize to zero. The incentive compatibility constraint \((IC_i)\) is imposed on the problem in order to guarantee that the \( i\)-type agent reports the truth. \((ex-PC_{i,y})\) means that \( i\)-type agent has no wealth to afford the pecuniary cost \( \theta_i d \).

This problem is essentially equivalent to a standard adverse selection problem. Although the model differs from the standard adverse selection model in that a signal \( y \) is verifiable, the use of \( y \) does not improve the principal’s payoff as \( y \) is only a noisy signal of \( d \) and the latter is verifiable. Without the loss of generality, we assume that \( w(\theta, G) = w(\theta, B) \) for any \( \theta \) and thus we ignore \((ex-PC_{i,y})\).

By applying the standard procedure to this problem,\(^{12}\) the constraints in the problem are reduced to the binding \((PC1)\), binding \((IC0)\), and the so-called monotonic condition, i.e.,

\[ w(\theta_1, y) - \theta_1 d(\theta_1) = 0, \quad (PC1') \]

\[ w(\theta_0, y) - w(\theta_1, y) = \theta_0 (d(\theta_0) - d(\theta_1)), \quad (IC0') \]

\[ d(\theta_0) \geq d(\theta_1). \quad (M) \]

Before deriving an optimal contract, we consider an optimal compensation scheme \( w(\theta, y) \), given \( (d(\theta_0), d(\theta_1)) = (d_H, d_L) \). In this case, the principal bear the so-called information rent to utilize the agent’s private information. The reason for this is explained in Figure 1. If \( \theta \) is verifiable, the optimal compensation scheme is \( w(\theta_0, G) = w(\theta_0, B) = \theta_0 d_H \) and

\(^{12}\)See contract theory textbooks such as ? and ?.
$w(\theta_1, G) = w(\theta_1, B) = \theta_1 d_L$. The principal’s expected payoff is $f(v_p(d_H) - \theta_0 d_H) + (1 - f)(v_p(d_L) - \theta_1 d_L)$. When $\theta$ is not verifiable, the scheme is not incentive compatible, as the $\theta_0$-type agent obtains a rent $\Delta \theta d_L$ by reporting the false type $\theta_1$ (Region B), i.e., the scheme violates (IC0'). To maintain the agent’s truth-telling, the principal must increase the compensation to the $\theta_0$-type agent $w(\theta_0, y)$ by $\Delta \theta d_L$ (Region A) and thus bears information rent $(f \Delta \theta d_L)$.

**Lemma 1.** Suppose that a complete contract is feasible and the principal’s decision is $(d(\theta_0), d(\theta_1)) = (d_H, d_L)$. An optimal compensation scheme is $(w(\theta_0, y), w(\theta_1, y)) = (\theta_0 d_H + \Delta \theta d_L, \theta_1 d_L)$ for any $y$. The principal’s payoff $\pi_{HL}^*$ is given by

$$E_{y, \theta}[v_P(d(\theta)) - w(\theta, y)] = f(v_p(d_H) - \theta_0 d_H - \Delta \theta d_L) + (1 - f)(v_p(d_L) - \theta_1 d_L).$$

When the principal freely chooses a decision pair $(d(\theta_0), d(\theta_1)), (d_H, d_L)$ is not always optimal. If the marginal cost of $\theta_0$-type agent with respect to $d$ (i.e., $\theta_0 \Delta d$) exceeds the marginal benefit ($\Delta v_p$), the principal prefers $(d_L, d_L)$ to $(d_H, d_L)$. If the marginal benefit is sufficiently large, $(d_H, d_H)$ is optimal. As a result, the principal’s optimal payoff and an optimal contract are shown in the following proposition.\(^{13}\)

**Proposition 1.** Suppose that a complete contract is feasible. We denote two threshold values by $k_1 = \left(\theta_1 + \frac{1}{\theta_0} \Delta \theta\right) \Delta d$ and $k_2 = \theta_0 \Delta d$.

\(^{13}\)The restriction on the available decision set $D$ to a binary set $\{d_H, d_L\}$ is important for the result.
1. If $\Delta v_p > k_1$, it is optimal that $(d(\theta_0), d(\theta_1)) = (d_H, d_H)$ and $w(\theta, y) = \theta_1d_H$ for any $\theta, y$. The principal’s expected payoff is given by $v_p(d_H) - \theta_1d_H$.

2. If $k_1 \geq \Delta v_p \geq k_2$, it is optimal that $(d(\theta_0), d(\theta_1)) = (d_H, d_L)$ and $(w(\theta_0, y), w(\theta_1, y)) = (\theta_0d_H + \Delta \theta d_L, \theta_1d_L)$ for any $y$. The principal’s expected payoff is given by $f(v_p(d_H) - \theta_0d_H - \Delta \theta d_L) + (1 - f)(v_p(d_L) - \theta_1d_L)$.

3. If $k_2 > \Delta v_p$, it is optimal that $(d(\theta_0), d(\theta_1)) = (d_L, d_L)$ and $w(\theta, y) = \theta_1d_L$ for any $\theta, y$. The principal’s expected payoff is given by $v_p(d_L) - \theta_1d_L$.

**Proof.** Straightforward.

For the convenience, we denote the optimal expected payoff of the principal given $(d(\theta_0), d(\theta_1)) = (d_l, d_m)$ by $\pi_{lm}$.

Then we easily show that the principal is indifferent between delegation and centralization under the complete contract assumption.

**Proposition 2.** Suppose that a complete contract is feasible. Then the principal is indifferent between delegation and centralization.

**Proof.** Since the revelation principle implies that the principal’s payoff under delegation is always realized under centralization, we here show the converse (the principal’s optimal payoffs under centralization can also be achieved in the decentralized decision process). Suppose that the principal prefers to $(d(\theta_0), d(\theta_1)) = (d_H, d_L)$. Let $w(m, d)$ be the compensation contingent on the agent’s message and the agent’s decision. We set $w(m, d)$ as follows.

$$
w(\theta_0, d_H) = \theta_0d_H + \Delta \theta d_L, \quad w(\theta_0, d_L) = \theta_0d_L,
$$

$$
w(\theta_1, d_H) = \theta_1d_L, \quad w(\theta_1, d_L) = \theta_1d_L.
$$

We can show that the principal’s payoff under this compensation scheme is equivalent to one under centralization, while the agent tells the truth about his ability and chooses a pair of desirable decisions $(d(\theta_0), d(\theta_1)) = (d_H, d_L)$. Thus, the principal can mimics the outcome of centralization.

**Benchmark under the Incomplete Contract Assumption** In this case, any variable is not verifiable and hence the principal pays a fixed amount $w$. When the principal maintains authority, she maximizes $v_P(x) - w$ subject to (PC1) and (PC2). As a result, the principal always chooses $d_H$ and pays $\theta_1d_H$ to the agent. The principal’s payoff is $v_p(d_H) - \theta_1d_H$.\(^{14}\)

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\(^{14}\)In this setting, communication as a cheap-talk game is infeasible since the conflict of both parties is sufficiently large. In fact, the agent’s optimal report strategy is to always report $\theta_L$, as telling $\theta_H$ implies that the principal chooses $d_H$. Therefore, no separating equilibrium exists.
If authority is delegated to the agent, he always chooses $d_L$ to maximize $w - \theta d$ and the principal pays a compensation $\theta_1 d_L$ to satisfy (PC1) and (PC2). The principal’s payoff is $v(d_L) - \theta_1 d_L$. Therefore, we obtain the following proposition.\footnote{Alonso and Matouschek (2005), Dessein (2002), Holmstrom (1984), and Jensen and Meckling (1992) consider the incomplete assumption and obtain the similar results.}

**Proposition 3.** Suppose that the principal cannot write any contract. If $\Delta v_p < \theta_1 \Delta d$, delegation is strictly preferred to centralization; otherwise, the principal prefers centralization.

*Proof.* Straightforward. \qed

## 4 Imperfect Commitment Case

In this section, a trade-off between centralization and delegation is analyzed under the imperfect commitment assumption in which decision $(d)$ is not verifiable. We study each decision process separately and compare them. To show the trade-off clearly, we focus on the case where the principal implements $(d(\theta_0), d(\theta_1)) = (d_H, d_L)$. The other cases are considered in the next section.

### 4.1 Centralization

Suppose that the decision process is centralized. In this case, the principal faces a self-commitment problem: since the principal cannot commit to the decision *ex ante*, she chooses $d$ to maximize her *interim* payoff after receiving the agent’s report, i.e.,

$$v_p(d) - E_y[w(\theta, y)] = v_p(d) - w(\theta, B) - g(G; d)[w(\theta, G) - w(\theta, B)],$$

where we assume that $M = \Theta$ without loss of generality.\footnote{Bester and Strausz (2001) point out that the classic revelation principle does not hold if the principal cannot commit to some control (i.e., $d$ in our model). However, we can show that in our model where $D$ is binary, the revelation principle holds.} If $y$ is not utilized ($w(\theta, G) = w(\theta, B)$), the principal has an incentive to choose $d_H$ regardless of the agent’s report, as it brings a higher *interim* payoff to the principal ($v_p(d_H) > v_p(d_L)$ for any $\theta$).

By promising high incentive intensity to the agent *ex ante* (increasing $w(\theta, G) - w(\theta, B))$, the principal is less tempted to choose $d_H$ *interim* since the choice of $d_H$ increases not only $v(d)$ but also the expected payment. In order to implement $(d(\theta_0), d(\theta_1)) = (d_H, d_L)$, the principal has to devise a compensation scheme satisfying the following conditions which we call...
self-commitment constraints (SCC), i.e.,

\[ v_p(d_H) - \sum_{y \in Y} g(y; d_H)w(\theta_0, y) \geq v_p(d_L) - \sum_{y \in Y} g(y; d_L)w(\theta_0, y), \]

\[ v_p(d_L) - \sum_{y \in Y} g(y; d_L)w(\theta_1, y) \geq v_p(d_H) - \sum_{y \in Y} g(y; d_H)w(\theta_1, y), \]

or equivalently,

\[ w(\theta_0, G) - w(\theta_0, B) \leq S, \quad \text{(SCC0)} \]

\[ w(\theta_1, G) - w(\theta_1, B) \geq S, \quad \text{(SCC1)} \]

where \( S = \Delta v_p = \frac{v_p(d_H) - v_p(d_L)}{g(G; d_H) - g(G; d_L)} > 0. \)

The optimal problem becomes the following.

\[
\begin{align*}
\max_{\{w(\theta, y)\}} & \quad E_{y, \theta}[v_p(d) - w(\theta, y)] \\
\text{s.t.} & \quad (\text{PC}_i), (\text{IC}_i), (\text{ex-PC}_{iy}), \text{and} (\text{SCC}_i).
\end{align*}
\]

The principal designs a compensation scheme to maximize her \textit{ex ante} payoff subject to (PCi), (ICi), (ex-PCiy), and (SCCi).

By the following lemma, we confirms that only \( \theta_0 \)-type agent acquires the information rent like the complete contract case.

**Lemma 2.** In the solution of \([P-2]\), (i) (PCi) is slack for any \( \theta_i \); (ii) (IC1) is slack; (iii) (IC0) is binding.

**Proof.** See lemma 4 in appendix.

The reason for (i) in the lemma is that (PCi) is implied by (ex-PCiG) and (ex-PCiB). (ii) and (iii) are similar to the complete contract case. By telling a lie, \( \theta_0 \)-type agent expects the principal to choose \( d_L \), which entails less costs on the agent. Therefore, the \( \theta_0 \)-type agent drives the information rent.

The following proposition shows an optimal compensation scheme.

**Proposition 4.** Suppose that the ability of commitment is imperfect, the principal holds authority, and the principal implements \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\).

An optimal compensation scheme is

\[ w(\theta_0, G) = w(\theta_0, B) = \theta_0d_H + \Delta \theta d_L + g(G; d_L)S, \]
\[ w(\theta_1, G) = \theta_1d_L + S, w(\theta_1, B) = \theta_1d_L. \]

The principal’s payoff is given by

\[ E_{\theta, \theta}[v_p(d(\theta)) - w(\theta, y)] = \pi^*_H - g(G; d_L)S, \]

where \( \pi^*_H \) is the principal’s payoff under the complete contract assumption.
Proof. By the lemma 2, the principal maximize her payoff subject to the binding (IC0), (ex-PCI1y) and (SSC1) for any $\theta_1$ and $y$. We first show that in the solution, (i) (ex-PCI1G) is slack; (ii) (ex-PCI1B) is binding; and (iii) (SSC1) is binding.

(i): By some manipulation, (ex-PCI1G) becomes

$$w(\theta_1, G) - \theta_1 d_L = [w(\theta_1, B) - \theta_1 d_L] + (w(\theta_1, G) - w(\theta_1, B)) \geq 0.$$ 

Then, (ex-PCI1B) and (SSC1) imply (ex-PCI1G).

(ii) and (iii): By substituting the binding (IC0) into the objective function, it yields

$$E_\theta[v_p(d(\theta))] + f \theta_0 \Delta d - w(\theta_1, B) - [w(\theta_1, G) - w(\theta_1, B)].$$

This implies (ii) and (iii), as $w(\theta_1, B)$ and/or $w(\theta_1, G) - w(\theta_1, B)$ increase the expected payment without violating (ex-PC1G0) and (ex-PC1B). Therefore, (ex-PC1B) and (SSC1) are binding, i.e., $w(\theta_1, G) = 0 d_L + S$ and $w(\theta_1, B) = \theta_1 d_L$.

Second, we show that there are many $w(\theta_0, G)$ and $w(\theta_0, B)$ satisfying (ex-PC0G), (ex-PC0B), (SCC0), and the binding (IC0). By substituting $w(\theta_1, G) = \theta_1 d_H + S$ and $w(\theta_1, B) = \theta_1 d_L$ into the binding (IC0), it yields

$$\sum_{y \in Y} g(y; d_H)w(\theta_0, y) = \theta_1 d_H + \theta_0 \Delta d + g(G; d_L)S.$$ 

Since $\sum_{y \in Y} g(y; d_H)w(\theta_0, y) - \theta_0 d_H > 0$, there exists many $w(\theta_0, G)$ and $w(\theta_0, B)$. One simple example of the schemes is that $w(\theta_0, G) = w(\theta_0, B) = \theta_0 d_L + \Delta \theta d_L + g(G; d_L)S$.

In contrast to the complete contract case, this proposition shows that the principal should offer a compensation contingent on $y$ to the $\theta_1$-agent as she faces the self-commitment constraints, i.e.,

$$w(\theta_1, y) - w(\theta_1, y) \geq S (>) 0.$$ 

In the other words, the incentive intensity of the $\theta_1$-agent’s scheme should be non-negative.

This requirement for the high incentive intensity results in additional costs to the principal in two ways (see Figure 2). The first one is through the “direct effect.” To commit to $d_L$, the principal must increase the compensation $w(\theta_1, G)$, as the agent is protected by the ex post participation constraints. The increase of $w(\theta_1, G)$ decreases the principal’s payoff by $(1 - f)g(G; d_L)S$.

At the same time, the change in the compensation to the $\theta_1$-type agent results in the second cost to the principal. The increase of $w(\theta_1, G)$ increases the $\theta_1$-type agent’s expected payoff. This means that the $\theta_0$-type agent
obtains more rent (Region B in Figure 2) by sending a false message $\theta_1$. To maintain truth-telling, the principal additionally pays the $\theta_1$-type agent by an amount shown by Region A. Thus, the principal’s payoff decreases by $fg(G; d_L)S$, which is the “indirect effect.” In the aggregate, the principal bears a self-commitment cost $(1 - f)g(G; d_L)S + fg(G; d_L)S = g(G; d_L)S$.

Note that as the performance is more informative (a likelihood ratio of “good” performance $g(G; d_H)/g(G; d_L)$ increases) and the marginal benefit of $d$ is smaller ($\Delta v$ decreases), the self-commitment cost decreases. In either case, it is more easy for the principal to commit to choose $d_L$.

### 4.2 Delegation

Next, we consider delegation. In this case, as the agent has a discretion on $d$, the principal faces moral hazard problem, which requires the performance-based compensation scheme. The agent makes a decision to maximize his payoff, i.e.,

$$E_y[U_A(d, w; \theta, y)] = w(\theta, B) + g(G; d)[w(\theta, G) - w(\theta, B)] - \theta_1 d.$$

If $y$ is not utilized ($w(\theta, G) = w(\theta, B)$), the agent has no incentive to choose $d_H$ as $d_H$ is more costly ($\theta d_H > \theta d_L$ for any $\theta$). To implement the desirable decision $((d(\theta_0), d(\theta_1)) = (d_H, d_L))$, the principal has to design the compensation scheme satisfying

$$w(\theta_0, G) - w(\theta_0, B) \geq I_0, \quad \text{(DC0)}$$

$$w(\theta_1, G) - w(\theta_1, B) \leq I_1, \quad \text{(DC1)}$$
where \( I_0 = \frac{\theta_0 \Delta d}{\Delta g} \) and \( I_1 = \frac{\theta_1 \Delta d}{\Delta g} \).

In addition, the agent has more incentive to tell a lie under delegation. After telling a lie \( m = \theta_j \), the \( \theta_1 \)-type agent does not have to choose \( d(\theta_j) \) as he has a discretion on \( d \) to maximize \( \sum_{y \in Y} w(\theta_j; y)g(y; d) - \theta_id(\theta_i) \). Since this increases the benefit from telling a lie, \( (\text{ICi}) \) in the benchmark problem becomes more severe, i.e., for any \( \theta_i, \theta_j \),

\[
g(G; d(\theta_i))w(\theta_i, G) + g(B; d(\theta_i))w(\theta_i, B) - \theta_id(\theta_i) \\
\geq \max_d \left[ g(G; d)w(\theta_j, G) + g(B; d)w(\theta_j, B) - \theta_jd \right]. \quad (\text{D-ICi})
\]

Therefore, the principal’s optimization problem changes as follows.

\[
\begin{align*}
\max_{\{w(\theta, y)\}} & \quad E_{\theta, \theta}[v_p(d) - w(\theta, y)] \\
\text{s.t.} & \quad (\text{PCi}), (\text{D-ICi}), (\text{ex-PCiy}), \text{ and (DCi}).
\end{align*}
\]

To solve this problem, we show the following lemma.

**Lemma 3.** Suppose that the ability of commitment is imperfect, authority is delegated to the agent, and the principal implements \( (d(\theta_0), d(\theta_1)) = (d_H, d_L) \). If \( \{w(\theta, y)\} \) is a solution to following \([P-3']\), then it is also a solution to \([P-3]\).

\[
\begin{align*}
\max_{\{w(\theta, y)\}} & \quad E_{\theta}[v_p(x) - \sum_{y \in Y} g(y; x)w(\theta, y)] \\
\text{s.t.} & \quad w(\theta_1, B) = w(\theta_0, B) + g(G; d_L)I_0, \quad (\text{D-IC'}') \\
& \quad w(\theta_0, G) - w(\theta_0, B) = I_0, \quad (\text{DC0'}') \\
& \quad w(\theta_1, G) - w(\theta_1, B) = 0, \quad (\text{DC1'}') \\
& \quad (\text{ex-PCiy}).
\end{align*}
\]

**Proof.** See Appendix. \(\square\)

\(\text{(DC0'}')\) means that the principal has to devise a performance-based compensation scheme to lead the \( \theta_0 \)-type agent to choose \( d_H \), while \( \text{(DC1}') \) means that the principal does not have to do that for the \( \theta_1 \)-type.

Under \( \text{(DC0}') \) and \( \text{(DC1}') \), \( \text{(D-IC0)} \) and \( \text{(D-IC1)} \) are equivalent to \( \text{(D-IC}') \). This implies that unlike the complete contract assumption we cannot exclude the possibility that the \( \theta_1 \)-type agent tells a lie (i.e., \( \text{(D-IC1)} \) is binding).\(^{17}\) While the principal has to pay a large amount to him in order to give the \( \theta_0 \)-type agent an incentive (as he is protected by \text{ex post} participation

\(^{17}\)Note that \( \text{(DC0}') \) and \( \text{(DC1}') \) are only sufficient condition of a solution to \([P-3]\). Therefore, some compensation scheme may not satisfy either \( \text{(D-IC0)} \) or \( \text{(D-IC1)} \).
constraints), this scheme strengthens the \( \theta_1 \)-type agent’s incentive to report \( m = \theta_0 \) as described below.

By solving the [P-3'], we obtain one of the optimal compensation schemes.

**Proposition 5.** Suppose that the ability of commitment is imperfect, authority is delegated to the agent, and the principal implements \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\). If \( \Delta d_L \geq g(G; d_H)I_0 \), the following compensation scheme is optimal.

\[
\begin{align*}
\text{w}(\theta_0, G) &= \theta_0 d_H + \Delta d_L + (1 - g(G; d_H))I_0, \\
\text{w}(\theta_0, B) &= \theta_0 d_H + \Delta d_L - g(G; d_H)I_0, \\
\text{w}(\theta_1, G) &= \text{w}(\theta_1, B) = \theta_1 d_L.
\end{align*}
\]

The principal achieves the same payoff as in the complete contract benchmark \( \pi_{HL}^* \). If \( \Delta d_L < g(G; d_H)I_0 \), the following compensation scheme is optimal.

\[
\begin{align*}
\text{w}(\theta_0, G) &= \theta_0 d_H + I_0, \\
\text{w}(\theta_0, B) &= \theta_0 d_H, \\
\text{w}(\theta_1, G) &= \text{w}(\theta_1, B) = g(G; d_L)I_0 + \theta_0 d_L.
\end{align*}
\]

The principal’s payoff is \( \pi_{HL}^* - [g(G; d_H)I_0 - \Delta d_L] \) where \( \pi_{HL}^* \) is the principal’s payoff under the complete contract assumption.

**Proof.** By substituting (D-IC'), (DC0') and (DC1') into the objective function, we obtain

\[
\begin{align*}
f(v_p(d_H) - w(\theta_0, B) - g(G; d_H)I_0) \\
+ (1 - f)(v_p(d_L) - w(\theta_0, B) - g(G; d_L)I_0).
\end{align*}
\]

\( w(\theta_0, B) \) is constrained by only (ex-PC0B) and (ex-PC1B) as we can easily show that (ex-PCiB) implies (ex-PCiG) (since (DC0') and (DC1')). By substituting (DC0') and (DC1') into (ex-PC0B), (ex-PC1B) and (D-IC'), we obtain

\[
\begin{align*}
w(\theta_0, B) &\geq \theta_0 d_H, \quad \text{(ex-PC0B')}
\end{align*}
\]

\[
\begin{align*}
w(\theta_0, B) &\geq \theta_0 d_H + [\Delta d_L - g(G; d_H)I_0]. \quad \text{(ex-PC1B')}
\end{align*}
\]

If \( \Delta d_L - g(G; d_H)I_0 \geq 0 \), (ex-PC1B') is binding \( w(\theta_0, B) = \theta_0 d_H + [\Delta d_L - g(G; d_H)I_0] \). If \( \Delta d_L - g(G; d_H)I_0 < 0 \), (ex-PC0B') is binding \( w(\theta_0, B) = \theta_0 d_H \). Therefore, we obtain an optimal compensation scheme. \( \square \)

If the principal delegates authority to the agent and prefers the \( \theta_0 \)-type agent to choose \( d_H \), a compensation to \( \theta_0 \)-agent should be contingent on
performance, as $d_H$ is costly for the agent. Therefore, the principal gives the $\theta_0$-type agent the incentive to choose $d_H$, i.e.,

$$w(\theta_0, G) - w(\theta_0, B) \geq I_0.$$ 

One might think that this additional constraint brings the principal an additional cost, as the agent is protected by ex post participation constraints. However, the principal’s payoff does not change if $\Delta\theta d_L \geq g(G; d_H)I_0$. $\theta_0$-type agent already earns information rent represented by the shaded area in Figure 3. By redistributing it, the principal can give the incentive to the $\theta_0$-type without changing the expected payment like Figure 3. Therefore, the principal can avoid the additional cost to lead the $\theta_0$-type agent to choose $d_H$.

If the information rent is not large enough to give the incentive to the $\theta_0$-type agent, delegation requires the additional cost which we call “incentive cost” (see Figure 4). Like the self-commitment cost under centralization, this cost comes through “direct effect” and “indirect effect.” Firstly, the principal must increase the expected compensation to the $\theta_0$-type agent by $g(G; d_H)I_0 - \Delta\theta d_L$, as the principal cannot give him the incentive only by redistributing the information rent and the agent is protected by ex post participation constraints. The principal’s expected payoff thus decreases by Region A ($f[g(G; d_H)I_0 - \Delta\theta d_L]$).

This causes “indirect effect” through (D-IC1). The increases of the principal’s expected payment means that the $\theta_1$-type agent has more incentive to report $\theta_0$. If the $\theta_1$-type agent reports the false message $\theta_0$, he obtains
\[ \theta_0 d_H + g(G; d_L)I_0 - \theta_1 d_L, \text{ which is equal to the area of Region A. To maintain the } \theta_1 \text{-type agent’s truth-telling, the principal has to pay more to the } \theta_1 \text{-type agent as in Region B } ((1 - f)[g(G; d_H)I_0 - \Delta \theta d_L]). \text{ As the result, the principal has to bear } g(G; d_H)I_0 - \Delta \theta d_L \text{ as incentive cost.}

Note that as the performance is more informative (a likelihood ratio of “good” performance \( g(G; d_H)/g(G; d_L) \) increases), the incentive cost decreases because it is more easy for the principal to give the agent an incentive.

### 4.3 Delegation vs Centralization

We compare delegation with centralization in the case where the ability of commitment is imperfect. Basically, the allocation of authority is determined by a trade-off between the incentive cost due to delegation and the self-commitment cost due to centralization. The difference between the principal’s payoffs in both decision processes is given as follows

\[
\begin{align*}
(\text{Delegation}) - (\text{Centralization}) &= [\pi_{HL} - \max\{ (g(G; d_H)I_0 - \Delta \theta d_L), 0 \}] - [\pi_{HL} - g(G; d_L)S], \\
&= - \max\{ [g(G; d_H)I_0 - \Delta \theta d_L], 0 \} + g(G; d_L)S.
\end{align*}
\]

(1)

The first term is the incentive cost, while the second term is the self-commitment cost. If the self-commitment cost outweighs the incentive cost, it is optimal for the principal to delegate authority to the agent; otherwise, the centralized decision process is optimal.
Proposition 6. Suppose that the ability of commitment is imperfect and the principal implements \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\). If \(g(G; d_L)S \geq (g(G; d_H)I_0 - \Delta \theta d_L)\), the principal prefers to delegate authority than to retain it; otherwise, the principal prefers to maintain authority than to delegate it.

We emphasize that our model reveals a new advantage of delegation over centralization. While the centralized decision process always causes the self-commitment cost \(g(G; d_L)S\) (see proposition 4), the principal bears no incentive cost if \(\Delta \theta d_L \geq g(G; d_H)I_0\) (see proposition 5). This is because (i) the \(\theta_0\)-type agent has information rent as in the standard adverse selection model and (ii) under delegation, by redistributing the rent, the principal can give the \(\theta_0\)-type agent the incentive to choose a difficult project without any additional cost.

We obtain the following comparative statics.

Corollary 1. Suppose that the ability of commitment is imperfect and the principal implements \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\). As \(\Delta v\) increases, or \(\theta_1\) decreases, the principal increasingly prefers a decentralized decision process to a centralized one.

First, inequality (1) shows that the principal increasingly prefers delegation to centralization as the information is more important (\(\theta_1\) increases \(\Delta \theta\) for the fixed \(\theta_0\)). The reason in our model is that the increase of \(\Delta \theta\) enlarges the agent’s capacity to tolerate the movement in compensation by increasing the \(\theta_0\)-agent’s information rent. Dessein (2002) also points out a similar result for the different reason, which the increase of \(\Delta \theta\) aggravates the loss of information as a demerit of centralization. However, we cannot observe his logic in our model such that while the centralized decision process requires the information rent, the decentralized decision process does not require it. Under the imperfect commitment case, the principal bears the information rent in either decision process, because even in the decentralized decision process the private information is useful to evaluate the agent’s performance.

Second, the delegation is beneficial for the principal as \(\Delta v\) increases as the increase of \(\Delta v\) makes the self-commitment problem more serious under centralization.

Third, an impact of improvements in the performance measure on the optimal allocation of authority is ambiguous, as that improvement reduces both the incentive cost and the self-commitment cost. As described in the next section, which also consider \((d_H, d_H)\) and \((d_L, d_L)\), we will show that delegation is more likely to be optimal as the performance measure is more informative.
5 Indirect Control and an Advantage of Delegation

In the previous section, we have showed a trade-off between centralization and delegation and found an advantage of delegating authority if the principal implements \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\). However \((d_H, d_L)\) may not be always \textit{ex ante} optimal for the principal. If the cost to implement \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\) is very high, she may give up the advantage and attempt to implement \((d_H, d_H)\) or \((d_L, d_L)\). In this section, we analyze all the possibilities of choice, including \((d_H, d_H)\) and \((d_L, d_L)\).

This section provides two main results. First, we confirm that the principal actually derives an advantage from delegating authority. In other words, delegation is optimal under the condition where the principal prefers \((d_H, d_L)\). Second, the comparative statics on a likelihood of \(y\) and \(\Delta v_P\) changes: delegation is more likely to be optimal as the performance measure is more informative and/or marginal benefit is greater. As described in next section, these results is consistent with empirical findings.

Instead of \(g(G; d_L)/g(G; d_H)\), we treat \(\Delta g/g(G; d_H)\) as the informativeness of performance, i.e.,

\[
\frac{\Delta g}{g(G; d_H)} = 1 - \frac{g(G; d_L)}{g(G; d_H)},
\]

where \(g(G; d_L)/g(G; d_H)\) is the reciprocal of a likelihood ratio of performance “G.” If performance provides no information on the decision, i.e., \(g(G; d_H) = g(G; d_L)\), \(\Delta g/g(G; d_H)\) is equal to zero. This measure is equal to one if the principal perfectly observes the decision of the authority holder through performance, i.e., \(g(G; d_H) = 1\) and \(g(G; d_L) = 0\).

Although the economic intuitions of these results is simple, there are many cases that we must separately handled. We explain the results and their intuition in this section and their formal procedures are given in appendix. Like the previous section, we study each decision process separately and compare them.

5.1 Centralization

Suppose that the decision process is centralized. The next proposition describes what it costs to implement each pair of decisions under delegation.

**Proposition 7.** Suppose that the ability of commitment is imperfect and the principal holds authority. Let \(\pi_{lm}\) be the optimal expected payoff of the principal for \((d(\theta_0), d(\theta_1)) = (d_l, d_m)\) under the complete contract assumption.

\(^{18}\Delta g/g(G; d_H) = 1\), however, does not imply perfect information since if \(g(G; d_L) = 0\) and \(g(G; d_H) > 0\), this measure is equal to zero but the performance is still noisy.
Figure 5: The Optimal Decision Pair under the Centralized Case

1. If the principal implements \((d_H, d_H)\), her expected payoff is \(\pi_{HH}^*\).

2. If the principal implements \((d_H, d_L)\), her expected payoff is \(\pi_{HL}^* - g(G; d_L)S\).

3. If the principal implements \((d_L, d_L)\), her expected payoff is \(\pi_{LL}^* - g(G; d_L)S\).

If the principal implements \((d_H, d_L)\) or \((d_L, d_L)\), the principal bears the self-commitment cost, while she does not in the other case \((d_H, d_H)\). Recall that the self-commitment cost comes from the ex post temptation of the principal to choose \(d_L\). Since the implementation of \((d_H, d_H)\) is congruent with the principal’s ex post temptation, centralization does not cause the self-commitment cost.

By comparing these payoffs, we obtain Figure 5 which represents the optimal decision pair. If performance is sufficiently informative \((\Delta g/g(G; d_H) > \theta_0/\theta_1)\), the optimal decision pair changes from \((d_L, d_L)\) to \((d_H, d_L)\), and from \((d_H, d_L)\) to \((d_H, d_H)\) as \(\Delta v_P\) increases, because the larger \(\Delta v_P\) means that \(d_H\) is more beneficial for the principal.

As performance becomes less informative \((\Delta g/g(G; d_H)\) decreases), the region of \((d_H, d_L)\) is gradually replaced by \((d_H, d_H)\). Since the self-commitment cost does not occur in the case \((d(\theta_0), d(\theta_1)) = (d_H, d_H)\), less informative performance makes it more difficult for the principal to commit in both case \((d_L, d_L)\) and case \((d_H, d_L)\), while the principal’s payoff in \((d_H, d_H)\) does not change. Therefore, if performance is sufficiently less informative...
(\(\Delta g/g(G;d_H) < \theta_0/\theta_1\)), the principal gives up using the agent’s private information. Eventually, \((d_H,d_H)\) is optimal for any \(\Delta v_P\).

### 5.2 Delegation

Next we next consider an optimal contract when authority is delegated to the agent. The following proposition describes what it costs to implement each pair of decisions under delegation.

**Proposition 8.** Suppose that the ability of commitment is imperfect and the principal holds authority. Let \(\pi_m\) be the optimal expected payoff of the principal for \((d(\theta_0),d(\theta_1)) = (d_l,d_m)\) under the complete contract assumption.

1. If the principal implements \((d_H,d_H)\), her expected payoff is \(\pi_{HH}^* - g(G;d_H)I_1\).

2. If the principal implements \((d_H,d_L)\), her expected payoff is \(\pi_{HL}^* - \max \{g(G;d_H)I_0 - \Delta \theta x_L, 0\}\).

3. If the principal implements \((d_L,d_L)\), her expected payoff is \(\pi_{LL}^*\).

If the principal implements \((d_L,d_L)\), delegation does not cause the incentive cost since the agent has an incentive to choose \(d_L\) without performance-based compensation. The implementation of \((d_H,d_H)\) requires some incentive cost as the principal has to use a noisy performance to give the agent an incentive.

By comparing these payoffs, we obtain Figure 6. If performance is sufficiently informative \((\Delta g/g(G;d_H) \geq \theta_0 \Delta x/\Delta \theta x_L)\), the optimal decision pair changes from \((d_L,d_L)\) to \((d_H,d_L)\), and from \((d_H,d_L)\) to \((d_H,d_H)\) as \(\Delta v_P\) increases, because the larger \(\Delta v_P\) means that \(d_H\) is more beneficial for the principal.

If performance is less informative \((\Delta g/g(G;d_H) < \theta_0 \Delta x/\Delta \theta x_L)\), the region of \((d_H,d_L)\) and \((d_H,d_H)\) moves rightwards as \(\Delta g/g(G;d_H)\) decreases. Note that the incentive cost occurs in both case \((d_H,d_H)\) and case \((d_H,d_L)\), while does not in case \((d_L,d_L)\). Since the less informative performance increases the incentive cost, the principal increasingly prefers \((d_L,d_L)\) as \(\Delta g/g(G;d_H)\) decreases.

### 5.3 Delegation vs Centralization

Now, we can completely compare delegation with centralization under the imperfect commitment assumption. By comparing the optimal payoffs in two decision processes, we obtain the following result.


Figure 6: The Optimal Decision Pair under the Decentralized Case

**Proposition 9.** Suppose that the ability of commitment is imperfect. The principal prefers to delegate authority than to retain it if

\[
\Delta v_p < \max\{\theta_1 \Delta d, \min\{k_1, k_1 - (I_0 - \Delta \theta d_L)/(1 - f)\}\}.
\]

Otherwise, the principal prefers to maintain authority than to delegate it.

Figure 7 shows the optimal allocation of authority for \(\Delta v_P\) and \(\Delta g/g(G; d_H)\). We remarks five points. First, we confirm that the principal derives an advantage from delegating authority if \(\Delta g/g(G; d_H) \geq \theta_0 \Delta d/\Delta \theta d_L\) and \(\Delta v_p \in [k_2, k_1]\). In this case, the incentive cost does not occur when the decision process is decentralized, while the self-commitment problem results in the principal bearing a cost. Therefore, the principal enjoys the advantage of delegation.

Second, delegation is not optimal even if the principal enjoys the advantage of delegation \((\Delta g/g(G; d_H) \geq \theta_0 \Delta d/\Delta \theta d_L)\). To implement \((d_H, d_H)\), the principal bears no self-commitment cost under the centralized decision process, while the incentive cost occurs under the decentralized decision process. If \(d_H\) is ex ante optimal \((k_1 < \Delta v_p)\), he principal prefers centralization.

From the first and the second point, we can observe the case that the principal achieves the second-best outcome even if the commitment ability is imperfect. For the large \(\Delta v_p\), the self-commitment cost does not occur under centralization, while there is no incentive problem under delegation for the small \(\Delta v_p\). In the middle range of \(\Delta v_p\), the principal enjoys the advantage of delegation and thus avoids the incentive cost under delegation.
As a result, the implementation cost under the imperfect commitment is the same as that under the complete contract assumption. Of course, this argument holds only if performance is sufficiently informative.

Fourth, as the performance measure is more informative, the larger is the range of $\Delta v_P$ for which delegation is optimal. In particular, this point is clearly shown in the case $\theta_1 \Delta d < \Delta v_P < k_1$, where $(d_H, d_L)$ is optimal. The reason is as follows. Recall that $\Delta v_P$ determines the extent of the self-commitment problem due to centralization (see (SC$i$)), while the moral hazard problem due to delegation relies on $\theta_0 \Delta d$ (see (D-IC$i$)). Since the optimality of $(d_H, d_L)$ requires that the marginal benefit of $d_H$ ($\Delta v_P$) is at least larger than its marginal cost ($\theta_0 \Delta d$), it implies that the principal more easily solves the incentive problem than the self-commitment problem. As a result, more informative performance measure make delegation more advantageous and thus we obtain the monotonic relationship between delegation and informativeness of performance.

Finally, if performance is so noisy, the principal gives up the performance-based compensation in either decision processes. In this case, the threshold value is the same as that under the incomplete contract assumption.

6 Discussions

6.1 Empirical Evidence

Proposition 9 implies a comparative statics in our model.
Corollary 2. Suppose that the ability of commitment is imperfect. The principal increasingly prefers a decentralized decision process to a centralized one as (i) $\Delta v_P$ decreases; (ii) $\theta_1$ increases; and (iii) $\Delta g/g(G; d_H)$ increase.

This means that delegation is more likely as (i) decisions are more important for the principal; (ii) local information is more important; and (iii) performance measure is more informative.

These results are consistent with the empirical findings.\(^{19}\) To capture the informativeness of performance measure, Moers (2006), by using the questionnaires, measures the extent to which performance measure is influenced by outside or internal uncontrollable factors such as economic conditions and the decisions made in the other parts of organization. He found that the improvement in quality of performance measure increases a likelihood of delegation.

Colombo and Delmastro (2004) tests (i), (ii) and (iii) by examining an allocation of authority among Italian manufacturing plants and their parent companies. They consider “Local Area Network” and/or on-line connection to be monitoring systems of headquarters, and find a positive correlation between an adaptation of such communication systems and delegation.\(^{20}\) They also find that delegation is enhanced by the informational advantage and importance of decisions. The former is measured by the number of hierarchical level of the plant and sizes of the plant, which the latter is measured by capital intensity.

Abernethy et al. (2004) examines a randomly selected sample of companies listed on the Amsterdam Stock Exchange to test (ii) and (iii). As the improved performance measure, they consider DSMs (divisional summary performance measure), which are designed to evaluate performance on decisions of division managers, and find that the introduction of that system increases the possibility of delegation. It is also found that asymmetrical information increases the extent to which the headquarters delegate authority to the manager.

6.2 The Optimal Allocation of Authority under Different Assumptions

Figure 7 also provides the robustness of the results under different assumptions. The complete contract assumption corresponds with a case $\Delta g/g(G; d_H) = 1$, while the imperfect contract assumption with a case

\(^{19}\)We explain only empirical evidence closely related to our results. See Rantakari (2007) for more detail who reviews empirical studies on relationships among incentive, risk, and authority.

\(^{20}\)While they recognize the possibility that the network reduces the informational advantage of the plant managers, they interpret them as the above on the base of their empirical result.
\[ \Delta g/g(G;d_H) = 0. \Delta g/g(G;d_H) \in (0,1) \] means the imperfect commitment case.

**Corollary 3.** 1. The principal is more likely to prefer delegation to centralization when the ability to commit is imperfect than when the ability to commit is complete.

2. The principal is more likely to prefer delegation to centralization when the ability to commit is imperfect than when there is no ability to commit.

Under the complete contract assumption, centralization and delegation are indifferent. This result is so sensitive since delegation is optimal even for slightly noisy performance. This result comes from self-commitment problem under centralization. On the other hand, the result under the incomplete contract assumption is robust. If performance is so noisy, the performance-contingent compensation scheme is so costly and thus the principal faces the trade-off under the incomplete contract assumption, i.e., between the loss of control due to a delegation and information loss due to centralization. In this sense, the incomplete contract assumption may be more appreciate for a researcher to analyze the situations where the targeted firms cannot use informative performance.

### 6.3 Message-contingent Delegation

Actual decision processes often employs a mix of centralization and delegation. For example, an allocation of authority among the headquarters and plant managers may be dependent on the manager’s report.\(^{21}\) If such message-contingent delegation is feasible with sufficiently small fixed costs, how does the optimal allocation of authority change? While under the complete contract assumption the message-contingent is never optimal, it is optimal under the imperfect commitment assumption. The optimal allocation of authority is modified as in figure 8.

Suppose that the principal implements \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\) and performance measure is not informative \((\Delta g/g(G;d_H) < \theta_0 \Delta d/\Delta d_L)\). In this case, the unconditional delegation causes the incentive problem of the \(\theta_0\)-type agent, while the unconditional centralization induces the self-commitment problem when message is \(\theta_1\). These problems are completely settled by using the following message-contingent delegation: the principal maintains authority if \(m = \theta_0\) and she delegates authority to the agent if \(m = \theta_1\). Both the principal’s \textit{ex post} temptation under centralization and the agent’s choice under delegation accord with the \textit{ex ante} optimal choice. Therefore, while

\(^{21}\)Krahmer (2006) considers the message-contingent delegation, but it has not analyzed a relationship between performance and the message-contingent delegation.
it is costly for the principal to write the message-contingent delegation, she achieves the second-best payoff.

If the performance is sufficiently informative, the principal does not have to design the message-contingent delegation. Under the unconditional delegation, the principal achieves the second-best payoff as the redistribution of the information rents solves the incentive problem without any additional costs. With consideration for the writing cost, the message-contingent delegation is not optimal. As the result, we obtain Figure 8.

**Proposition 10.** Suppose that the ability of commitment is imperfect and a message-contingent delegation is feasible with sufficiently small fixed costs. If \( k_2 < \Delta v_P < k_1 \) and \( \frac{\Delta g}{g(G; d_H)} > \theta_0 \frac{\Delta d}{\Delta \theta d_L} \), the message-contingent delegation is optimal.

This proposition implies that the message-contingent delegation is more likely as performance measure is less informative. This is a testable implication which the existing literature has not examined.

7 Concluding Remarks

We examine the optimality of delegation under the imperfect commitment assumption. Our conclusions are that (i) the principal strictly prefers delegation to centralization if (a) decisions are more important for the principal; (b) the local information is more important; and (c) performance measure is
more informative; (ii) the principal should adopt a performance-based compensation scheme under both delegation and centralization; however, the structures of the optimal compensation schemes are rather different, and (iii) the principal prefers delegation to centralization to greater extent than she does in the incomplete contract or the complete contract case. Furthermore, we find a new advantage from delegation.

In our model, we has assumed that the agent is protected by ex post participation constraints. This assumption plays an important role in that the use of performance measures endogenously causes costs, both the self-commitment cost due to centralization or the incentive cost due to delegation. As modeling of the endogenous costs in the other manners, a limited liability constraint is also possible.

Even if the principal faces a participation constraint and the limited liability constraints, most of our results holds. This solely means that more severe compensation is feasible, i.e., the principal can offer the compensation below the agent’s cost \( \theta d > w(\theta, y) \geq 0 \). This relaxed constraints causes the case that the principal can avoid the self-commitment cost, as the fluctuation of compensation in the range of \( \theta d > w(\theta, y) \geq 0 \) does not cause any cost. Of course, this argument is also applied to the decentralized case and thus our results of the comparative statics and the advantage of delegation hold.

Finally, we close this paper with a discussion of some possible extensions. In our model, we focus on the interaction between the allocation of authority and compensation schemes. There, however, are the other incentive systems in organizations, such as promotions and the addressing of career concerns. How these incentive systems affect the optimal allocation of authority is an interesting future direction for our research.

Appendix

A Proof of lemma 3

Lemma 3. Suppose that the ability of commitment is imperfect, authority is delegated to the agent, and the principal implements \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\). If \( w \) is an optimal compensation scheme of the following \([P-3']\),
then it is also an optimal compensation scheme of \([P-3]\).

\[
[P-3']
\]

\[
\max_w E_\theta[v_p(x) - \sum_{y \in Y} g(y; x)w(\theta, y)]
\]

s.t.
\[
w(\theta_1, B) = w(\theta_0, B) + g(G; d_L)I_0, \quad \text{(D-IC')}
\]
\[
w(\theta_0, G) - w(\theta_0, B) = I_0, \quad \text{(DC0')}
\]
\[
w(\theta_1, G) - w(\theta_1, B) = 0, \quad \text{(DC1')}
\]
\[
(\text{ex-PCIy}).
\]

**Proof.**  

- \(w(\theta_1, G) - w(\theta_1, B) = 0\)
  
  Suppose that there exists an optimal compensation \(\{w(\theta, y)\}\) satisfying \(w(\theta_1, G) - w(\theta_1, B) > 0\) and all the other constraints of problem \([P-3]\). We define a new scheme \(\{\tilde{w}(\theta, y)\}\) as follows.

\[
\tilde{w}(B, \theta_1) = w(B, \theta_1) + g(G; d_L)(w(\theta_1, G) - w(\theta_1, B)),
\]
\[
\tilde{w}(G, \theta_1) = w(G, \theta_0), \quad \tilde{w}(B, \theta_0) = w(B, \theta_0).
\]

In this scheme, we observe that (i) \(\{\tilde{w}(\theta, y)\}\) does not change the principal’s payoff (since \(\sum_{y \in Y} \tilde{w}(y, \theta)g(y; d(\theta)) = \sum_{y \in Y} w(y, \theta)g(y; d(\theta))\)); (ii) \(\{\tilde{w}(\theta, y)\}\) satisfies (ex-PCIy); (iii) \(\{\tilde{w}(\theta, y)\}\) satisfies (D-IC1) since \(\tilde{w}(\theta_1, y) = w(\theta_1, y)\).

A remaining part of the proof is that \(\{\tilde{w}(\theta, y)\}\) satisfies (D-IC0), i.e.,

\[
\tilde{w}(\theta_0, B) + g(G; d_H)(\tilde{w}(\theta_0, G) - \tilde{w}(\theta_0, B)) - \theta_0d_H \geq \max_d \{\tilde{w}(\theta_1, B) + g(G; d)(\tilde{w}(\theta_1, G) - \tilde{w}(\theta_1, B)) - \theta_0d\}.
\]

By \(\tilde{w}(\theta_1, G) - \tilde{w}(\theta_1, B) = 0\), (D-IC0) is rewritten by

\[
\tilde{w}(\theta_0, B) + g(G; d_H)(\tilde{w}(\theta_0, G) - \tilde{w}(\theta_0, B)) - \theta_0d_H \geq \tilde{w}(\theta_1, B) - \theta_0d_L,
\]

By using the definition of \(\{\tilde{w}(\theta, y)\}\),

\[
w(\theta_0, B) + g(G; d_H)(w(\theta_0, G) - w(\theta_0, B)) - \theta_0d_H \geq w(B, \theta_1) + g(G; d_L)(w(\theta_1, G) - w(\theta_1, B)) - \theta_0d_L.
\]

Since \(\{w(\theta, y)\}\) is an optimal compensation scheme, the above inequality holds. Therefore, \(\{\tilde{w}(\theta, y)\}\) is also an optimal compensation of \([P-3]\).
• \( w(\theta_0, G) - w(\theta_0, B) = I_0 \)

Suppose that there exists an optimal compensation \( \{w(\theta, y)\} \) satisfying \( w(\theta_0, G) - w(\theta_0, B) > I_0 \) and all the other constraints of problem [P-3]. Without loss of generality, we assume \( w(\theta_1, G) = w(\theta_1, B) \). By using this scheme \( \{w(\theta, y)\} \), We define a new scheme \( \{\tilde{w}(\theta, y)\} \) as follows.

\[
\begin{align*}
\tilde{w}(\theta_0, B) &= \tilde{w}(\theta_1, B) - g(G; d_L)I_0, \\
\tilde{w}(\theta_0, G) - \tilde{w}(\theta_0, B) &= I_0, \\
\tilde{w}(\theta_1, G) &= \tilde{w}(\theta_1, B) = w(\theta_1, G) = (\theta_1, B).
\end{align*}
\]

We observe that \( \{\tilde{w}(\theta, y)\} \) satisfies (D-IC0), (DCi), and (ex-PCiy). Furthermore, \( \{\tilde{w}(\theta, y)\} \) increases the principal’s payoff. By subtracting the expected payment in \( \{w(\theta, y)\} \) from that in \( \{\tilde{w}(\theta, y)\} \), we obtain

\[
E_{y,\theta}[\tilde{w}(\theta, y)] - E_{y,\theta}[w(\theta, y)]
\]

\[
= f[\tilde{w}(\theta_0, B) + g(G; d_H)(\tilde{w}(\theta_0, G) - \tilde{w}(\theta_0, B)) - w(\theta_0, B) + g(G; d_H)(w(\theta_0, G) - w(\theta_0, B))]
\]

\[
= f[w(\theta_1, B) - w(\theta_0, B) - \theta_0\Delta d - g(G; d_H)(w(\theta_0, G) - w(\theta_0, B))]
\]

\[
\leq f[g(G; d_H)(w(\theta_0, G) - w(\theta_0, B)) - \theta_0\Delta d + \theta_0\Delta d - g(G; d_H)(w(\theta_0, G) - w(\theta_0, B))]
\]

The last inequality comes from the fact that \( \{w(\theta, y)\} \) satisfies (D-IC0). Therefore, we assume \( w(\theta_0, G) - w(\theta_0, B) = I_0 \) without loss of generality.

• \( w(\theta_1, B) - w(\theta_0, B) = g(G; d_L)I_0 \)

By substituting (DC0') and (DC1') into (D-IC0) and (D-IC1) respectively, and rearranging them, we obtain

\[ g(G; d_L)I_0 \geq w(\theta_1, B) - w(\theta_0, B) \geq g(G; d_L)I_0. \]

So we can replace (D-IC0) and (D-IC1) with (D-IC').

\[ \square \]

**B Proof of Proposition 9**

In this appendix, we analyze all the possibilities of choice, including \( (d_H, d_H) \) and \( (d_L, d_L) \). We compute an optimal compensation scheme under each decision process and prove proposition 9 by comparing them.

**B.1 Centralization**

Suppose that the decision process is centralized. All kinds of decisions namely, \( ((d_H, d_H), (d_H, d_L) \) and \( (d_L, d_L)) \), are considered. To address this,
we use \( d(\theta_i) \) to denote a decision that the principal implements if the agent’s message is \( \theta_i \). In this case, the self-commitment constraints are extended as follows. For each \( \theta_i \),

\[
d(\theta_i) = \arg \max_d E_y[v_p(d) - w(\theta_i, y)]; \quad (SCC_i-E)
\]
equivalently, for \( d' \neq d(\theta_i) \)

\[
(g(G; d(\theta_i)) - g(G; d'))[w(\theta_i, G) - w(\theta_i, B)] \geq v_p(d(\theta_i)) - v_p(d').
\]

If \( (d(\theta_0), d(\theta_1)) = (d_H, d_L) \), (SSC0-E) and (SSC1-E) are equivalent to (SSC0) and (SSC1).

The optimal problem becomes the following.

\[
\begin{align*}
\text{[P-2E]} \\
\max_{d, w} & \quad E_y, \theta_i[v_p(d(\theta_i) - w(\theta_i, y)) \\
\text{s.t.} & \quad (PC_i), (IC_i), (ex-PC_iy), \text{ and } (SCC_i-E).
\end{align*}
\]

In this problem, the principal designs a pair of decisions \( (d(\theta_0), d(\theta_1)) \) and a compensation scheme \( (w(\theta_i, y)) \) to maximize her \textit{ex ante} payoff subjected to the extended self-commitment constraints.

We first show that \([P-2E]\) can be replaced by the following \([P-2E']\).

**Lemma 4.** Suppose that the ability of commitment is imperfect and the principal holds authority. \([P-2E]\) is equivalent to the following problem.

\[
\begin{align*}
\text{[P-2E']} \\
\max_{d, w} & \quad E_y, \theta_i[v_p(d(\theta_i) - w(\theta_i, y)) \\
\text{s.t.} & \quad E_y[U_A(d(\theta_0), w(\theta_0, y); \theta_0, y)] = E_y[U_A(d(\theta_1), w(\theta_1, y); \theta_0, y)], \\
& \quad d(\theta_0) \geq d(\theta_1), \\
& \quad \text{(ex-PCiy) and } (SCC_i-E).
\end{align*}
\]

**Proof.** It is easily shown that (i) (IC0) and (IC1) → (M); (ii) (M) and (IC0') → (IC0) and (IC1); (iii) (IC0) and (PC1) → (PC0); and (iv) (LLC1G) and (LLC1B) → (PC1). By showing that an optimal \( H \) in \([P-2]\) satisfies (IC0'), we obtain the lemma.

Suppose that \( E_y[U_A(d(\theta_0), w(\theta_0, y); \theta_0, y)] > E_y[U_A(d(\theta_1), w(\theta_1, y); \theta_0, y)] \).

Then, we obtain

\[
E_y[U_A(d(\theta_0), w(\theta_0, y); \theta_0, y)] > E_y[U_A(d(\theta_1), w(\theta_1, y); \theta_0, y)] \\
\geq E_y[U_A(d(\theta_1), w(\theta_1, y); \theta_1, y)] \geq 0,
\]

where the last inequality is obtained from (PC1). Decreasing \( E_{y \in Y}[w(\theta_0, y)] \) slightly reduces the principal’s expected payment, while relaxing (IC1) without violating (PC1), (SCCi), and (LLCiY). This contradicts the assumption. \( \square \)
Similar to the complete contract case, (IC0) and (IC1) are replaced by (IC0') and (M), where (IC0') is the binding (IC0) and (M) is the so-called monotonic constraint.

The following proposition shows an optimal contract and the principal’s payoff when the principal maintains authority.

**Proposition 11.** Suppose that the ability of commitment is imperfect and the principal holds authority. The optimal contract and the principal’s payoff are as follows.

1. \( \frac{g}{g_H} > \frac{\theta_0}{\theta_1} \)
   
   (a) If \( \Delta v_p > k_1 - \frac{g(G; d_L)S}{1-f} \), \((d(\theta_0), d(\theta_1)) = (d_H, d_H)\) is optimal. The principal’s expected payoff is given by \( \pi_{HH}^* \).
   
   (b) If \( k_1 - \frac{g(G; d_L)S}{1-f} \geq \Delta v_p \geq k_2 \), \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\) is optimal. The principal’s expected payoff is given by \( \pi_{HL}^* - g(G; d_L)S \).
   
   (c) If \( k_2 > \Delta v_p \), \((d(\theta_0), d(\theta_1)) = (d_L, d_L)\) is optimal. The principal’s expected payoff is given by \( \pi_{LL}^* - g(G; d_L)S \).

2. \( \frac{g}{g_H} \leq \frac{\theta_0}{\theta_1} \)
   
   (a) If \( \Delta v_p \geq \theta_1 \frac{\Delta g}{g_H} \), \((d(\theta_0), d(\theta_1)) = (d_H, d_H)\) is optimal. The principal’s expected payoff is given by \( \pi_{HH}^* \).
   
   (b) If \( \Delta v_p < \theta_1 \frac{\Delta g}{g_H} \), \((d(\theta_0), d(\theta_1)) = (d_L, d_L)\) is optimal. The principal’s expected payoff is given by \( \pi_{LL}^* - g(G; d_L)S \).

**Proof.** We only consider \( d(\theta_0) = d(\theta_1) \) since \((d_H, d_L)\) has been already examined in proposition 4. By \( d(\theta_0) = d(\theta_1) \), (IC0') becomes

\[
E_y[w(\theta_0, y)] = E_y[w(\theta_1, y)]. \tag{2}
\]

By using this equation, we separately compute a solution in each case.

1. \((d(\theta_0), d(\theta_1)) = (d_H, d_H)\)
   
   In this case, we can easily demonstrate two things. First, (SCCi) is not binding as the optimal compensation scheme under the complete contract assumption satisfies (SCCi). Second, the optimal compensation scheme satisfies

\[
E_y[w(\theta_1, y)] = \theta_1 d_H. \tag{3}
\]

since \( E_y[w(\theta_1, y)] \geq \theta_1 d_H \) and (IC0') imply \( E_y[w(\theta_0, y)] > \theta_1 d_H \). From (2) and (3), we obtain

\[
E_y[w(\theta_0, y)] = E_y[w(\theta_1, y)] = \theta_1 d_H.
\]
Although there are many compensation schemes satisfying these constraints, all these schemes give the principal the same payoff, i.e., $v_p(d_H) - \theta_1 d_H$. A simple example of the schemes is $w(\theta_i, y) = \theta_1 d_H$ for any $i, y$.

2. $(d(\theta_0), d(\theta_1)) = (d_L, d_L)$
   
   By using (2), the objective function becomes
   
   $$v_p(d_L) - E_y[w(\theta_0, y)].$$

   Since it is shown that (i) (SCC1) and (SCC2) are binding and (ii) (ex-PC0B) and (ex-PC1B) are binding, the optimal scheme is $w(\theta, G) = \theta_1 d_L + S$ and $w(\theta, B) = \theta_1 d_L$ for any $\theta$.

By comparing the principal’s payoffs in these cases, we obtain the proposition.

B.2 Delegation

Next we consider an optimal contract when authority is delegated to the agent. Since all kinds of decisions $((d_H, d_H), (d_H, d_L),$ and $(d_L, d_L))$ are considered, for any $d' \neq d(\theta_i)$, $(DC_i)$ becomes

$$\left(g(G; d(\theta_i)) - g(G; d')\right)[w(\theta_i, G) - w(\theta_i, B)] \geq \theta_i (d(\theta_i) - d').$$

$(DC_i)$

If $(d(\theta_0), d(\theta_1)) = (d_H, d_L)$, $(DC_i)$ is equivalent to $(DC_i)$.

Therefore, the principal’s optimization problem changes as follows.

$$\begin{align*}
&P-3E \\
&\max_w E_\theta[v_p(x) - \sum_{y \in Y} g(y; x)w(\theta, y)] \\
&\text{s.t.} \quad (PC_i), (D-IC_i), (\text{ex-PC}_i), \text{ and } (DC_i). 
\end{align*}$$

In this problem, the principal designs a pair of decisions $(d(\theta_i))$ and a compensation scheme $(w(\theta, y))$ to maximize her ex ante payoff subjected to the extended incentive constraints.

By solving this problem, we obtain the following proposition.

Proposition 12. Suppose that the ability of commitment is imperfect and authority is delegated to the agent. The optimal contract and the principal’s expected payoff are as follows.

1. $\Delta g/g(G; d_H) \geq \theta_0 \Delta d/\Delta d_L$

   (a) If $\Delta v_p > k_1 + \frac{g(G; d_H)}{1 - f} I_1$, $(d(\theta_0), d(\theta_1)) = (d_H, d_H)$ is optimal. The principal’s expected payoff is given by $\pi_{HH}^* - g(G; d_H)I_1$. 

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By using this equation, we separately compute each solution. We consider case (\(D-IC1\)) become

\[\text{Proof.} \]

2. \(\Delta g/g(G; d_H) < 0\Delta d/\Delta \theta d_L\)

(a) If \(\Delta v_p > k_1 + g(G; d_H)[I_1 - I_0] + \Delta \theta x_L\), \((d(\theta_0), d(\theta_1)) = (d_H, d_H)\) is optimal. The principal’s expected payoff is given by \(\pi_{HH}^* - g(G; d_H)I_1\).

(b) If \(k_1 + g(G; d_H)[I_1 - I_0] + \Delta \theta x_L \geq \Delta v_p \geq k_2\), \((d(\theta_0), d(\theta_1)) = (d_H, d_L)\) is optimal. The principal’s expected payoff is given by \(\pi_{HL}^* - [g(G; d_H)I_0 - \Delta \theta x_L]\).

(c) If \(k_2 - [g(G; d_H)I_0 - \Delta \theta x_L] > \Delta v_p\), \((d(\theta_0), d(\theta_1)) = (d_L, d_L)\) is optimal. The principal’s expected payoff is given by \(\pi_{LL}^*\).

\(\text{Proof.}\) We consider case \((d_H, d_H)\) and \((d_L, d_L)\). By \(d(\theta_0) = d(\theta_1)\), \((D-IC0)\) and \((D-IC1)\) become

\[E_y[w(\theta_0, y)] = E_y[w(\theta_1, y)].\] (4)

By using this equation, we separately compute each solution.

1. \((d_H, d_H)\)

In this case, it is shown that (i) \((\text{ex-PCiB})\) and \((\text{DCi-E})\) imply \((\text{ex-PCiG})\), (ii) \((\text{ex-PC1B})\) is binding; (iii) \((\text{DC1-E})\) is binding. Here, we prove only (iii) since (i) and (ii) are trivial. Suppose that \(w(\theta_1, G) - w(\theta_1, B) = I_1 + \epsilon\) for positive \(\epsilon > 0\). By using (4) and binding \((\text{ex-PC1B})\), we obtain

\[E_y[w(\theta_0, y)] = E_y[w(\theta_1, y)] = g(G; d_H)(I_1 + \epsilon).\]

By decreasing \(\epsilon\), the principal’s payoff is improved. Then, \(w(\theta_1, G) - w(\theta_1, B) = I_1\).

While there are many \((w(\theta_0, G), w(\theta_0, B))\) satisfying \((\text{LLC0})\), \((\text{DC0-E})\), their expected payment in the optimal schemes must be the same as \(E_y[w(\theta_1, y)]\) since (4) holds. Therefore, one of the optimal compensation scheme is \((w(\theta_1, G), w(\theta_1, B)) = (\theta_1 d_H + I_1, \theta_1 d_H)\).

By substituting this compensation scheme into the objective function, the principal’s payoff is \(\pi_{HH}^* - g(G; d_H)I_1\).

2. \((d_L, d_L)\)

We can easily show that the optimal compensation scheme under the complete contract assumption \((3\ in\ proposition\ 1)\) satisfies \((\text{DCi-E})\) and (4). Therefore, the compensation scheme is optimal. Under the compensation, the principal’s payoff is \(\pi_{LL}^*\).
By a comparison of the principal’s payoffs in these cases, we obtain the proposition.

\[ \frac{\theta_0 \Delta x}{\Delta v_P} \]

\[ \frac{\theta_0}{\theta_1} \]

\[ k_2 \]

\[ k_1 \]

\[ \Delta v_P \]

\[ \Delta g \]

\[ g(G; d_H) \]

B.3 Delegation vs Centralization

The figure 9 represents the results in proposition 11 and 12. Note that case (a) and (b) disappear if \( \theta_0 \Delta d / \Delta \theta d_L < \theta_0 / \theta_1 \).

If \( \Delta v_P \leq k_2 \), delegation is optimal as it achieves the second-best outcome (Note that (DC0) and (DC1) are not binding). Delegation also achieves the second-best outcome if \( k_2 < \Delta v_P < k_1 \) and \( \Delta g / g(G; d_H) \geq \theta_0 \Delta d / \Delta \theta d_L \). Similarly, centralization is optimal if \( k_1 \leq \Delta v_P \).

Therefore, we have four cases; (a) \((d_H, d_L)\) under delegation versus \((d_H, d_L)\) under centralization; (b) \((d_L, d_L)\) under delegation versus \((d_H, d_L)\) under centralization; (c) \((d_H, d_L)\) under delegation versus \((d_L, d_L)\) under centralization; and (d) \((d_L, d_L)\) under delegation versus \((d_L, d_L)\) under centralization. By comparing these payoffs in each case, we obtain proposition 9.

References


