Cheap Talk with the Exit Option: A Model of Exit and Voice*

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Abstract

The paper presents a formal model of the exit and voice framework originated by Hirschman [7]. To be more precise, we modify the cheap talk model of Crawford and Sobel [4] such that the sender of a cheap talk message has the exit option. We demonstrate that the existence of the exit option may increase the informativeness of cheap talk and improve welfare if the sender's exit value is comparatively large. This result suggest that the exit reinforces the voice in that the credibility of the exit increases the informativeness of the voice.

Keywords: Exit, Voice, Cheap Talk, Informativeness, Credibility

Journal of Economic Literature Classification Numbers: D23, D82, J5, L22

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1 Introduction

Since the publication of the book "Exit, Voice, and Loyalty" by Hirschman [7], exit-voice perspective has been widely adopted by studies in the field of political science, and has also been extended to various studies on relationships and organizations such as buyer-seller relationships, employer-employee (or union) relationships, hierarchies, public services, political parties, family, and adolescent development (See Hirschman [7] and [8]).

Broadly speaking, the exit and the voice are alternative means of dealing with problems that arises within an ongoing relationship or organization. For example, consider a buyer-seller relationship. Suppose some unobservable shock hits the production process and deteriorates the quality of the seller's product. The quality deterioration is immediately observed by the buyer, while it is hard to perceive for the seller due to, say, the complexity of the the production process. In this situation, the buyer usually has two options. One is to stop buying the good; this is the exit option. The other is to express his dissatisfaction directly to the seller; this is the voice option. Hirschman insists that the voice as well as the exit is important for the sustainability of relationships and organizations—a concept that has been neglected in economics thus far.

With regard to the workings of the exit and the voice in a real economy, a point of considerable interest concerns how the exit interacts with the voice; this also constitutes the main point of Hirschman's discussion. From one perspective, the exit works as a complement to the voice. Indeed, Hirschman briefly points out in Palgrave's dictionary [8], "[t]he availability and threat of exit on the part of important customer or group of members may powerfully reinforce their voice." However, it is not very clear why and how the exit can reinforce the voice. The present paper aims to clarify this by analyzing a formal model of exit and voice.

In this paper, the exit is regarded as a decision to terminate an ongoing relationship. On the other hand, the voice is interpreted as an activity involving sending a costless message that enables the improvement of the relationship. In other words, we identify the voice with "cheap talk" for transmitting useful information. Among others, the model by Crawford and Sobel [4] (hereinafter referred to as CS) is the most successful one describing cheap talk with private information. We employ the CS model as the basis of the environment we consider in the present paper, and extend it to the situation in which the exit option is available.

The CS model has two players. One player possesses private information about the current state of the relationship which is randomly drawn. In order to transmit the information, he/she sends a costless message to his/her partner, and the latter responds with a decision affecting both the agents' payoffs. In CS, the latter is called the Receiver (R) while the former is called the Sender (S). CS shows that the incongruence between both agents' preferences restricts the informativeness of cheap talk; in particular, they demonstrate that perfect information transmission via cheap talk is impossible as an equilibrium behavior unless the agents' preferences completely coincide.

In the present paper, we assume that S has the exit option after he observes R's decision. When S exercises the exit option, both agents obtain some fixed values. Consider the case where S's exit value is large and R's exit value is small. In this case, R has strong incentive to prevent S from choosing the exit option, and therefore, R will make a decision that is desirable for S even if both agents' preferences differ. Expecting R's response, S has a strong incentive to transmit more accurate information via cheap talk. It follows that the existence of S's exit option increases the informativeness of cheap talk, which in turn may increase not only S's payoff but also *R's*. Moreover, we show that, as S's exit value approaches the largest payoff he/she can obtain from the current relationship, the information transmission via cheap talk in the most informative equilibrium becomes almost perfect. In other words, the exit reinforces the voice in that the existence of the exit increases the informativeness of the voice. This is the main result of the present paper.

CS also shows that the more congruent both agents' preferences are, the more informative is cheap talk on the most efficient equilibrium. In other words, in the situation without the exit option, the informativeness of cheap talk is determined mainly by the degree of incongruence between the agents' preferences. However, in the situation with the exit option, there is another determinant of the informativeness: *the credibility of the exit*. A larger exit value for S makes his/her choice of the exit option more credible, which in turn enables a more informative cheap talk transmission on the equilibrium. Thus, we show that informative cheap talk transmission can be carried out even if S's preference is not exactly similar to R's.

To the author's knowledge, the exit-voice perspective has seldom been analyzed in any formal model in economics despite the vast citations.[1] Banerjee and Somanthan [1] is an

[1]For efforts in political science, see, for example, the survey by Dowding et al. [6].

exception in that they present a game-theoretical model of voice. However, their model differs form ours in a few respects. First, they do not take the exit option into account, and therefore, they do not investigate the interplay between exit and voice, which the present paper places an emphasis on. On the other hand, they consider the collective aspect of voice formation, which is abstracted out from our model. In this regard, the present paper can be considered as a complement to theirs.

Apart from the exit-voice perspective, the CS model *per se* has still attracted considerable attention and has been extended to various directions.[2] However, the effect of the exit option on cheap talk has hardly been analyzed. As an exception, Matthews [11] deals with a cheap talk game with a congress and a president—the receiver and sender, respectively—with veto, which is a means similar to the exit option in our model. In particular, the timing of events in his model is approximately the same as ours. However, there is a large difference with respect to what private information pertains to. In Matthews, private information concerns the sender's preference, while in our model, it pertains to the current state of the relationship. One may consider such a difference to be small, but it leads to very different outcomes: in Matthews, the informativeness of cheap talk is constrained on a strict upper bound, independent from the exit value. On the other hand, we show that in our model, an equilibrium can be close to that with perfect information transmission to any degree. In other words, Matthews does not emphasize that the existence of the exit increases the informativeness of cheap talk, which is the main claim of the present paper.

The rest of the paper is organized as follows: in Section 2, we present a formal model of exit and voice. In Section 3, we analyze a somehow specific model, and present the main claim of this paper. In Section 4, we present a sufficient condition for the main claim to hold in a general model. In Section 5, we discuss a few assumptions in our model, and we conclude the paper in Section 6.

2 Model

There are two players, the sender (S) and the receiver (R). At the beginning of the game, Nature chooses a current state of the relationship between S and R, $t \in T$, according to a probability distribution $F(t)$. A realized state is observed by S, but not by R. Based on

[2] For example, see Krishna and Morgan [9], Battaglini [2], Dessein [5], and Chen et al. [3], among others.

this observation, S chooses a message $m \in M$ sent to R. This message is cheap talk in that it is payoff-irrelevant. After R receives S's message, R chooses an action $a \in A$ relevant to both players' payoffs. For $i = S, R$, let i's payoff be $y^i(t, a)$. We assume $T = M = [0, 1]$ and $A = \mathbb{R}$. For $i = S, R$, $y^i(t, a)$ is defined on $[0, 1] \times \mathbb{R}$. $F(t)$ has a continuous density $f(t)$ where $f(t) > 0$ for any $t \in T$. We assume that for $i = S, R$, $y^i(t, a)$ is twice continuously differentiable and

$$\begin{aligned} &\forall t\ \exists a \text{ such that } \frac{\partial y^i(t, a)}{\partial a} = 0, \\ &\forall t, \forall a,\ \frac{\partial^2 y^i(t, a)}{\partial a^2} < 0, \\ &\forall t, \forall a,\ \frac{\partial^2 y^i(t, a)}{\partial a \partial t} > 0. \end{aligned}$$

Up to this point, the ingredients are the same as in CS. We introduce the concept of exit. After observing R's action, S chooses whether to exit or stay. If S chooses to exit, S and R's payoffs are U^S and U^R. If S chooses to stay, S and R's payoffs are given by $y^S(t, a)$ and $y^R(t, a)$. S and R's equilibrium payoffs are denoted by $V^S(t, a)$ and $V^R(t, a)$. In other words, for $i = S, R$,

$$V^i(t, a) = \begin{cases} U^i, & \text{if S chooses to exit,} \\ y^i(t, a), & \text{if S chooses to stay.} \end{cases}$$

We assume that $U^S \leq Y^S$ and $U^R \leq Y^R$. Let $D^i = Y^i - U^i$ for $i = R, S$.

To simplify the analysis, we mainly use a more specific model. *A uniform-quadratic model (with constant bias)* is a model in which $F(t)$ is a uniform distribution function on $[0, 1]$ and R and S's stay payoffs are expressed as

$$\begin{aligned} y^R(t, a) &= Y^R - (t - a)^2, \\ y^S(t, a) &= Y^S - (t + b - a)^2, \end{aligned}$$

for some $b > 0$. b is called *a bias* which represents a degree of incongruence between S and R's optimal actions. A uniform-quadratic model was originally analyzed in Section 4 of CS. Moreover, we assume that $b < \frac{1}{2\sqrt{3}}$.

We consider a perfect Bayesian equilibrium as equilibrium concept. We also restrict our attention to the class of equilibria with pure strategies. A pure strategy perfect Bayesian equilibrium is defined by $(\mu, P, \alpha, \epsilon)$ in which

- $\mu : T \to M$: S's message strategy,
- $P : M \times T \to [0,1]$: R's posterior belief distribution function over T on the observation of m,
- $\alpha : M \to A$: R's action choice strategy, and
- $\epsilon : T \times A \to \{0,1\}$: S's exit strategy. To be more precise, $\epsilon = 1$ refers to exit, and $\epsilon = 0$ to stay.

The equilibrium conditions are

$$\mu(t) \in \arg\max_{m \in M} \left\{ \epsilon(t, \alpha(m)) U^S + (1 - \epsilon(t, \alpha(m))) y^S(t, \alpha(m)) \right\}, \quad \forall t \in T,$$

$$P(m,t) = \frac{\lambda(\{\tilde{t} | m = \mu(\tilde{t})\} \cap [0,t])}{\lambda(\{\tilde{t} | m = \mu(\tilde{t})\})}, \quad \forall m \text{ such that } \{\tilde{t} | m = \mu(\tilde{t})\} \neq \emptyset \ (\lambda\text{: Lebesgue measure}),$$

$$\alpha(m) \in \arg\max_{a \in A} \int_{t \in T} \left\{ \epsilon(t,a) U^R + (1 - \epsilon(t,a)) y^R(t,a) \right\} P(m, dt), \quad \forall m \in M,$$

$$\forall t \in T, \forall a \in A, \begin{cases} U^S > y^S(t,a) & \Rightarrow \epsilon(t,a) = 1, \\ U^S < y^S(t,a) & \Rightarrow \epsilon(t,a) = 0. \end{cases}$$

The 1st line refers to the condition that $\mu(t)$ is an optimal message for type t of S given R's strategy and S's exit strategy. The 2nd line refers to the condition that R's posterior belief is updated by adhering as much as possible to the Bayesian approach. The 3rd line refers to the condition that $\alpha(m)$ is an optimal action for R given R's posterior updated based on the observation of m and S's exit strategy. The last line refers to the condition that $\epsilon(t,a)$ is an optimal exit choice for type t of S given a realized action a.

We say an action a is *induced on the equilibrium path* if there exists $t \in T$ such that $a = \alpha \circ \mu(t)$ and $\epsilon(t,a) = 0$. For ease of exposition, given $\underline{t} < \overline{t}$, denote the uniform distribution function on interval $[\underline{t}, \overline{t}]$ by $U_{\underline{t}}^{\overline{t}}$, i.e.,

$$U_{\underline{t}}^{\overline{t}}(t) = \begin{cases} 0, & \text{if } t < \underline{t}, \\ \frac{t - \underline{t}}{\overline{t} - \underline{t}}, & \text{if } \underline{t} \leq t \leq \overline{t}, \\ 1, & \text{if } t > \overline{t}. \end{cases}$$

With a slight abuse of notation, denote the distribution function with unit mass on point

$\bar{t}$ by $U_{\bar{t}}^{\bar{t}}$, i.e.,

$$U_{\underline{t}}^{\bar{t}}(t) = \begin{cases} 0, & \text{if } t < \bar{t}, \\ 1, & \text{if } t \geq \bar{t}. \end{cases}$$

3 Uniform-Quadratic Model

We first analyze a uniform-quadratic model. In Section 3.1, we revisit CS's results in an environment without exit. In Section 3.2, we consider an illustrative case of an environment with exit. In Section 3.3, we present a main claim in a general case of the uniform-quadratic model. In Section 3.4, we characterize the set of equilibria in which the exit option is never chosen on the equilibrium path, and discuss two determinants of informativeness of cheap talk.

3.1 Environment without Exit

We revisit CS's results in an environment without exit. If the exit option is not available, perfect information transmission via cheap talk does not occur. This is because S has no incentive to truthfully report the current state because of the fear of R exploiting the information.

To be more precise, CS shows that any equilibrium is characterized as a triple $(\hat{\mu}, \hat{P}, \hat{\alpha})$ such that, for $n = 1, \ldots, N$,[3]

$$\begin{aligned} \hat{\mu}(t) &= \hat{m}_n, \quad \forall t \in (\hat{t}_{n-1}, \hat{t}_n), \\ \hat{P}(\hat{m}_n, t) &= U_{\hat{t}_{n-1}}^{\hat{t}_n}(t), \\ \hat{\alpha}(\hat{m}_n) &= \frac{\hat{t}_{n-1} + \hat{t}_n}{2}, \end{aligned}$$

where $\hat{m}_1, \ldots \hat{m}_N$ are all distinct[4] and $\hat{t} = (\hat{t}_0, \ldots, \hat{t}_N)$ is determined such that

$$\begin{aligned} &0 = \hat{t}_0 < \cdots < \hat{t}_N = 1, \\ &(\hat{t}_n + b) - \hat{m}_n = \hat{m}_{n+1} - (\hat{t}_n + b), \quad n = 1, \ldots, N-1, \end{aligned} \tag{1}$$

[3] R's response to a message unsent on the equilibrium path is irrelevant. For example, it suffices to pick up any $\hat{m}_n$ and specify as $\alpha(m) = \alpha(\hat{m}_n)$ for any $m \neq \hat{m}_1, \ldots, \hat{m}_N$.

[4] Throughout the paper, any two different symbols for messages are thought to be distinct unless specified otherwise.

are satisfied. In other words, any equilibrium has a finite interval $\hat{T}_1, \ldots, \hat{T}_n$ where $\hat{T}_n = (\hat{t}_{n-1}, \hat{t}_n)$, and each interval corresponds to one message. If a realized state t lies in n's interval $\hat{T}_n$, S sends a message $\hat{m}_n$. Then, after observing any equilibrium message, the posterior belief is uniform on the corresponding interval and R's optimal action is the midpoint of the interval, which minimizes the quadratic loss. Moreover, (1) is equivalent to

$$y^S(\hat{t}_n, \hat{m}_n) = y^S(\hat{t}_n, \hat{m}_{n+1}), \quad n = 1, \ldots, N,$$

i.e., at a boundary point $\hat{t}_n$ between two adjoining intervals, S must be indifferent between sending $\hat{m}_n$ and $\hat{m}_{n+1}$. This ensures that $\mu(\cdot)$ is an optimal message strategy.

The fact that any equilibrium has a finite interval directly implies that perfect information transmission cannot be sustained as an equilibrium behavior. Information transmission via cheap talk is more or less noisy.

It is easily verified that there is always an equilibrium with one interval, i.e., a babbling equilibrium. As for $N \geq 2$, by (1), we obtain

$$\hat{t}_n = n\hat{t}_1 + 2n(n-1)b, \quad n \geq 1,$$

where

$$\hat{t}_1 = \frac{1 - 2N(N-1)b}{N}.$$

Then, the necessary and sufficient condition for the existence of the equilibrium with N (≥ 2) intervals is $\hat{t}_1 > 0$, or equivalently

$$b < \frac{1}{2N(N-1)}. \tag{2}$$

Henceforth, we restrict our attention to the most informative equilibrium or the equilibrium with the most intervals. Indeed, CS shows that the equilibrium with the most intervals is Pareto superior to any other equilibria with less intervals. We define $\hat{N}(b)$ such that $\hat{N}(b) = 1$ if $b \geq 1/4$, and $\hat{N}(b)$ is the largest natural number satisfying (2) if $b < 1/4$. Then, $\hat{N}(b)$ is the largest number of intervals given b. It is clear that $\hat{N}(b)$ is nonincreasing in b. In other words, in the environment without exit, the informativeness of cheap talk is mainly determined by a degree of incongruence between the agents' preferences.

3.2 Environment with Exit: Case of of $b > 1/4$

Hereafter, we consider the environment in which the exit option is available for S. First, we consider the case where R's exit value, U^R, is small. A small U^R implies that R has strong incentive to prevent S from choosing the exit option.

As an example, we consider the case of $b > 1/4$. As noted in Section 3.1, if there is no exit option, there is only the equilibrium with one interval, as follows:

$$\hat{\mu}(t) = \frac{1}{2}, \quad \forall t,$$
$$\hat{P}(m,t) = U_0^1(t), \quad \forall m,$$
$$\hat{\alpha}(m) = \frac{1}{2}, \quad \forall m.$$

S and R's ex ante equilibrium payoffs are

$$\hat{V}^S = Y^S - \frac{1}{12} - b^2,$$
$$\hat{V}^R = Y^R - \frac{1}{12}.$$

We now turn to the environment with exit. As an extreme case, suppose $Y^S = U^S$. If U^R is sufficiently small such that $Y^R \geq U^R + b^2$, then the following constitutes an equilibrium:

$$\mu(t) = t, \quad \forall t,$$
$$P(m,t) = U_m^m(t), \quad \forall m$$
$$\alpha(m) = m + b, \quad \forall m,$$
$$\epsilon(t,a) = 0, \quad \text{iff } a = t + b.$$

In other words, perfect information transmission is realized via cheap talk. This equilibrium gives S and R's ex ante payoffs as follows:

$$V^S = Y^S,$$
$$V^R = Y^R - b^2.$$

Then, the existence of S's exit option definitively increases R's as well as S's ex ante payoffs.

The intuition is simple. Since $Y^S = U^S$, S will choose the exit option unless the best action for S is chosen. On the other hand, since $Y^R \geq U^R + b^2$, R wants to continue the relationship, and therefore, R will make the greatest effort to keep S in the relationship by choosing the best action for S. Expecting this, S truthfully reports a current state without fear of exploitation by R.

Even if $Y^S > U^S$, the larger U^S—or equivalently, the smaller D^S— is, the more accurate is the information sent on the equilibrium, provided U^R is sufficiently small. For example, we find the following sequence of equilibria in which S never exits on the equilibrium path:[5]

Case 1: When $D^S \geq \left(\frac{1}{2}+b\right)^2$,

$$\begin{aligned}
&N = 1,\\
&\mu(t) = m_1, \quad \forall t,\\
&\alpha(m_1) = \frac{1}{2}.
\end{aligned}$$

Case 2: When $\left(\frac{1}{2}+b\right)^2 > D^S \geq \frac{1}{4}$,

$$\begin{aligned}
&N = 1,\\
&\mu(t) = m_1, \quad \forall t,\\
&\alpha(m_1) = 1 + b - \sqrt{D^S}.
\end{aligned}$$

Case 3: When $\frac{1}{4} > D^S \geq \frac{1}{4}\left(\frac{1}{2}+b\right)^2$,

$$\begin{aligned}
&N = 2,\\
&\mu(t) = \begin{cases} m_1, & \text{if } t \in \left(0, \frac{2(1-b-\sqrt{D^S})}{3}\right), \\ m_2, & \text{if } t \in \left(\frac{2(1-b-\sqrt{D^S})}{3}, 1\right), \end{cases}\\
&\alpha(m) = \begin{cases} \frac{1-b-\sqrt{D^S}}{3}, & \text{if } m = m_1, \\ 1 + b - \sqrt{D^S}, & \text{if } m = m_2. \end{cases}
\end{aligned}$$

[5]Precisely, we assume $U^R < Y^R - (1/2+b)^2$.

Case 4: When $\frac{1}{4}\left(\frac{1}{2}+b\right)^2 > D^S \geq \frac{1}{16}$,

$$N = 2,$$

$$\mu(t) = \begin{cases} m_1, & \text{if } t \in \left(0, 1-2\sqrt{D^S}\right), \\ m_2, & \text{if } t \in \left(1-2\sqrt{D^S}, 1\right). \end{cases}$$

$$\alpha(m) = \begin{cases} 1+b-3\sqrt{D^S}, & \text{if } m = m_1, \\ 1+b-\sqrt{D^S}, & \text{if } m = m_2. \end{cases}$$

Case 5: When $\frac{1}{16} > D^S \geq \frac{1}{36}$,

$$N = 3,$$

$$\mu(t) = \begin{cases} m_1, & \text{if } t \in \left(0, 1-4\sqrt{D^S}\right), \\ m_2, & \text{if } t \in \left(1-4\sqrt{D^S}, 1-2\sqrt{D^S}\right), \\ m_3, & \text{if } t \in \left(1-2\sqrt{D^S}, 1\right). \end{cases}$$

$$\alpha(m) = \begin{cases} 1+b-5\sqrt{D^S}, & \text{if } m = m_1, \\ 1+b-3\sqrt{D^S}, & \text{if } m = m_2, \\ 1+b-\sqrt{D^S}, & \text{if } m = m_3. \end{cases}$$

The graph of S's ex post payoff functions in these equilibria are illustrated by Figures 1–5.[6] These figures show that the smaller D^S is, the more intervals does the equilibrium have. The value of D^S is negatively related to the credibility of S's exit, implying that the credibility of the exit increases the informativeness of the cheap talk or the voice.

3.3 Environment with Exit: General Case

Even in the case of an arbitrary b, if $Y^S = U^S$ and $Y^R \geq U^R + b^2$, the equilibrium with perfect information transmission, described in the previous subsection, still constitutes an equilibrium. Then, it is easily verified that $V^S > \hat{V}^S$ and $V^R > \hat{V}^R$, i.e., the existence of S's exit option is Pareto improving.

Next, consider the case in which $Y^S > U^S$ but U^S is sufficiently close to Y^S. Although an equilibrium payoff is not continuous in U^S, we can identify a sequence of equilibria in

[6] In Figures 1–5 we fix $Y^S = 1$ and $b = \sqrt{10}/12$.

which S and R's payoffs are approaching those in the equilibrium with perfect information, Y^S and $Y^R - b^2$. The next theorem is the formal result.

Theorem 1 For any natural number N such that $N \geq 1 + \frac{1}{2b}$, if

$$\begin{aligned} &\frac{1}{2(N-1)} > \sqrt{D^S} \geq \frac{1}{2N}, \\ &\sqrt{D^R} \geq b + \sqrt{D^S}, \end{aligned}$$

then the following $(\mu, P, \alpha, \epsilon)$ is an equilibrium:

$$\begin{aligned} &\mu(t) = m_n, \quad \text{if } t \in (t_{n-1}, t_n) \text{ for } n = 1, \ldots, N, \\ &P(m_n, t) = U_{t_{n-1}}^{t_n}(t), \quad \text{for } n = 1, \ldots, N, \\ &\alpha(m) = \begin{cases} a_1 = \frac{t_0 + t_1}{2} + \beta, & \text{if } m = m_1, \\ a_n = \frac{t_{n-1} + t_n}{2} + b, & \text{if } m = m_n \text{ for } n = 2, \ldots, N, \end{cases} \\ &\epsilon(t, a) = 0, \quad \text{iff } Y^S - (t + b - a)^2 \geq U^S. \end{aligned}$$

where

$$\begin{aligned} &\beta = \frac{1}{2} + b - N\sqrt{D^S}, \\ &t_n = \begin{cases} 0, & \text{if } n = 0, \\ \frac{n - 2(N-n)(b-\beta)}{N}, & \text{if } n = 1, \ldots, N. \end{cases} \end{aligned}$$

In this equilibrium, there are N intervals in which distinctive actions are induced and the exit option is never chosen on the equilibrium path.

Proof:
It is easily verified that

$$\begin{aligned} &\beta \in [0, b], \\ &0 = t_0 < t_1 < \cdots < t_N = 1. \end{aligned}$$

Then, $(\mu, P, \alpha, \epsilon)$ defined above makes a sense. It is also clear that P is consistent with μ. Since

$$\begin{aligned} &y^S(t_n, a_n) = y^S(t_n, a_{n+1}), \quad \text{for } n = 1, \ldots, N-1, \\ &V^S(t) \geq U^S, \quad \forall t, \end{aligned}$$

S has no incentive to deviate from μ and ϵ.

Moreover, $D^R \geq (b+\sqrt{D^S})^2$ implies that R is willing to prevent S from choosing the exit option as much possible. On the other hand,

$$V^S(t) = U^S, \quad \text{for } t = t_1, \ldots, t_N.$$

It follows that for any equilibrium action specified above, if R would reduce the level of a, some types of S would exit; on the other hand, if R would increase the level of a, R's quadratic loss increases. To be more precise, denote S's expected payoff of choosing a on receiving a message m_n by $V^R(a|m_n)$; then,

$$
\begin{aligned}
&(t_1-t_0)(V^R(a|m_1)-U^R)\\
&= \begin{cases}
0, & \text{if } a \in (-\infty, b-\sqrt{D^S}],\\
(a-b+\sqrt{D^S}-t_0)D^R - \int_{t_0}^{a-b+\sqrt{D^S}}(t-a)^2dt, & \text{if } a \in (b-\sqrt{D^S}, 1+b-(2N-1)\sqrt{D^S}],\\
(t_1-t_0)D^R - \int_{t_0}^{t_1}(t-a)^2dt, & \text{if } a \in (1+b-(2N-1)\sqrt{D^S}, b+\sqrt{D^S}],\\
(t_1-a+b+\sqrt{D^S})D^R - \int_{a-b-\sqrt{D^S}}^{t_1}(t-a)^2dt, & \text{if } a \in (b+\sqrt{D^S}, 1+b-(2N-3)\sqrt{D^S}],\\
0, & \text{if } a \in (1+b-(2N-3)\sqrt{D^S}, \infty).
\end{cases}
\end{aligned}
$$

It is verified that a_1 defined above is an optimizer. Similarly,

$$
\begin{aligned}
&(t_n-t_{n-1})(V^R(a|m_1)-U^R)\\
&= \begin{cases}
0, & \text{if } a \in (-\infty, 1+b-(2N-2n+3)\sqrt{D^S}],\\
(a-b+\sqrt{D^S}-t_{n-1})D^R - & \int_{t_{n-1}}^{a-b+\sqrt{D^S}}(t-a)^2dt,\\
& \text{if } a \in (1+b-(2N-2n+3)\sqrt{D^S},\\
& \quad 1+b-(2N-2n+1)\sqrt{D^S}],\\
(t_n-a+b+\sqrt{D^S})D^R - & \int_{a-b-\sqrt{D^S}}^{t_n}(t-a)^2dt,\\
& \text{if } a \in (1+b-(2N-2n+1)\sqrt{D^S},\\
& \quad 1+b-(2N-2n-1)\sqrt{D^S}],\\
0, & \text{if } a \in (1+b-(2N-2n-1)\sqrt{D^S}, \infty).
\end{cases}
\end{aligned}
$$

It is verified that a_n defined above is an optimizer. Thus, it follows that $(\mu, P, \alpha, \epsilon)$ constitutes an equilibrium. ∎

By a direct implication of the previous theorem, we obtain the following main two result of the present paper: one is that approximately perfect information transmission may be done via cheap talk in the presence of S's exit option. Another is that the existence of S's exit option increases R's as well as S's payoff.

Corollary 1 Suppose that $Y^R - b^2 > U^R$. Then, as U^S approaches Y^S, there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ pointwisely converges to $t + b$.

Proof:

Direct form the sequence of the equilibria in the previous theorem. ∎

Corollary 2 Suppose that $Y^R - b^2 > U^R$. Then, if U^S is sufficiently large, there exists an equilibrium in the environment with exit, in which S and R's ex ante payoffs are both larger than those in any equilibrium in the environment without exit.

Proof:

Consider the sequence of the equilibria in the previous theorem. It is obvious that the sequence of S's ex ante payoffs converges to Y^S as $D^S \to 0$. R's equilibrium payoff is

$$\begin{aligned} V^R =& Y^R - \frac{(N-1)[1+2(b-\beta)]^3 + [1-2(N-1)(b-\beta)]^3}{12N^3} \\ &- \frac{(N-1)[1+2(b-\beta)]b^2 + [1-2(N-1)(b-\beta)]\beta^2}{N}. \end{aligned}$$

Then, it converges to $Y^R - b^2$ as $N \to \infty$ since

$$\frac{1}{2(N-1)} > b - \beta \geq 0.$$

On the other hand, S and R's equilibrium ex ante payoffs in the environment without exit are derivated as follows:

$$\begin{aligned} \hat{V}^S &= Y^S - \frac{4N^2(N^2+2)b^2+1}{12N^2}, \\ \hat{V}^R &= Y^R - \frac{4N^2(N^2-1)b^2+1}{12N^2}. \end{aligned}$$

By a direct calculation,

$$\begin{aligned} Y^S &> \hat{V}^S, \\ Y^R - b^2 &> \hat{V}^R. \end{aligned}$$

It complete the proof. ∎

Remark 1 Corollary 1 implies that equilibrium actions converge to the one best for S. This is an outcome realized if R can commit to delegate the choice of the action to S.[7] In other words, the previous result implies that, even if it is the case that a commitment to delegation is impossible, the credibility of the exit option can brings about a similar outcome.

3.4 Characterization

In this section, we characterize the equilibria in which the exit option is never chosen on the equilibrium path. Throughout this section, we assume that $Y^S > U^S$ and $Y^R > U^R$.

First, we present basic properties of equilibria.

Proposition 1 There is an equilibrium.

Proposition 2 In any equilibrium, there are only finite actions induced on the equilibrium path.

The proofs are relegated to Appendixes A and B. It directly follows from the latter proposition that an equilibrium in which the exit option is never chosen on the equilibrium path is characterized by finite intervals.

Now, fix an interval $\hat{T} = (\underline{t}, \overline{t})$ and the corresponding message $\hat{m}$ and action $\hat{a}$. Then, by R's best response, at least one of the following conditions must hold:

(i) $\hat{a} = \frac{\underline{t}+\overline{t}}{2}$, or

(ii) $y^S(\overline{t}, \hat{a}) = U^S$.

Then, any interval is belonging to exactly one of the following three categories:

Interval $\mathcal{F}$: $\overline{t} - \underline{t} = 2\sqrt{D^S}$:

$$\hat{a} = \overline{t} - \sqrt{D^S} + b,$$
$$y^S(\underline{t}, \hat{a}) = y^S(\overline{t}, \hat{a}) = U^S.$$

[7] For a situation in which a commitment of delegation is possible, see Dessein [5] among others. Ottaviani [13] also analyzes a situation in which a partial commitment of delegation is possible.

Interval $\mathcal{A}$: $2\sqrt{D^S} > \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$:

$$\hat{a} = \bar{t} - \sqrt{D^S} + b,$$
$$y^S(\underline{t}, \hat{a}) > U^S,$$
$$y^S(\bar{t}, \hat{a}) = U^S.$$

Interval $\mathcal{N}$: $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$:

$$\hat{a} = \frac{\underline{t} + \bar{t}}{2},$$
$$y^S(\underline{t}, \hat{a}) > U^S,$$
$$y^S(\bar{t}, \hat{a}) > U^S.$$

On the marginal point of two neighboring intervals, S must be indifferent between sending actions corresponding to the intervals. Then, any possible equilibrium configuration of intervals with $N \geq 2$ is either of the followings:

(I) $\mathcal{N}, \ldots, \mathcal{N}$

(II) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}$

(III) $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$

(IV) $\mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$

(V) $\mathcal{F}, \ldots, \mathcal{F}$

Then, the equilibrium condition for each configuration is[8]

(I) $b < \frac{1}{2N(N-1)}$

(II) $\frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + Nb$ and $\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b$

(III) $\vee_{i=2}^{N-1} \left[\frac{1}{2N} + \frac{(i-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N} b \text{ and } \sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1} \right]$

[8]For the derivation, see Appendix C.

(IV) $\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N}b$ and $\sqrt{D^S} < \frac{1}{2(N-1)}$

(V) $\sqrt{D^S} = \frac{1}{2N}$

These conditions are illustrated in Figure 6. In summary, we obtain the following equilibrium condition:

Theorem 2 Suppose Y^R is sufficiently large. A sufficient and necessary condition for an equilibrium in which there are N intervals and the exit option is never chosen on the equilibrium path is

$$b < \frac{1}{2N(N-1)}, \text{or}$$

$$\left[\vee_{i=2}^{N}\left[\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}\right] \vee \left[\sqrt{D^S} < \frac{1}{2(N-1)}\right]\right] \wedge \left[\sqrt{D^S} \geq \frac{1}{2N}\right].$$

Corollary 3 Suppose Y^R is sufficiently large. A sufficient and necessary condition for an equilibrium in which there are intervals more than or equal to N and the exit option is never chosen on the equilibrium path is

(1) $b < \frac{1}{2N(N-1)}$, or

(2) $\vee_{i=2}^{N}\left[\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}\right] \vee \left[\sqrt{D^S} < \frac{1}{2(N-1)}\right]$.

While (1) is the same condition as in CS, (2) is a newly obtained condition. These conditions are respectively corresponding to two determinants of informativeness of cheap talk: congruence of preferences and credibility of exit.

4 General Model

In this section, in order to show that our main result holds in more broad environment, we analyses a general setting beyond uniform-quadratic one.

By the single-peakedness, we can find a unique maximizer of $y^i(t,\cdot)$ for any t. It is denoted by $\sigma^i(t)$. By the assumption, it is verified that

$$\frac{d\sigma^i(t)}{dt} > 0.$$

Moreover, we consider the following assumptions:

Assumption 1 There exists $b > 0$ such that

$$\sigma^S(0) - \sigma^R(0) \geq b, \text{ or,}$$
$$\sigma^R(1) - \sigma^S(1) \geq b.$$

Assumption 2 There exists Y^S such that

$$y^S(t, \sigma^S(t)) = Y^S, \quad \forall t.$$

Note that the uniform-quadratic model satisfies Assumptions 1 and 2. We show that these assumptions are sufficient for our main result.

Theorem 3 Suppose $\max_a y^R(t, a)$ is sufficiently large and Assumptions 1 and 2 hold. Then, for any natural number N, there exists $\underline{U}^S$ such that for any $U^S \in (\underline{U}^S, Y^S)$, there exits an equilibrium in which there are intervals more than or equal to N in which distinctive actions are induced and the exit option is never chosen on the equilibrium path. Moreover, there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ pointwisely converges to $\sigma^S(t)$ as U^S approaches Y^S.

A formal proof is relegated to Appendix D. The equilibrium is constructed in a similar way as in the uniform-quadratic model (Theorem 1). See the following example:

Example:

The previous theorem includes various models besides the uniform-quadratic one. Consider the case [9] in which $F(t)$ is a uniform distribution function and

$$y^R = Y^R - (ct - b - a)^2,$$
$$y^S = Y^S - (t - a)^2,$$

where $b, c > 0$. Since

$$\sigma^R(t) = ct - b,$$
$$\sigma^S(t) = t,$$

[9] This case is a variant of Melumad and Shibano [12].

it is verified that Assumptions 1 and 2 are satisfied. Note that when $c > 1 + b$, we obtain

$$\sigma^R(0) < \sigma^S(0),$$
$$\sigma^S(0) > \sigma^S(1),$$

in other words, the sign of the incongruence between S and R's preferences is reversed.

Define N such that

$$\frac{1}{2(N-1)} > \sqrt{D^S} \geq \frac{1}{2N}. \tag{3}$$

Then, as U^S approaches Y^S, N goes to infinity.

We define $t_0 = 0$ and

$$t_n = 1 - 2(N-n)\sqrt{D^S}, \quad n = 1, \ldots, N,$$
$$a_n = 1 - (2N - 2n + 1)\sqrt{D^S}, \quad n = 1, \ldots, N.$$

We construct a candidate for an equilibrium as follows:

$$\mu(t) = m_n, \quad t \in (t_{n-1}, t_n),$$
$$P(m_n, t) = U_{t_{n-1}}^{t_n}(t),$$
$$\alpha(m_n) = a_n,$$
$$\epsilon(t, a) = 0, \quad \text{iff } y^S(t, a) \geq U^S.$$

Since $y^S(t, \alpha \circ \mu(t)) \geq U^S$, the exit option is never chosen on the equilibrium path. On the other hand, for $n = 2, \ldots, N$, since $y^S(t_{n-1}, a_n) = y^S(t_n, a_n) = U^S$, if R would choose $\tilde{a} \neq a_n$, some types of S belonging to (t_{n-1}, t_n) would choose the exit option. Therefore, R with a sufficiently large Y^R has no incentive to deviate from the equilibrium action a_n.

Consider R's incentive after receiving a signal m_1. Since $y^S(t_1, a_1) = U^S$ and $a_1 > \sigma^S(t_1)$, if R would choose $\tilde{a} < a_1$, some types of S close to t_1 would choose the exit option. Therefore, R with sufficiently large Y^R has no incentive to choose $\tilde{a} < a_1$. On the other hand, R's expected payoff function *when the exit option is never chosen* is single-peaked in which the maximum is attained at

$$a^* = \frac{c}{2}\left[1 - 2(N-1)\sqrt{D^S}\right] - b.$$

Suppose U^S is sufficiently close to Y^S such that

$$N \geq \frac{c}{2b}.$$

Then, by (3),

$$\begin{aligned} a^* &= \frac{c}{2}\left[1 - 2(N-1)\sqrt{D^S}\right] - b \\ &\leq \frac{c}{2N} - b \\ &\leq 0 \\ &< 1 - (2N-1)\sqrt{D^S} \\ &= a_1. \end{aligned}$$

This implies that a deviation $\tilde{a} > a_1$ is never beneficial for R with a sufficiently large Y^S. Then, it is clear that the above candidate indeed constitutes an equilibrium.

Example:

Assumption 2 is crucial for the previous result. Consider the case[10] in which $F(t)$ is a uniform distribution function and

$$\begin{aligned} y^R &= (1+t)\sqrt{a} - a, \\ y^S &= (1+t)\sqrt{a} - \frac{1}{\sqrt{1+4b}}a. \end{aligned}$$

Since

$$\begin{aligned} \sigma^S(t) &= \left(\frac{1+t}{2}\right)^2 (1+4b), \\ \sigma^R(t) &= \left(\frac{1+t}{2}\right)^2, \end{aligned}$$

then

$$y^S(t, \sigma^S(t)) = \left(\frac{1+t}{2}\right)^2 \sqrt{1+4b}.$$

Assumption 1 is satisfied, but Assumption 2 is not.

In this example , if $y^S(\hat{t}, \hat{a}) = U^S$, then for $t < \hat{t}$, $y^S(t, \hat{a}) < U^S$. This implies that S receives a positive payoff in any threshold. Therefore, the informativeness is mostly restricted by a bias b, not by the credibility of the exit.

[10]This case is a special case of Marino [10].

5 Discussion

5.1 On the Assumption that Only S Has the Exit Option

Our model assumes that only S has the exit option. This assumption is satisfied in various examples. For example, in a seller-buyer relationship, S and R denote a buyer and a seller, respectively. As stated in the Introduction, when the buyer identifies a defect to his/her purchase, he/she chooses whether to directly lodge a complaint to the seller or to stop buying in silence. On the other hand, the seller usually does not refuse to sell to any particular buyer. Also, in the employer-employee relationship, S represents an employee who chooses whether he/she claims an improvement in the workspace or quits the firm, while R is an employer who is prohibited from firing employees without verifying any significant necessity, as is usually the case in most developed countries.

Even if R also has the exit option, our results would not change provided S keeps the exit option, for R can virtually induce S to choose the exit option by choosing some extreme action in our model. However, it is easily seen that things would change dramatically if S no longer has an exit option. This consideration suggests that whether or not the agent who has the voice option has the exit option is a relevant factor.

5.2 How to Realize a Small D^S and a Large D^R

Our main result Theorem 1 indicates that if D^S is sufficiently small and D^R is sufficiently large, there exists an equilibrium with a large volume of information transmitted. Then, how can S and R realize a combination of such a small D^S and a large D^R?

Initially, it seems likely to realize such a good combination in the situation where R obtains some constant fraction of the joint surplus from the relationship and S obtains the residual. For example, consider a seller-buyer relationship in which a buyer constantly buys some differentiated good from a seller for a fixed price. Suppose the quality of the good turns out to be bad due to some exogenous shock. As long as the buyer does not stop buying, the seller, represented by R, still obtains the same profit because of the fixed price, while the surplus of the buyer, interpreted as S, becomes smaller through quality deterioration. It follows that D^R is comparatively large and D^S is comparatively small. Then, it increases the credibility of the buyer's exit and the informativeness of cheap talk.

Another possible situation is that some relation-specific investment is crucial for the relationship, and it is done at the expense of R. In this situation, R appreciates their relationship much more than S, and thus, it makes S's exit more credible. Indeed, relation-specific investments are often found in various long-term relationships.

5.3 Monetary Transfer and Verifiability

For our results, it is crucial to make the implicit assumption that money cannot be transferred in a relationship. This assumption can be justified when R's action is not verifiable. Generally, when R's response to S's voice is highly complicated, it cannot be described well even if it can be observed. In this time, both agents cannot agree on any contract with monetary transfer.

Moreover, when an environment is dynamic and it cannot be verified whether or not there is a problem in a relationship, monetary transfer is rendered more difficult. In this situation, any monetary transfer dependent on R's action is never agreed upon. This is because, for instance, if both agents agree upon a contract in which some amount of money is transferred from R to S in the event of a problem, R surely insists that there is nothing wrong; therefore, such a contract is not credible at all.

6 Conclusion

This paper investigates the interplay between exit and voice by analyzing a modified version of the Crawford and Sobel [4] model in which the sender has the exit option after the receiver makes a decision. We obtain the result that, in the case where the sender's exit value is large and the receiver's exit value is small, the former's exit is so credible that the latter makes a decision desirable to the sender so as to prevent him/her from exercising the exit option; through this, accurate information can be transmitted via cheap talk on the equilibrium. In other words, it is shown that the informativeness of cheap talk is determined by not only the degree of incongruence between both agents' preferences, but also the credibility of the sender's exit. To the author's knowledge, this result is unprecedented in the literature on cheap talk with private information.

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Appendix: Proofs

A Proof of Proposition 1

Consider a triple

$$\begin{aligned} &\mu(t) = m_1, && \forall t, \\ &P(m,t) = U_0^1(t), && \forall m, \\ &\epsilon(t,a) = 0, && \text{iff } y^S(t,a) \geq U^S, \end{aligned}$$

for some $m_1 \in M$. It is clear that P is consistent with μ. Given the triple, let $V^R(a)$ be the R's expected payoff when R chooses a. Since

$$\epsilon(t,a) = 0 \;\Leftrightarrow\; a \in [b - \sqrt{D^S}, 1 + b + \sqrt{D^S}],$$

then

$$V^R(a) = U^R, \quad a \notin [b - \sqrt{D^S}, 1 + b + \sqrt{D^S}].$$

Let

$$a^* = \arg \max_{a \in [b - \sqrt{D^S}, 1 + b + \sqrt{D^S}]} V^R(a).$$

There must exist such a^* and

$$V^R(a^*) \geq V^R(b - \sqrt{D^S}) = U^R.$$

Then, combined with the above triple,

$$\alpha(m) = a^*, \quad \forall m,$$

constitutes an equilibrium. ∎

B Proof of Proposition 2

On the contrary, suppose that there exists an equilibrium with infinite actions induced on the equilibrium path. Then, there must be actions a_1, a_2, and a_3 induced on the equilibrium path such that $a_1 < a_2 < a_3$ and $a_3 - a_1 < \min\{\sqrt{D^R}, b\}$. Taking S's incentive into consideration, the following must hold:

$$\{t|\mu(t) = a_2\} \subseteq (\max\{a_1 - b, 0\}, a_3 - b) \neq \emptyset.$$

However, it can be easily verified that for any $t \in (\max\{a_1 - b, 0\}, a_3 - b)$,

$$Y^S - (t + b - a_1)^2 > U^S,$$
$$Y^S - (t + b - a_2)^2 > U^S.$$

It follows that there is no type of S who sends a message inducing a_2 on the equilibrium and would choose the exit option if R chooses a_1 or a_2. Since

$$-\int_{\{t|\mu(t)=a_2\}} (t - a_1)^2 dt > -\int_{\{t|\mu(t)=a_2\}} (t - a_2)^2 dt,$$

R would deviate to choosing a_1 after receiving a message inducing a_2. This is a contradiction. ∎

C Derivation of Equilibrium Conditions in Section 3.4

Derivations of (I) and (V) is omitted since those are immediately obtained from the previous analysis.

Consider (II). In this type of equilibrium,

$$t_n = \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \ldots, N-1, \\ 1, & n = N, \end{cases}$$

$$a_n = \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \ldots, N-1, \\ 1 - \sqrt{D^S} + b, & n = N. \end{cases}$$

The equilibrium condition is

$$y^S(t_{N-1}, a_{N-1}) = y^S(t_{N-1}, a_N),$$
$$2\sqrt{D^S} > t_N - t_{N-1} \geq 2\sqrt{D^S} - 2b,$$
$$t_1 - t_0 > 0,$$
$$t_N - t_{N-1} > 0.$$

Then we obtain

$$\frac{1}{2N}+\frac{(N-1)^2}{N}b<\sqrt{D^S}\le\frac{1}{2N}+Nb,$$
$$\sqrt{D^S}<1-(2N^2-4N+1)b,$$

where

$$t_1=\frac{2-2\sqrt{D^S}-2(2N^2-4N+1)b}{2N-1}.$$

Consider (III). Given any $i=2,\dots,N-1$, consider the following configuration:[11]

$$\underbrace{\mathcal{N},\dots,\mathcal{N}}_{i-1\text{ times}},\mathcal{A},\underbrace{\mathcal{F},\dots,\mathcal{F}}_{N-i\text{ times}}.$$

In this type of equilibrium,

$$t_n=\begin{cases}nt_1+2n(n-1)b, & n=0,\dots,i-1,\\ 1-2(N-n)\sqrt{D^S}, & n=i,\dots,N,\end{cases}$$
$$a_n=\begin{cases}\frac{t_{n-1}+t_n}{2}, & n=1,\dots,i-1,\\ t_n-\sqrt{D^S}+b, & n=i,\dots,N.\end{cases}$$

The equilibrium condition is

$$y^S(t_{i-1},a_{i-1})=y^S(t_{i-1},a_i),$$
$$2\sqrt{D^S}>t_i-t_{i-1}\ge 2\sqrt{D^S}-2b,$$
$$t_1-t_0>0,$$
$$t_i-t_{i-1}>0.$$

Then we obtain

$$\frac{1}{2N}+\frac{(i-1)^2}{N}b<\sqrt{D^S}\le\frac{1}{2N}+\frac{i^2}{N}b,$$
$$\sqrt{D^S}<\frac{1-(2i^2-4i+1)b}{2N-2i+1},$$

where

$$t_1=\frac{2-2(2N-2i+1)\sqrt{D^S}-2(2i^2-4i+1)b}{2i-1}.$$

[11] We implicitly assume $N\ge 3$.

Consider (IV). In this type of equilibrium,

$$t_n = \begin{cases} 0, & n = 0, \\ 1 - 2(N-n)\sqrt{D^S}, & n = 1, \ldots, N. \end{cases}$$

The equilibrium condition is

$$2\sqrt{D^S} > t_1 - t_0 \geq 2\sqrt{D^S} - 2b,$$
$$t_1 - t_0 > 0.$$

Then we obtain

$$\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N}b,$$
$$\sqrt{D^S} < \frac{1}{2(N-1)}.$$

D Proof of Theorem 3

In this proof we suppose that there exists $b > 0$ such that $\sigma^S(0) - \sigma^R(0) \geq b$. As for the case of $\sigma^R(1) - \sigma^S(1) \geq b$, we can prove the proposition by reversing all the variables in the following proof at the center of point 1/2.

When U^S is sufficiently close to Y^S, we can uniquely define $\gamma^i_-(t)$ and $\gamma^i_+(t)$ such that

$$\gamma^i_-(t) < \gamma^i_+(t),$$
$$y^i(t, \gamma^i_-(t)) = y^i(t, \gamma^i_+(t)) = U^S,$$

for any t. It is easily verified that $\gamma^i_-(t)$ is strictly increasing in t and

$$\gamma^i_-(t) < \sigma^i(t) < \gamma^i_+(t), \quad \forall t.$$

Moreover, as $U^S \uparrow Y^S$, $\gamma^i_+(t) - \gamma^i_-(t) \downarrow 0$ for any t.

We recursively define a sequence $\{\hat{t}_n\}_{n=0}^N$ in T as follows: first, we define $\hat{t}_0 = 1$. For $n \geq 1$,

(I) if $\hat{t}_{n-1} = 0$, we stop the recursive process and name $n-1$ as N,

(II) if $\hat{t}_{n-1} > 0$ and there exists $\hat{t} \in T$ such that $\gamma^S_+(\hat{t}) = \gamma^S_-(\hat{t}_{n-1})$, we define $\hat{t}_n = \hat{t}$, and

(III) if $\hat{t}_{n-1} > 0$ and there exist no $\hat{t} \in T$ such that $\gamma_+^S(\hat{t}) = \gamma_-^S(\hat{t}_{n-1})$, then we define $\hat{t}_n = 0$.

It is easily verified that $\hat{t}_n < \hat{t}_{n-1}$ if $\hat{t}_{n-1} > 0$. In other words, $\{\hat{t}_n\}_{n=0}^N$ is a strictly decreasing sequence. Also, it is easily verified that as $U^S \uparrow Y^S$, we can find $\hat{t}_n$ sufficiently close to $\hat{t}_{n-1}$ given any $\hat{t}_{n-1} > 0$. Then for any n, there exists $\underline{U}^S$ such that for any $U^S \in (\underline{U}^S, Y^S)$, $\hat{t}_n$ exists.

By the construction of $\{\hat{t}_n\}$, we obtain the following result:

Lemma 1

$$\forall n = 1, \ldots, N,\ \forall t \in [\hat{t}_n, t_{n-1}],\ y^S(t, \gamma_-^S(\hat{t}_{n-1})) \geq U^S,$$
$$\forall n = 1, \ldots, N-1,\ \forall \hat{a} \neq \gamma_-^S(\hat{t}_{n-1}),\ \exists \hat{t} \in [\hat{t}_n, \hat{t}_{n-1}] \text{ such that } y^S(\hat{t}, \hat{a}) < U^S.$$

Let $\Pi^R(a; \ell, u) = \int_\ell^u y^R(t, a) f(t) dt$ and $\mathcal{E}(\ell, u, U^S) = \{\tilde{a} | y^S(t, \tilde{a}) \geq U^S\ \forall t \in [\ell, u]\}$.

Lemma 2 There exists $\underline{U}^S < Y^S$ such that for any $U^S \in (\underline{U}^S, Y^S)$, if N is finite, then

$$\gamma_-^S(\hat{t}_{N-1}) = \arg \max_{a \in \mathcal{E}(\hat{t}_N, \hat{t}_{N-1}, U^S)} \Pi^R(a; \hat{t}_N, \hat{t}_{N-1}).$$

Proof:

It is easy for the case in which $\hat{t}_N = 0$ and $\gamma_+^S(\hat{t}_N) = \gamma_-^S(\hat{t}_{N-1})$, since $\mathcal{E}(\hat{t}_N, \hat{t}_{N-1}, U^S) = \{\gamma_-^S(\hat{t}_{N-1})\}$.

Consider the case in which $\hat{t}_{N-1} > 0$, there exist no $\hat{t} \in T$ such that $\gamma_+^S(\hat{t}) = \gamma_-^S(\hat{t}_{N-1})$, and $\hat{t}_N = 0$. It is verified that $\mathcal{E}(\hat{t}_N, \hat{t}_{N-1}, U^S) = [\gamma_-^S(\hat{t}_{N-1}), \gamma_+^S(\hat{t}_N)]$. Then it suffices to show that

$$\frac{\partial \Pi^R(a; \hat{t}_N, \hat{t}_{N-1})}{\partial a} < 0, \quad \forall a \in [\gamma_-^S(\hat{t}_{N-1}), \gamma_+^S(\hat{t}_N)].$$

By the property of y^R, it is verified that there exists the unique maximizer of $\Pi^R(a; \hat{t}_N, \hat{t}_{N-1})$. We denote it by $\sigma^R(\hat{t}_N, \hat{t}_{N-1})$. Moreover, $\Pi^R(a; \hat{t}_N, \hat{t}_{N-1})$ is strictly decreasing at $a > \sigma^R(\hat{t}_N, \hat{t}_{N-1})$. On the other hand, we can find $\underline{U}^S$ sufficiently close to Y^S such that

$$\gamma_-^S(\hat{t}_{N-1}) > \sigma^S(\hat{t}_{N-1}) - \frac{b}{4},$$
$$\sigma^S(\hat{t}_{N-1}) - \sigma^R(\hat{t}_{N-1}) > \frac{b}{2},$$
$$\sigma^R(\hat{t}_N, \hat{t}_{N-1}) < \sigma^R(\hat{t}_N) + \frac{b}{4}.$$

Then we obtain

$$\begin{aligned}\gamma_-^S(\hat{t}_{N-1}) &> \sigma^S(\hat{t}_{N-1}) - \frac{b}{4}\\ &> \sigma^R(\hat{t}_{N-1}) + \frac{b}{4}\\ &> \sigma^R(\hat{t}_N) + \frac{b}{4}\\ &> \sigma^R(\hat{t}_N, \hat{t}_{N-1}).\end{aligned}$$

This complete the proof. ∎

We define $\{a_n\}_{n=1}^N$ as follows:

$$\hat{a}_n = \gamma_-^S(\hat{t}_{n-1}).$$

Then we construct a candidate for an equilibrium, $(\mu, P, \alpha, \epsilon)$, as follows:

$$\begin{aligned}&\mu(t) = m_n, \quad \text{if } t \in [\hat{t}_n, \hat{t}_{n-1}],\\ &P(m_n, t) = U_{\hat{t}_n}^{\hat{t}_{n-1}}(t),\\ &\alpha(m_n) = \hat{a}_n,\\ &\epsilon(t, a) = 0, \quad \text{iff } y^S(t, a) \geq U^S.\end{aligned}$$

¿From Lemmas 1 and 2, R has no incentive deviate from α as long as R's equilibrium payoff is sufficently larger than R's exit payoff. Moreover, it is easily verified that S in $t \in [\hat{t}_n, \hat{t}_{n-1}]$ has no incentive to send message $m_{\tilde{n}}$ for $\tilde{n} \neq n$. Then $(\mu, P, \alpha, \epsilon)$ constitutes an equilibrium.

∎

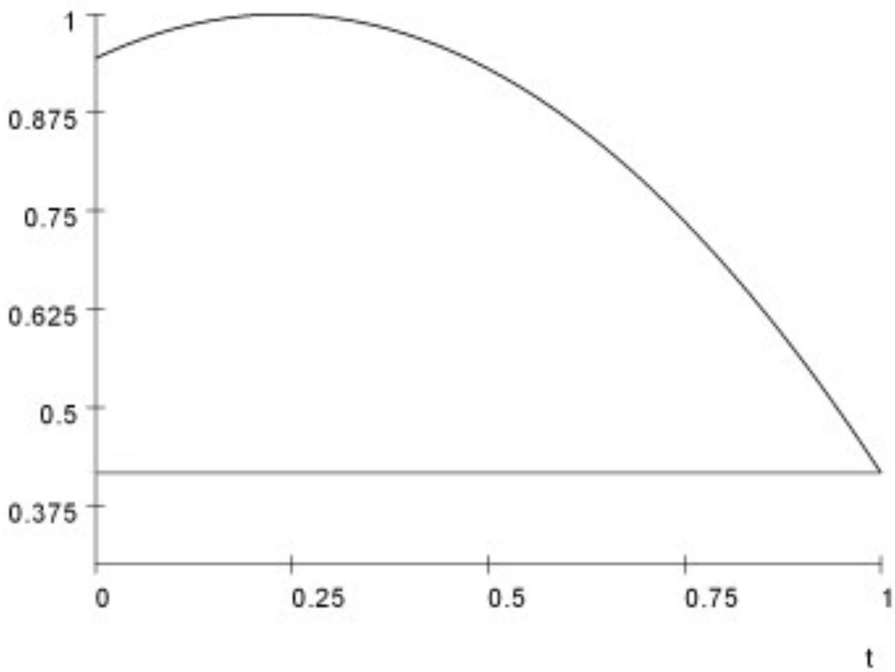

Figure 1: S's payoff in Case 1

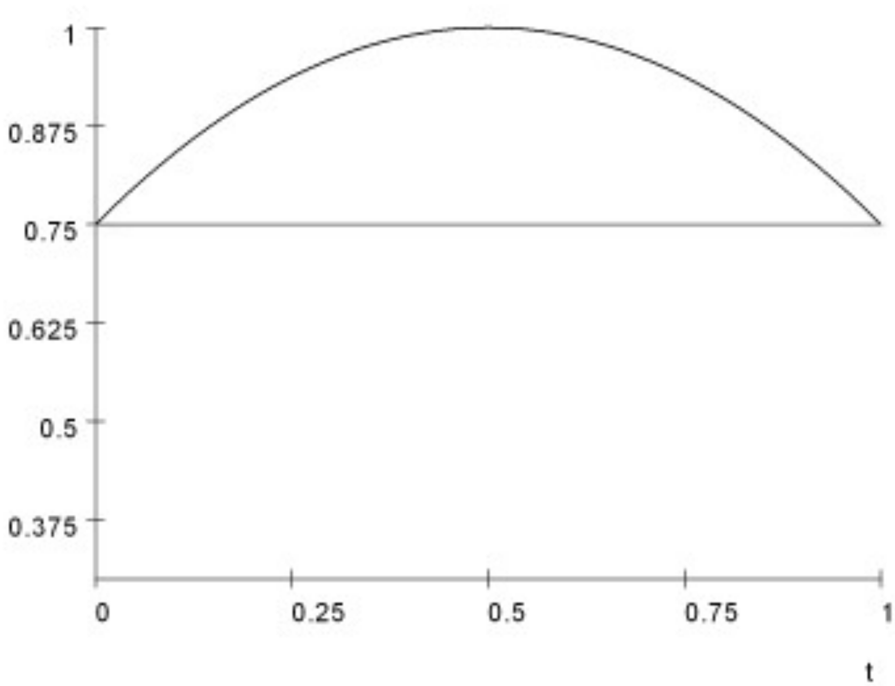

Figure 2: S's payoff in Case 2

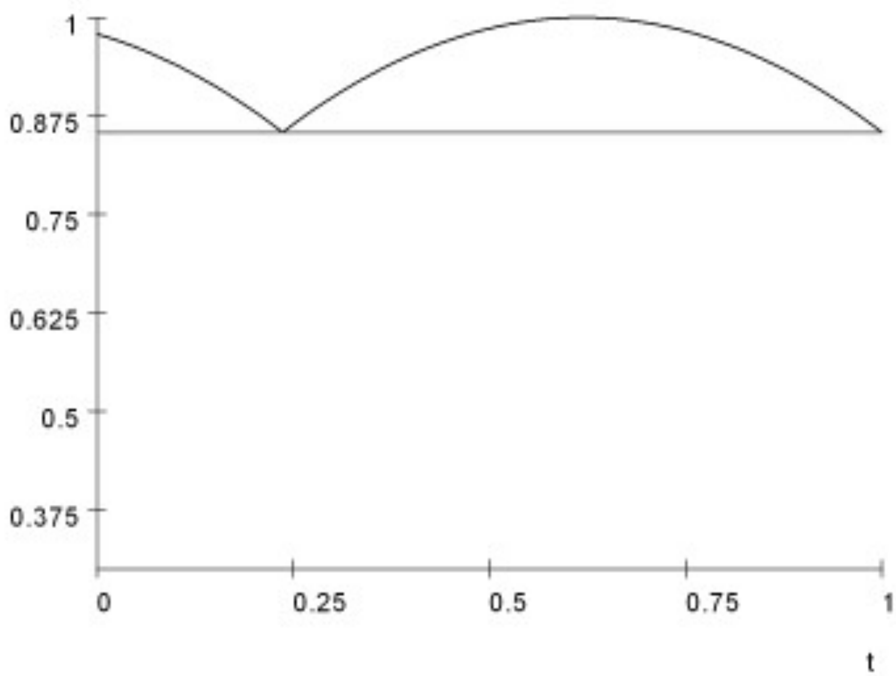

Figure 3: S's payoff in Case 3

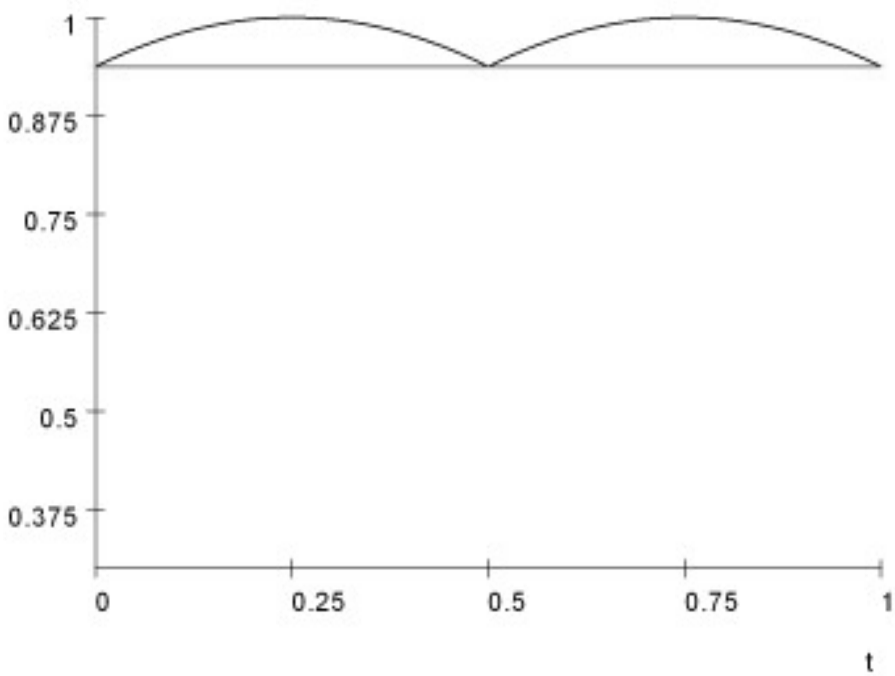

Figure 4: S's payoff in Case 4

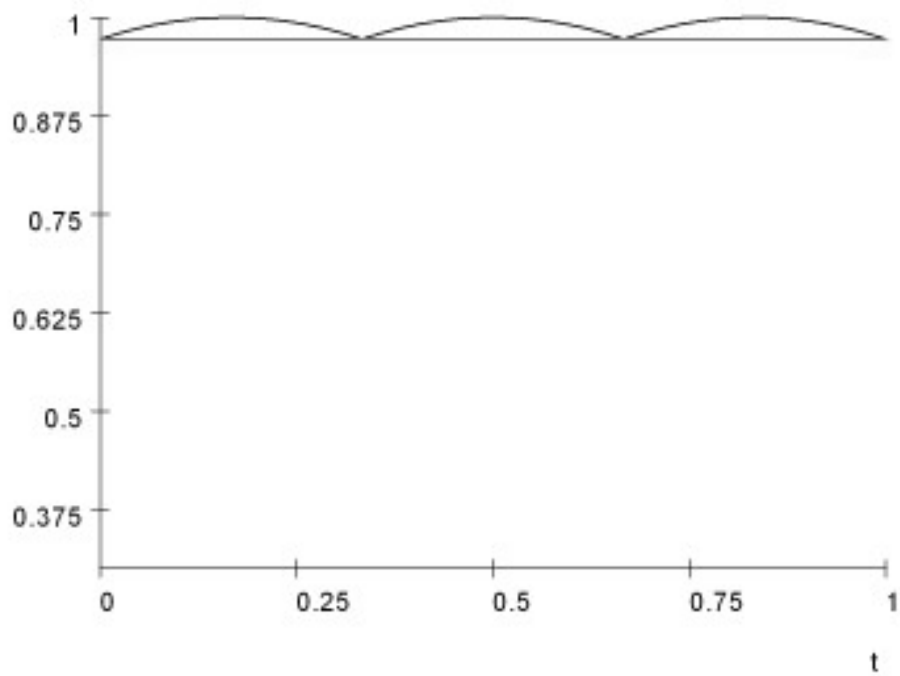

Figure 5: S's payoff in Case 5

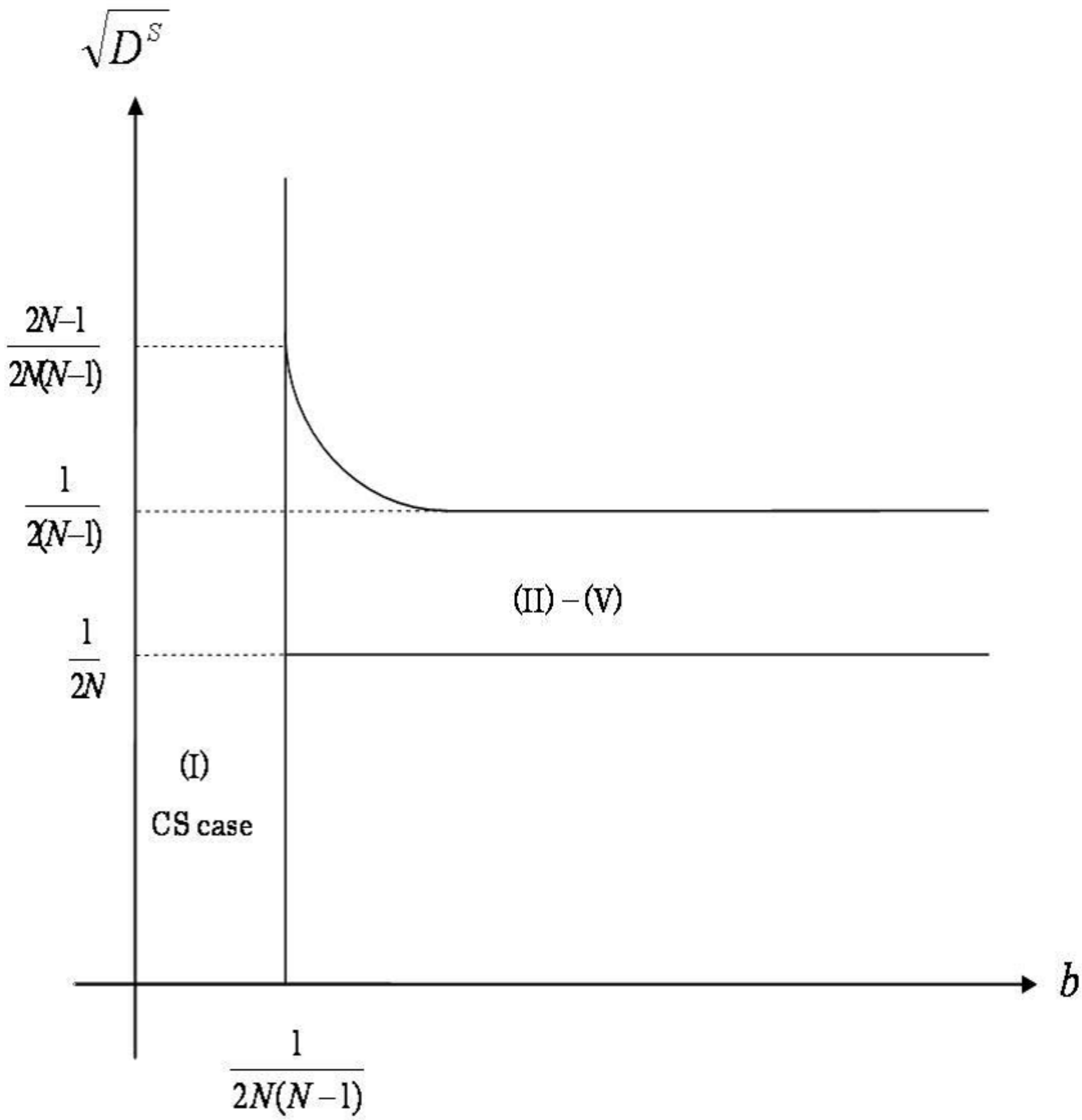

Figure 6: Equilibrium with N intervals