A Citizen-Candidate Model with Sequential Entry*

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Abstract

This article studies a citizen-candidate model where the entry decision is sequential rather than simultaneous. We focus on a situation where there exist three potential candidates including a minor citizen who never wins in a vote unless she is the unique candidate. This sequential entry model has several contrasts to the simultaneous entry models: (i) strategic candidacy occurs on a two-candidate equilibrium; and (ii) the minor citizen may be pivotal in that her more preferred competitor becomes the winner. Furthermore, a Condorcet winner is no longer a dominant candidate in that the entrant on a one-candidate equilibrium may not be a Condorcet winner and there may exist no one-candidate equilibrium even if there exists a Condorcet winner among the potential candidates.

1 Introduction

A vast literature on analysis of electoral competition has argued that electoral outcomes heavily rely on the candidates' decision whether to stand for the election as well as their ideological positions, motivation for office, and ability to commit to policy announcement. On the one hand, the classic spatial models (or Downsian models) rule out the endogenous choice of entry, and assume that each candidate announces and commits to policy. On the other hand, the citizen-candidate approach, initiated by Osborne and Slivinski (1996) and Besley and Coate (1997), considers each citizen's endogenous choice to stand for election or not with entry cost. This approach typically assumes that each candidate cannot commit to their policy announcements, and then the ideal policy of the winning candidate is implemented after the election. The citizen-candidate approach has provided new insights such as policy divergence and strategic candidacy.

Nevertheless, while the endogenous entry decision is an innovative feature of the citizen-candidate model departing from the classic Downsian analysis, little is known

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about how the structure of entry decisions influences the equilibrium. Specifically, in standard setups, the citizens' entry decisions are simultaneous so that each of them must make a decision on entry without knowing the others' decision. However, some citizens can make a decision with knowing the others' decisions in the real world elections since the timing of entry decisions might be different among citizens.

For instance, there is a recent phenomenon that is specific to sequential structure of entry decision, which is observed in the 2012 presidential election of the Liberal Democratic Party (hereafter, LDP) in Japan. Before the election, Sadakazu Tanigaki, the incumbent president of LDP, strongly intended to enter because he believed nobody entering from the executive. However, Nobuteru Ishihara, No. 2 person in that executive, decided to enter the election before Tanigaki finalized his decision. After several rounds of negotiation, Tanigaki bowed out since he learned that Ishihara committed to enter. Interestingly, Tanigaki's decision to withdraw influenced the outcome of the vote later. As a result of Tanigaki's exit, Ishihara bore negative image on his competence since his entry was regarded as a betray to his boss, and then Ishihara lost the election.

In order to make clear how such a sequential structure of entry decision affects electoral outcomes, this article studies a political competition of citizen candidates where the entry decision is sequential rather than simultaneous. As a first step of understanding the impact of sequential entry decisions, we consider one of the simplest environments of sequential entry decisions: there are three citizens (potential candidates) and the timing of entry decisions is exogenously fixed. Furthermore, throughout the main analysis, we focus on cases where there exists a minor citizen, who never wins in a vote unless she is the unique candidate. In such a situation, there also exists a (strict) Condorcet winner who beats any other candidate in a pairwise vote. We demonstrate that if the grand winner, a candidate who can win when all the three candidate stand, does not coincide with the Condorcet winner, then political competition with sequential entry decisions can induce a different outcome from the scenario of simultaneous entry decisions.[1]

One of our main results is on strategic candidacy, that is, entry with no chance to win. We show that strategic candidacy occurs if and only if the minor citizen prefers the grand winner to the Condorcet winner, and makes a decision on entry before the Condorcet winner. When strategic candidacy occurs, the minor citizen is the player who attempts strategic candidacy to deter the entry by the Condorcet winner and to guarantee the grand winner to win in the election. It should be pointed out that in this case there

[1]If the grand winner coincides with the Condorcet winner, then she is the unique candidate on equilibrium, which is the same outcome as the scenario of simultaneous entry decisions.

are only two candidates, the minor citizen and the grand winner, which is never observed in the scenario of simultaneous entry decisions. In the simultaneous scenarios, given that there are only two candidates, the losing candidate strictly prefers withdrawing the entry since it can save the entry cost.[2] However, when the entry decisions are sequential, commitment to entry by a leader could crowd out less preferred followers, and induce another favorite citizen to win. Hence, the losing candidate has an incentive to attempt strategic candidacy. The condition for strategic candidacy on two-candidate equilibrium further implies that the order of the decision making matters; if the Condorcet winner makes an entry decision before the minor citizen, then such strategic candidacy never happens.

Second, we show that the minor citizen can be a "kingmaker" in the sequential entry model. That is, although the minor citizen never wins in election, her strategic entry decision can always induce the preferred rival candidate, either the Condorcet winner or the grand winner, to win. The minor citizen cannot have such influential power in the scenario of simultaneous entry decisions. In our setting where the grand winner is different from the Condorcet winner, the minor citizen can guarantee the grand winner to win by the minor citizen's entry (or expectation of the minor's entry) while she can guarantee the Condorcet winner to win by the minor citizen's withdrawal. A little bit surprisingly, as long as the entry decisions are sequential, the order of the decisions is irrelevant to this result. In other words, the observability of the other's entry decision is essential for supporting the kingmaker position of the minor citizen.

Furthermore, we see that, unlike in the scenario of simultaneous entry decisions, the Condorcet winner is no longer a dominant candidate. That is, (i) there may be a one-candidate equilibrium on which the unique candidate is not a Condorcet winner; and (ii) there may be no one-candidate equilibrium even if there is a strict Condorcet winner. We demonstrate them by examples where the unique candidate is the grand winner, and the grand winner and the minor citizen are the candidate. In both cases, the Condorcet winner is significantly weak citizen in the election compared to the models of simultaneous entry decisions.

Our political model is based on the seminal works by Osborne and Slivinski (1996) and Besley and Coate (1997), who develop the citizen-candidate model where each citizen endogenously chooses whether to stand for election with entry costs, and each of them cannot commit to her policy announcement before voting. We consider a citizen-

[2]This logic of impossibility of strategic candidacy on two-candidate equilibria applies for more broader settings as pointed out by Ishihara (2013).

candidate model with sequential decision making of entry and show that the structure of the entry decision making can qualitatively change the equilibrium outcome.[3] Political competition of sequential decision making is studied by Palfrey (1984) and Callander (2005). However, both of them adopt a Downsian framework where each candidate can commit to her policy platform and their main interest is policy divergence in spatial competition, which rarely emerges in standard Downsian models.[4]

The issue of strategic candidacy is also investigated by Osborne and Slivinski (1996) and Besley and Coate (1997) though they do not support strategic candidacy on two-candidate equilibria.[5] Recent studies by Asako (2013) and Ishihara (2013) successfully demonstrate strategic candidacy on two-candidate equilibria by introducing partial commitment to platform (in Asako (2013)) and repeated interaction (in Ishihara (2013)). We demonstrate the similar phenomenon of strategic candidacy by using a different factor: commitment to entry to crowd out the less preferred rivals.

The rest of the paper is organized as follows. The next section describes the basic components of the model. Section 3 shows several results when the entry decisions are simultaneous, and Section 4 demonstrates that such results are not valid when the entry decisions are sequential. The final section concludes. All the proofs are in the Appendix.

2 The Environment

The political competition proceeds according to the following three steps. First, potential candidates make a decision whether to enter to election or not. Second, the voting procedure decides the winner from the set of the candidates. Finally, the winner implements a policy. Our main interest is to make clear that when the potential candidates make a decision sequentially, how the outcome is different from the case of simultaneous decision making.

There are three potential candidates (hereafter called citizens) and denote the set of the citizens by $\mathcal{N} = \{1, 2, 3\}$. Each citizen makes a decision to stand for election or not. Let $A \equiv \{E, N\}$ be the action set of each citizen where E (resp. N) means entry (resp. no entry). Denote $S \subset \mathcal{N}$ be the set of the candidates, citizens entering the election.[6]

[3] Unlike Osborne and Slivinski (1996) and Besley and Coate (1997), our model drops some of generality such as the number of citizens. A more comprehensive analysis of generalized setups with sequential entry decisions is left for future research.

[4] See Osborne (1995) for robustness of the median voter theorem.

[5] There is another literature from a slight different interest where they investigate whether a voting procedure is immune to the threat of strategic candidacy (Dutta et al., 2001, 2002; Ehlers and Weymark, 2003; Eraslana and McLennan, 2004; Samejima, 2007).

[6] When $i \in S$, we use term "candidate i" or "citizen i" interchangeably.

Given S, the voting procedure determines the winner of the election.

We assume that as in citizen-candidate frameworks, candidates cannot commit to a policy platform before voting so that the electoral outcome is characterized by the identity of the winner. When candidate $i \in S$ wins in the election, citizen $j \in \mathcal{N}$ obtains political benefit $v^j(i)$. Assume that if there is no candidate in the election, then citizen j's political benefit is given by $v^j(0)$ where 0 means a status quo policy. When a citizen stands for election, she incurs an entry cost $d > 0$. Then, given that candidate i wins in the election, the *ex post* payoff for citizen j is $v^j(i) - d$ if she stood for election and $v^j(i)$ if not.

We take a reduced form approach in the voting procedure as follows: given $S \neq \emptyset$, a set of voters chooses the winner according to function $C(S)$. Formally, $C(\cdot)$ is a function from $2^{\mathcal{N}}$ to $\mathcal{N} \cup \{0\}$ where $C(S) \in S$ for any $S \in 2^{\mathcal{N}} \setminus \{\emptyset\}$ and $C(\emptyset) = 0$. Thanks to the reduced form approach, the political competition is described as a simple game in which an action chosen by each player is to choose either to enter or not. There are some remarks on the choice function. First, since we assume $C(S)$ is a function but not a correspondence, ties in a vote are excluded for any cases. Second, since $C(S) \neq \emptyset$ for any $S \neq \emptyset$, there is always a winner whenever there is a candidate. Third, although we define the outcome of no candidates as $C(\emptyset) = 0$, this never emerges on equilibrium under the assumptions we made above.

We define three kinds of citizens, minor citizen, Condorcet winner, and grand winner, as follows.

Definition 1 *1. Citizen i is minor (denoted by M) if $C(S) \neq i$ for any $S \in 2^{\mathcal{N}} \backslash \{i\}$ such that $i \in S$.*

2. Citizen i is the Condorcet winner (denoted by CW) if $C(\{i, j\}) = i$ for any $j \in \mathcal{N} \backslash \{i\}$.

3. Citizen i is the grand winner (denoted by GW) if $C(\mathcal{N}) = i$.

A minor citizen is a weak player in that she cannot win as long as there is a rival in the vote. A Condorcet winner beats any other candidate in a pairwise vote. Especially, in scenarios of simultaneous entry decisions, she is a influential citizen in that there always exists an equilibrium where she is the unique candidate and then the winner. Finally, the grand winner is a citizen who can win if all citizens stand for election. By definition, there always and uniquely exists a grand winner. We abuse notation M, CW, and GW such that, for instance, $CW = 1$ means that citizen 1 is the Condorcet winner,

$CW = GW$ means that the Condorcet winner is also the grand winner, and so on. Denote the political benefit of the minor citizen, the Condorcet winner, and the grand winner by $v^M(\cdot)$, $v^{CW}(\cdot)$, and $v^{GW}(\cdot)$, respectively. Throughout the analysis, we assume that (i) $v^j(j) - v^j(i) > d$ for all $j \in \mathcal{N}$ and $i \in (\mathcal{N} \backslash \{j\}) \cup \{0\}$; and (ii) $v^M(CW) \neq v^M(GW) - d$. The first assumption means that each citizen wants to win in the first place, and the second one means that the minor citizen strictly prefers either the Condorcet winner with exit or the grand winner with entry.[7]

For most of the analysis, we assume that there is a (unique) minor citizen.

Assumption 1 *There exists a unique minor citizen.*

Since there are three citizens in our model, we can show that there is also a Condorcet winner if there is a minor citizen.

Lemma 1 *Under Assumption 1, there (uniquely) exists a Condorcet winner.*

Lemma 1 implies that under Assumption 1, all M, CW, and GW exist. Note that the Condorcet winner may coincide with or be different from the grand winner. In our main analysis, we demonstrate that the political outcome in the scenario of sequential entry decisions is qualitatively different from that of simultaneous entry decisions if CW is different from GW. Also, it is worthwhile to remark that the voting outcomes represented by choice function $C(\cdot)$ under Assumption 1 are replicated by a one-dimensional spatial model with sincere voting.

Before proceeding to the equilibrium analysis, we define strategic candidacy as follows. Let σ be the profile of the citizens' strategies, which will be formally defined later, and let $\hat{S}(\sigma)$ be the set of the citizens who enter on the equilibrium.

Definition 2 *We say that*

- *strategic candidacy occurs if* $\{C(\hat{S}(\sigma))\} \neq \hat{S}(\sigma)$*, and*
- *citizen* i *attempts strategic candidacy if* $i \in \hat{S}(\sigma) \backslash \{C(\hat{S}(\sigma))\}$.

Strategic candidacy basically means that a citizen stands for election without any possibility to win. One of our main results states that strategic candidacy can occur when there are exactly two candidates, which never emerges if the entry decisions are simultaneous.

[7] To simplify the analysis, we exclude that indifference case.

3 Simultaneous Entry Decision: Benchmark

As a benchmark, we first suppose that the entry decisions are simultaneous and highlight the result in Osborne and Slivinski (1996) and Besley and Coate (1997).[8] When the entry decisions are simultaneous, the political competition is defined as a normal form game of three players where each citizen i chooses strategy $\sigma_i \in A$. We focus on pure strategy Nash equilibria. Let $\hat{S}(\sigma)$ be the set of the citizens who choose E on equilibrium when the strategy profile is $\sigma \equiv (\sigma_1, \sigma_2, \sigma_3)$. Then under Assumption 1, there is a unique pure strategy Nash equilibrium $\hat{\sigma} \equiv (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ such that only the Condorcet winner enters.

Proposition 1 *Consider a simultaneous-entry model. Under Assumption 1, there exists a unique Nash equilibrium $\hat{\sigma}$ such that $\hat{S}(\hat{\sigma}) = \{CW\}$.*

Proposition 1 is actually a special case of Corollary 1 of Besley and Coate (1997), which basically states that the existence of a Condorcet winner is almost the necessary and sufficient condition for existence of one-candidate equilibrium. In political competition with simultaneous entry, the equilibrium requires each candidate not to deviate from the equilibrium strategy unilaterally. If a Condorcet winner has already stood on equilibrium, then no one has an incentive to enter since the entry cannot change the winner. Furthermore, there is no equilibrium where a citizen other than the Condorcet winner enters unopposed. In such cases, the Condorcet winner has an incentive to enter since she can win against the standing candidate.

The proposition also points out no possibility of strategic candidacy. It should be especially noted that in political competition with simultaneous entry decision, strategic candidacy happens only when there are at least three candidates standing for election. If the equilibrium satisfies $C(S) \neq S$, then there must exist a candidate $i \in C(S) \setminus S$ who actually enters due to a strategic incentive to change the winner rather than to win in election. Then, strategic candidacy requires two entrants other than herself.[9]

Finally, we note that the minor citizen has no impact on the electoral outcome in that the winner is always the Condorcet winner no matter which rival the minor prefers. We will show that, a little bit surprisingly, the minor citizen can choose either of the rivals as the winner if the entry decision is sequential.

[8] Our model of simultaneous entry is slightly different from both Osborne and Slivinski (1996) and Besley and Coate (1997). Specifically, Osborne and Slivinski (1996) restricts their attention to one-dimensional spatial competition and Besley and Coate (1997) assume that there are finite voters who attempt strategic voting.

[9] This result somewhat relies on the assumptions that the winner cannot credibly choose a policy different from her ideal point. When the winner can credibly implement a policy other than her ideal point, strategic candidacy may arise on two-candidate equilibrium since entry may change the policy implemented by the rival. (Asako, 2013; Ishihara, 2013)

4 Sequential Entry Decision

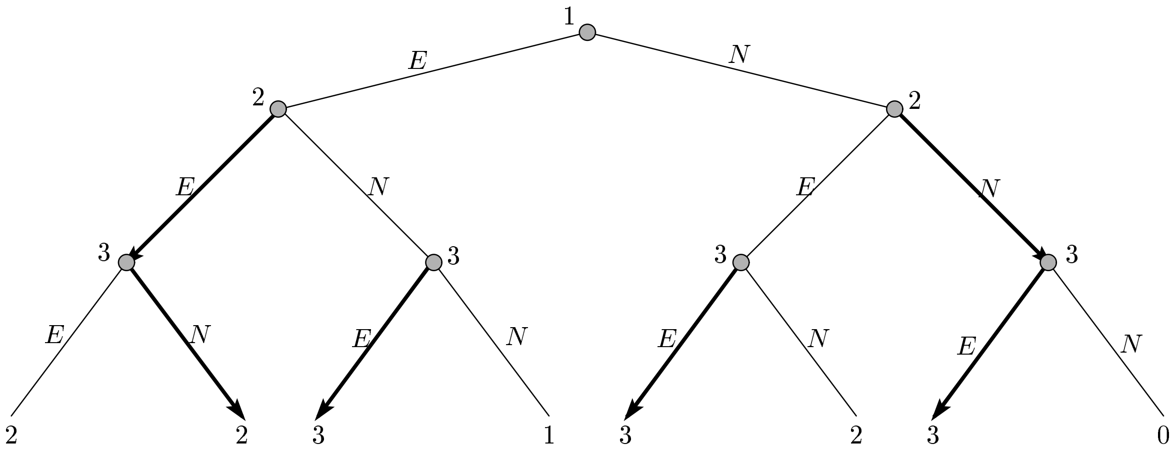
We now consider the case of sequential entry decision. The political competition of sequential entry decision is an extensive form game with perfect information as follows. First, citizen 1 decides whether to stand for election or not. Second, after observing citizen 1's decision, citizen 2 decides whether to stand for election or not. Finally, after observing citizen 1's and 2's decisions, citizen 3 decides whether to stand for election or not. For $i = 2, 3$, let $h_i \subset \{1, \ldots, i-1\}$ be the set of candidates who have decided to enter when citizen i makes a decision. Denote citizen i's strategy by $\sigma_i(h_i) \in \{E, N\}$, meaning citizen i's action after history h_i. Similar to simultaneous-entry models, let $\hat{S}(\sigma)$ be the set of the citizens who choose E on the equilibrium path when the strategy profile is $\sigma \equiv (\sigma_1, \sigma_2, \sigma_3)$. We look at subgame perfect equilibria of the extensive form game.

As we will see the detail later, the minor citizen plays an important role in our model. Especially, (i) the minor citizen is the unique citizen who may attempt strategic candidacy; and (ii) if the Condorcet winner is different from the grand winner, then a citizen preferred by the minor citizen always wins in election. To understand the result and the intuition, we first look at an example.

4.1 Examples

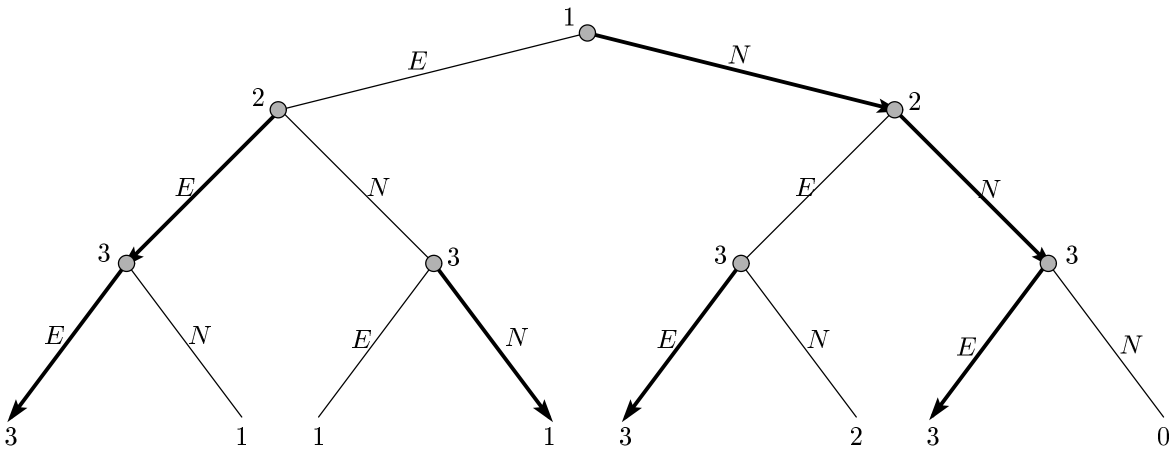
Consider an example where $C(\{1,2\}) = 2$, $C(\{1,3\}) = 3$, $C(\{2,3\}) = 3$, and $C(\mathcal{N}) = 2$. These voting outcomes imply that citizen 1 is minor, citizen 2 is the grand winner, and citizen 3 is the Condorcet winner. The fact that each citizen wants to win in election at the first place implies that the best responses of citizens 2 and 3 are fully characterized by the backward induction. Figure 1 describes the decision procedure and the best responses of citizens 2 and 3 according to the backward induction. The backward induction implies that citizen 2 wins if citizen 1 enters while citizen 3 wins if citizen 1 does not enter.

If $v^1(2) - d > v^1(3)$, then citizen 1 prefers entering with citizen 2 winning. In this case, citizen 1 actually attempts strategic candidacy, which never occurs in simultaneous-entry models. Once citizen 1 enters, it is impossible for the Condorcet winner to face two-candidate competition, and the grand winner can take an advantageous position. Since citizen 1 prefers the grand winner to the Condorcet winner, it is favorable for citizen 1 to induce the grand winner to win by entry even if the entry is costly and there is no chance to win. On the other hand, if $v^1(2) - d < v^1(3)$, then citizen 1 prefers staying away and citizen 3 wins. This outcome is same as that of the simultaneous-entry model. Nevertheless, together with the case of $v^1(2) - d > v^1(3)$, we can see that the winner is



(The terminal nodes represent the winner.)

Figure 1: Decision Procedure when $1 = M$, $2 = GW$, and $3 = CW$



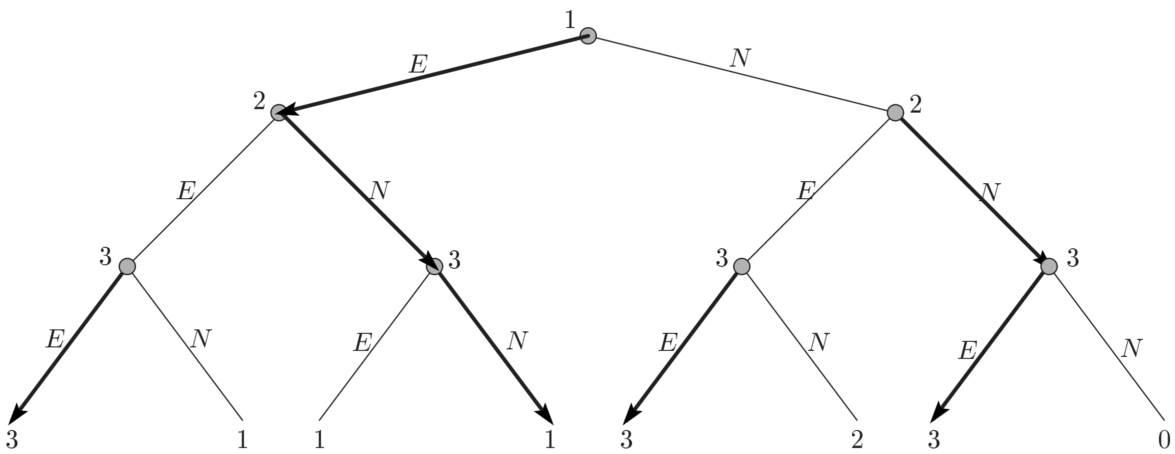
(The terminal nodes represent the winner.)

Figure 2: Decision Procedure when $1 = CW$, $2 = M$, $3 = GW$, and $v^2(3) - d > v^2(1)$

determined according to citizen 1's preference.

Consider another example where $C(\{1,2\}) = 1$, $C(\{1,3\}) = 1$, $C(\{2,3\}) = 3$, and $C(\mathcal{N}) = 3$. These voting outcomes imply that citizen 1 is the Condorcet winner, citizen 2 is minor, and citizen 3 is the grand winner. Figure 2 and 3 illustrate the decision procedure and the subgame perfect equilibrium when $v^2(3) - d > v^2(1)$ and $v^2(3) - d < v^2(1)$, respectively. In contrast to the previous example, the minor citizen is now a follower of the Condorcet winner. Nevertheless, the identity of the winner still relies on the minor citizen's preference; whether $v^2(3) - d$ is greater than or less than $v^2(1)$.

When $v^2(3) - d > v^2(1)$, the backward induction implies that the grand winner is the unique candidate and the winner, which is preferred to the Condorcet winner being elected by the minor citizen. In this case, the minor citizen follows the Condorcet winner and can monitor the Condorcet winner's decision. Specifically, if the Condorcet winner enters, then the minor citizen can prevent the Condorcet winner winning by enter, which



(The terminal nodes represent the winner.)

Figure 3: Decision Procedure when $1 = CW$, $2 = M$, $3 = GW$, and $v^2(3) - d < v^2(1)$

can induce three-candidate competition and guarantee the grand winner to win. The Condorcet winner then must be crowded out by expecting the minor's entry. On the other hand, when $v^2(3) - d < v^2(1)$, as Figure 3 shows, the Condorcet winner is the unique candidate. Combining these two observations implies that the minor citizen is still a kingmaker as in the previous example.

We will show below that the kingmaker property of the minor citizen holds more generally. Specifically, suppose that the Condorcet winner is different from the grand winner. Then, in sequential-entry models, the winner is one of the minor citizen's rival candidates who is preferred by the minor citizen. A little bit surprisingly, this property holds regardless of the order of decision making.

4.2 When Strategic Candidacy Occurs

The first example discussed above suggests a possibility of strategic candidacy in cases of sequential entry decisions. The necessary and sufficient condition for strategic candidacy is characterized as follows.

Proposition 2 *Consider a sequential-entry model. Under Assumption 1, there exists a subgame perfect equilibrium σ^* where strategic candidacy occurs if and only if the following conditions hold:*

1. *citizen M is a leader of citizen CW; and*

2. *$v^M(CW) < v^M(GW) - d$.*

Furthermore, if strategic candidacy occurs, then $\hat{S}(\sigma^*) = \{M, GW\}$.

This proposition implies that both the timing of the minor citizen's decision and her preference are relevant to strategic candidacy. First, the minor citizen must be a leader of the Condorcet winner when strategic candidacy occurs. Second, the minor citizen must prefer the grand winner to the Condorcet winner. Furthermore, the set of the candidates on the equilibrium tells us that strategic candidacy is attempted only by the minor citizen to crowd out the Condorcet winner. We also conclude that from Condition 2, the grand winner must be different from the Condorcet winner when strategic candidacy occurs.

The conditions in Proposition 2 tells us an intuition of strategic candidacy. When the minor citizen makes a decision before the Condorcet winner, the minor citizen attempts strategic candidacy as a commitment to deter the Condorcet winner's entry. The commitment to enter can eliminate the possibility of two-candidate competition, which makes it impossible for the Condorcet winner to win in the vote. Condition 2 guarantees that since the minor citizen prefers the grand winner to the Condorcet winner, the minor citizen has certainly an incentive to crowd out the Condorcet winner.

A sharp contrast to the simultaneous scenarios is that strategic candidacy can be commitment of entry by the minor citizen in the sequential model. As shown in the example of Figure 1, the minor citizen's commitment to enter makes it difficult to support a situation where the Condorcet winner wins, which deters the entry by the Condorcet winner. In other words, this commitment to enter can crowd out the Condorcet winner, and induce the grand winner to win. In the scenarios of simultaneous entry, on the other hand, such strategic candidacy never occurs; the minor citizen cannot commit to enter with losing for certain. Given that the minor citizen predicts that she and the grand winner are the candidates on equilibrium, the minor citizen strictly prefers to exit since it can save the entry cost without changing the electoral outcome. Thus, strategic candidacy never occurs in two-candidate equilibrium in the simultaneous scenarios.

4.3 Minor Citizen as a Kingmaker

It is worthwhile to remark that the minor citizen's preference is crucial to electoral outcomes when the minor citizen is a leader of the Condorcet winner. When the minor citizen prefers the grand winner, the grand winner wins because of strategic candidacy by the minor citizen as mentioned above. On the other hand, when the minor citizen prefers the Condorcet winner, the minor citizen never attempts strategic candidacy. She can induce the Condorcet winner's entry by committing to exit. In this case, the Condorcet winner wins because three-candidate competition never occurs.

Interestingly, the minor citizen being pivotal is a general property irrelevant to the order of decision making as long as the Condorcet winner is different from the grand winner. In order to argue this property, we next look at the subgame perfect equilibria when the minor citizen is a follower of the Condorcet winner. As shown in the following proposition, only the citizen preferred by the minor citizen enters on equilibrium when the minor citizen is a follower of the Condorcet winner.

Proposition 3 *Consider a sequential-entry model with Assumption 1. Suppose that citizen M is a follower of citizen CW. Then, for any subgame perfect equilibrium σ^*,*

1. $\hat{S}(\sigma^*) = \{GW\}$ *holds if* $v^M(CW) < v^M(GW) - d$*; and*

2. $\hat{S}(\sigma^*) = \{CW\}$ *holds if* $v^M(CW) > v^M(GW) - d$*.*

First, suppose that the minor citizen prefers the grand winner to the Condorcet winner. In this scenario, the minor citizen is interpreted as a "monitor" of the Condorcet winner. That is, if the minor citizen observes the Condorcet winner's entry, then she can trigger her entry to guarantee the grand winner to win. Expecting the minor citizen's entry decision, the Condorcet winner gives up to enter since her entry induces the three-candidate competition. Expecting or observing such behaviors of the minor citizen and the Condorcet winner, the grand winner always enters. As a result, the grand winner becomes the unopposed winner in equilibrium. In other words, the minor citizen can support the grand winner to be elected by acting as a monitor of the Condorcet winner instead of strategic candidacy.

Next, suppose that the minor citizen prefers the Condorcet winner being elected. The outcome is the same as that in the scenario of simultaneous entry decisions. That is, it is more beneficial for the minor citizen not to enter, which avoids three-candidate competition. Predicting the minor citizen's behavior, the Condorcet winner always enters since she never loses. Expecting or observing such behaviors of the minor and the Condorcet winner, the grand winner also gives up to enter. Thus, the Condorcet winner becomes the unopposed winner in equilibrium.

Combining the results so far, the winner in this model is characterized by the following proposition.

Proposition 4 *Consider a sequential-entry model with Assumption 1. Then* $C(\hat{S}(\sigma^*)) = CW$ *for any subgame perfect equilibrium* σ^* *if and only if* $v^M(CW) > v^M(GW) - d$*.*

Note that this result itself does not depend on whether $CW \neq GW$. When the Condorcet winner is same as the grand winner, $v^M(CW) > v^M(GW) - d$ is always satisfied. and the Condorcet winner is always the unique candidate.[10] In other words, if the Condorcet winner is same as the grand winner, then she can always win as long as she stands for election. It means that the rest of the citizens cannot influence the electoral outcome by their entry decisions. In such cases, the structure of the decision making, whether simultaneous or sequential, does not matter.

By contrast, when the Condorcet winner is different from the grand winner, the rest of the citizens, the minor citizen and the grand winner, hold an influential power in the election. Especially, the electoral outcome is reflected by the minor citizen's preference in the following sense.

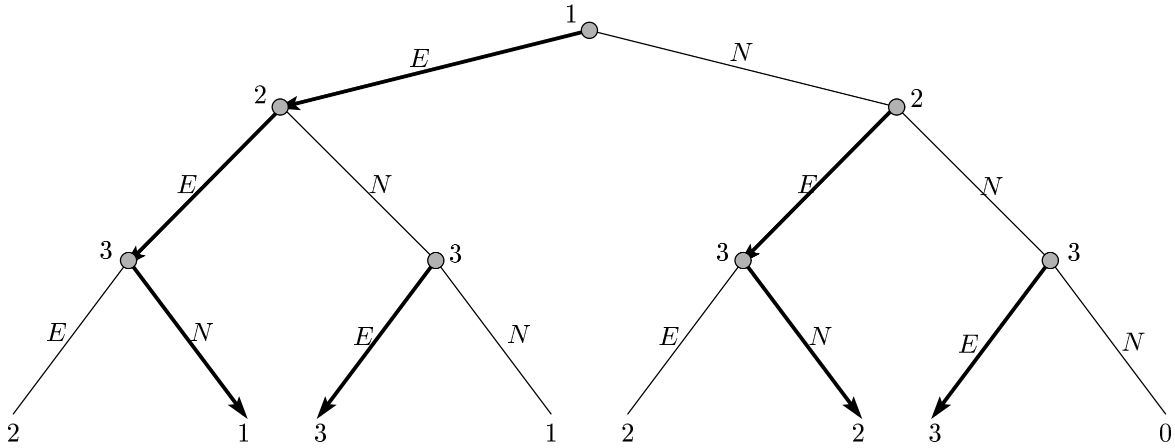
Corollary 1 *Consider a sequential-entry model with Assumption 1. In addition suppose* $CW \neq GW$*. Then* $C(\hat{S}(\sigma^*)) = CW$ *(resp.* GW*) for any subgame perfect equilibrium* σ^* *if and only if* $v^M(CW) > v^M(GW) - d$ *(resp.* $v^M(CW) < v^M(GW) - d$*).*

The corollary argues that the minor citizen is a "kingmaker" in that she is a pivotal player who seldom wins in a vote. The minor citizen receives payoff $v^M(CW)$ when the Condorcet winner wins without entry by the minor citizen. On the other hand, $v^M(GW) - d$ is the payoff for the minor citizen when she enters and the grand winner wins. That is, the corollary states that the minor citizen can choose a preferable candidate by using her entry decision. However, the minor citizen is not a dictator since her most favorite choice is herself to be elected, which can never be realized. Nevertheless, the minor citizen is regarded as a kingmaker since she can pick up the winner from the rivals.

The order of the entry decisions is irrelevant for the kingmaker property as long as the entry decisions are sequential. Actually, the minor citizen cannot be the kingmaker in cases of simultaneous entry decisions since the grand winner cannot win in election. However, if the decision making is sequential, then the minor citizen can induce the grand winner to win by strategic candidacy (when she is a leader) or by monitoring the Condorcet winner (when she is a follower).

The equilibrium analysis further implies that the Condorcet winner is not as influential as in simultaneous-entry models. As mentioned in the last section, whenever there is a Condorcet winner in simultaneous-entry models, there is a political equilibrium in which only the Condorcet winner is the candidate. It is sharply contrasted with sequential-entry models where the existence of Condorcet winner does not necessarily imply the existence of one-candidate equilibrium with the Condorcet winner.

[10]This is formally argued by Lemma 2 in the Appendix.



$C(\{1,2\}) = 1$, $C(\{1,3\}) = 3$, $C(\{2,3\}) = 2$, $C(\mathcal{N}) = 2$,
$v^2(1) - d > v^2(3)$, and $v^3(1) > v^3(2) - d$
(The terminal nodes represent the winner.)

Figure 4: Example without Minor Citizens nor Condorcet Winners

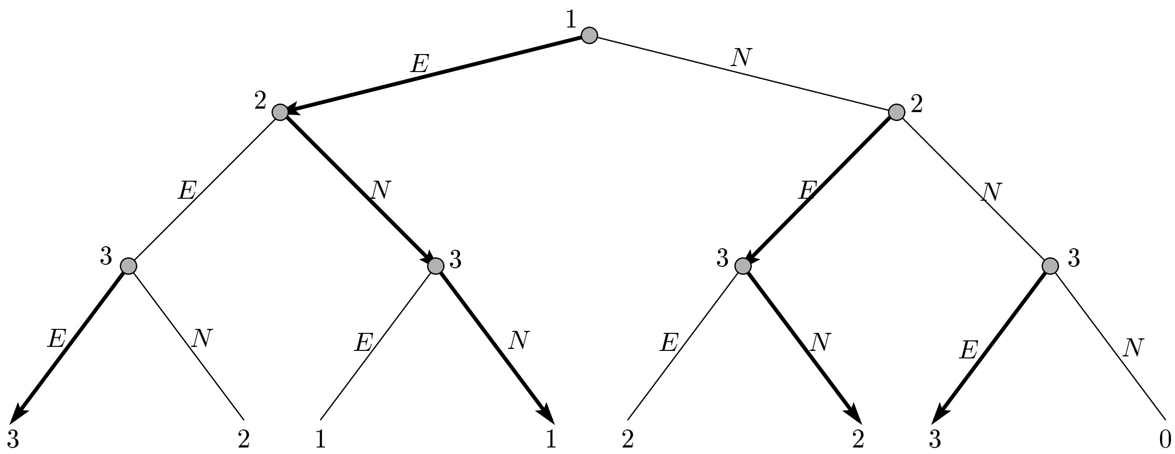
4.4 When There Is No Minor Citizen

Throughout the main analysis, we have assumed that there are a minor citizen and a Condorcet winner (Assumption 1). Nevertheless, the contrasts from the simultaneous models may emerge without minor citizen.

Strategic candidacy may occur in a two-candidate equilibrium without minor citizen. In Figure 4, we draw the decision procedure of an example where there is no minor citizen (nor the Condorcet winner). The backward induction shows that only citizen 1 and 2 enter, and citizen 1 is elected on the equilibrium path. Then, citizen 2 attempts strategic candidacy. In this example, citizen 2 prefers citizen 1 to citizen 3 (i.e., $v^2(1) - d > v^2(3)$). If citizen 2 withdraws entry, then citizen 3 gains a chance to win against citizen 1 (i.e., $C(\{1,3\}) = 3$). Hence citizen 2 has an incentive to enter without a chance to win since she wants to crowd out citizen 3 and guarantees citizen 1 to win.

We can also show a possibility that the Condorcet winner cannot win even without minor citizens. In Figure 5, we draw another example where there is no minor citizen and citizen 2 is a Condorcet winner. The backward induction shows that only citizen 1 enters and wins on the equilibrium path, implying that the Condorcet winner, citizen 2, does not win. In this example, in order to preclude citizen 3 from being elected, citizen 2 abandons to enter even if she is the Condorcet winner. Expecting citizen 2's withdraw, citizen 1 can enter unopposed.

These examples point out possibilities that strategic candidacy may occur and a citizen other than the Condorcet winner may be a unique candidate without Assumption 1. It must be noted that such outcomes rely on the order of decision making if Assumption



$C(\{1,2\}) = 2$, $C(\{1,3\}) = 1$, $C(\{2,3\}) = 2$, $C(\mathcal{N}) = 3$, and $v^2(1) > v^2(3) - d$
(The terminal nodes represent the winner.)

Figure 5: Example without a Minor Citizen

1 is dropped. Nevertheless, the existence of minor citizens is not necessarily a crucial assumption for our main arguments on the electoral outcome.

5 Concluding Remarks

This article highlights that change of the structure of decision making on entry can substantially change the equilibrium behaviour in a citizen-candidate model. We consider a citizen-candidate model with sequential entry and show that some well-known results in the simultaneous decision model are not necessarily valid. Specifically, we show that (i) strategic candidacy may occur on a two-candidate equilibrium; (ii) it may be the case that there is no one-candidate equilibrium even if there is a strict Condorcet winner among the potential candidates; and (iii) there may be a one-candidate equilibrium on which the unique candidate is not a Condorcet winner.

Our propositions rely more or less on assumptions that there are three potential candidates and the timing of decision making is fixed. Thus, the natural question is how robust our result is when there are four or more citizens or the timing of decision making is endogenous. Both extensions are seemingly challenging. When we add an extra citizen in our model, the electoral outcome would highly depend on the order of decision making, citizens' preferences, and voting outcomes. It makes it much more difficult for us to induce general property on electoral outcomes in sequential-entry models. If we try an analysis of endogenous timing, it is necessary to find an appropriate approach: what model should be constructed for characterizing robust and realistic results. These issues are left for future research.

A Proofs

A.1 Proof of Lemma 1

Without loss of generality, assume that citizen 1 is minor. That is, $C(\{1,2\}) = 2$ and $C(\{1,3\}) = 3$ hold. By definition of choice function $C(\cdot)$, $C(\{2,3\}) \in \mathcal{N}$. That is, if $C(\{2,3\}) = 2$, then citizen 2 is the Condorcet winner; otherwise, citizen 3 is the Condorcet winner. ■

A.2 Proof of Proposition 1.

Without loss of generality, assume that citizen 1 is the Condorcet winner.

Existence Show that $\hat{\sigma} \equiv (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3) = (E, N, N)$ is a Nash equilibrium. First, it is obvious that citizen 1 has no incentive to deviate from $\hat{\sigma}_1 = E$ given $\hat{\sigma}_2 = \hat{\sigma}_3 = N$ since she wins for certain by entering. Next, consider citizen 2's strategy given $\hat{\sigma}_1 = E$ and $\hat{\sigma}_3 = N$. If $a_2 = E$, then her payoff is $v^2(1) - d$ since citizen 1 is the Condorcet winner. If $a_2 = N$, then her payoff is $v^2(1)$. That is, citizen 2 has no incentive to deviate from $\hat{\sigma}_2 = N$. The same argument can apply for citizen 3. Thus, $\hat{\sigma}$ is a Nash equilibrium.

Uniqueness We will show that any other pure strategy profile $\tilde{\sigma} \neq (E, N, N)$ is not a Nash equilibrium. Note that $\#(\hat{S}(\tilde{\sigma})) \neq 0$; otherwise, any citizen has an incentive to enter since she wins for certain by entering the election.

Case 1: $\#(\hat{S}(\tilde{\sigma})) = 1$. Because $\tilde{\sigma} \neq (E, N, N)$, $\tilde{\sigma}_1 = N$ holds. However, citizen 1 has an incentive to enter since she is the Condorcet winner, which is a contradiction.

Case 2: $\#(\hat{S}(\tilde{\sigma})) = 2$. Without loss of generality, assume $\hat{S}(\tilde{\sigma}) = \{i, j\}$ and $C(\{i, j\}) = i$. Note that citizen j has an incentive to exit for saving the entry cost, which is a contradiction.

Case 3: $\#(\hat{S}(\tilde{\sigma})) = 3$. Let citizen j be the loser other than citizen M. If $a_j = E$, then citizen j's payoff is $v^j(GW) - d$. If $a_j = N$, then citizen j's payoff is $v^j(GW)$ since the entrants are citizens GW and M. Thus, citizen j has an incentive to exit for saving the entry cost, which is a contradiction. ■

A.3 Proofs of Propositions 2, 3, and 4

Let $\hat{h}_i(\sigma)$ citizen i's history (i.e., set of the candidates citizen i observes) under strategy profile σ on the equilibrium path.

The proof consists of a series of the following Lemmas.

Lemma 2 *Suppose $CW = GW$. Then, for any subgame perfect equilibrium σ^*, $\hat{S}(\sigma^*) = \{CW\}$.*

Proof of Lemma 2. First, we show $\sigma^*_{CW}(h_{CW}) = E$ for any h_{CW}. Consider CW's arbitrary history h_{CW}. If CW chooses E, then by assumption of $C(\cdot)$, she wins in the election regardless of the rival candidates and gains $v^{CW}(CW) - d$. If, on the other hand, CW chooses N, then either another candidate wins or no one is elected, which yields payoffs for CW strictly less than $v^{CW}(CW) - d$. Then, CW prefers to enter.

Consider, next, an action chosen by another citizen i. Suppose that citizen i is a leader of the CW. Then, since CW will enter and be elected regardless of citizen i's decision, citizen i prefers to stay away for saving the entry costs. Suppose, on the other hand, that citizen i is a follower of the CW. Then, since CW has already entered on the equilibrium path, CW will be elected regardless of citizen i's decision. Hence, citizen i again prefers to stay away for saving the entry costs on the equilibrium path.

Therefore, a subgame perfect equilibrium must satisfy that on the equilibrium path only CW enters. ■

Lemma 3 *For any subgame perfect equilibrium σ^*, neither $\hat{S}(\sigma^*) = \emptyset$, $\hat{S}(\sigma^*) = \{M\}$, $\hat{S}(\sigma^*) = \mathcal{N}$, nor $\hat{S}(\sigma^*) = \{M, CW\}$.*

Proof of Lemma 3. Suppose $\hat{S}(\sigma^*) = \emptyset$. Then, citizen 3 chooses N on the equilibrium path. If she deviates to enter on the equilibrium path, then she is the unique candidate and elected, which is strictly better off than not to enter. Therefore, it is a contradiction.

Suppose $\hat{S}(\sigma^*) = \{M\}$. There are two subcases to be considered.

(i) $M = 1$ **or** 2. On the equilibrium path, $\sigma^*_3(\hat{h}_3(\sigma^*)) = N$, and her payoff is $v^3(M)$. However, it is obvious that citizen 3 has an incentive to enter under history $\hat{h}_3(\sigma^*)$ since she always wins by entering, which is a contradiction.

(ii) $M = 3$. On the equilibrium path, $\sigma_2(\hat{h}_2(\sigma^*)) = N$ and citizen 2's payoff is $v^2(M)$. Suppose that she deviates to $a_2 = E$ under history $\hat{h}_2(\sigma^*) = \emptyset$. Since citizen 3 is minor, she never enters the election under history $h_3 = \{2\}$. As a result, citizen 2's payoff is $v^2(2) - d$ by this deviation, so citizen 2 has an incentive to deviate, which is a contradiction.

Suppose, next, $\hat{S}(\sigma^*) = \mathcal{N}$. By definition, $C(\hat{S}(\sigma^*)) = GW$ and citizen CW's payoff is $v^{CW}(GW) - d$. Now, suppose that citizen CW deviates to $a_{CW} = N$. There are two subcases to be considered.

(i) citizen GW follows citizen CW. In the subgame after $a_{CW} = N$, citizen GW still enters. Then, this deviation does not change the winner. Hence, citizen CW has an incentive to deviate for saving the entry cost, which is a contradiction.

(ii) citizen CW follows citizen GW. Note that citizen GW has already entered at citizen CW's decision node. Then, this deviation does not change the winner. Hence, citizen CW has an incentive to deviate for saving the entry cost, which is a contradiction.

Suppose, finally, $\hat{S}(\sigma^*) = \{M, CW\}$. Note that citizen M's equilibrium payoff is $v^M(CW) - d$. Suppose that citizen M deviates to $a_M = N$. There are two subcases to be considered.

(i) citizen M follows citizen CW. Note that $CW \in \hat{h}_M(\sigma^*)$, so $a_M = N$ induces that citizen CW is the winner since there exist at most two standing citizens. Hence, citizen M obtains $v^M(CW)$. Thus, citizen M has an incentive to exit for saving the cost, which is a contradiction.

(ii) citizen M is followed by citizen CW. Citizen CW observes that citizen M has already exited at her decision node. Given this history, citizen CW's optimal action is $a_{CW} = E$ since she definitely wins once entering the election. Hence, citizen M obtains payoff $v^M(CW)$. Thus, citizen M has an incentive to enter, which is a contradiction. ■

Lemma 4 *If citizen i attempts strategic candidacy in equilibrium σ^*, then citizen i is minor.*

Proof of Lemma 4. Suppose, in contrast, that there exists a subgame perfect equilibrium σ^* such that either (i) $CW \in \hat{S}(\sigma^*)\backslash\{C(\hat{S}(\sigma^*))\}$ or (ii) $GW \in \hat{S}(\sigma^*)\backslash\{C(\hat{S}(\sigma^*))\}$.

Case 1: $CW \in \hat{S}(\sigma^*)\backslash\{C(\hat{S}(\sigma^*))\}$**.** To support this scenario, $\hat{S}(\sigma^*) = \mathcal{N}$ should hold. However, it is impossible by Lemma 3, which is a contradiction.

Case 2: $GW \in \hat{S}(\sigma^*)\backslash\{C(\hat{S}(\sigma^*))\}$**.** To support this scenario, $\hat{S}(\sigma^*) = \{CW, GW\}$ should hold. Note that citizen GW obtains $v^{GW}(CW) - d$. Now, suppose that citizen GW deviates to $a_{GW} = N$ under history $\hat{h}_{GW}(\sigma^*)$. There are two cases to be considered.

(i) citizen CW follows citizen GW. Given $a_{GW} = N$, citizen CW still enters since she wins the election. That is, the winner is still citizen CW in this deviation. Hence, citizen GW has an incentive to exit for saving the entry cost, which is a contradiction.

(ii) citizen GW follows citizen CW. Note that citizen CW has already entered at citizen GW's decision node. Hence, this deviation does not change the winner. Then, citizen GW has an incentive to exit for saving the entry cost, which is a contradiction. ■

Lemma 5 *If the minor citizen M attempts strategic candidacy, then $v^M(CW) < v^M(GW) - d$.*

Proof of Lemma 5. Suppose, in contrast, that there exists a subgame perfect equilibrium σ^* such that $M \in \hat{S}(\sigma^*) \backslash \{C(\hat{S}(\sigma^*))\}$ when $v^M(CW) > v^M(GW) - d$. Lemma 3 implies $\hat{S}(\sigma^*) \neq \mathcal{N}$ and $\hat{S}(\sigma^*) \neq \{M, CW\}$. Then, we must have $\hat{S}(\sigma^*) = \{M, GW\}$ and $GW \neq CW$. Note that citizen M's equilibrium payoff is $v^M(GW) - d$. Suppose that citizen M deviates to $a_M = N$. Since $CW \neq M$, there are two subcases to be considered.

(i) citizen M follows citizen CW. Note that $CW \notin \hat{h}_M(\sigma^*)$, so $a_M = N$ induces that citizen GW is the winner since citizen CW exits. Hence, citizen M obtains $v^M(GW)$. Thus, citizen M has an incentive to exit for saving the cost, which is a contradiction.

(ii) citizen M is followed by citizen CW. Citizen CW observes that citizen M has already exited at her decision node. Then, citizen CW enters since she always wins for entering the election. That is, citizen M's payoff becomes $v^M(CW)$. By the hypothesis, citizen M has an incentive to exit, which is a contradiction.

Therefore, if the minor citizen M attempts strategic candidacy, then $v^M(CW) < v^M(GW) - d$ should hold. ■

Lemma 6 *Suppose that $CW \neq 1$ and $v^M(CW) < v^M(GW) - d$. Then, $\sigma^*_{CW}(\hat{h}_{CW}(E, \sigma^*_{-1})) = N$ holds for any subgame perfect equilibrium σ^*.*

Proof of Lemma 6. Note that the assumption $v^M(CW) < v^M(GW) - d$ implies $CW \neq GW$. Suppose, in contrast, that there exists a subgame perfect equilibrium σ^* such that $\sigma^*_{CW}(\hat{h}_{CW}(E, \sigma^*_{-1})) = E$. First, we show the following claim.

Claim 1 *If $CW \neq 1$ and $\sigma^*_{CW}(\hat{h}_{CW}(E, \sigma^*_{-1})) = E$, then $C(\hat{S}(E, \sigma^*_{-1})) = CW$.*

Proof of Claim 1. Suppose, in contrast, that $C(\hat{S}(E, \sigma^*_{-1})) \neq CW$. we have to consider the following two cases.

Case 1: $C(\hat{S}(E, \sigma^*_{-1})) = M$. In this scenario, $\hat{S}(E, \sigma^*_{-1}) = \{M\}$ should hold by definition of citizen M. However, $CW \in \hat{S}(E, \sigma^*_{-1})$, which is a contradiction.

Case 2: $C(\hat{S}(E, \sigma^*_{-1})) = GW$**.** Since $\sigma^*_{CW}(\hat{h}_{CW}(E, \sigma^*_{-1})) = E$, $\hat{S}(E, \sigma^*_{-1}) = \mathcal{N}$ should hold. However, by the same argument used in the proof of Lemma 3, we can show that citizen CW has an incentive to exit, which is a contradiction. □

Proof of Lemma 6 Continued. By Claim 1 and $CW \neq GW$, there should exist citizen $j \in \{M, GW\}$ such that $\sigma^*_j(\hat{h}_j(E, \sigma^*_{-1})) = N$. Hence, under strategy profile (E, σ^*_{-1}), citizen j's payoff is $v^j(CW)$. Note that $v^j(CW) < v^j(GW) - d$ holds since $v^M(CW) < v^M(GW) - d$. Now, suppose that citizen j deviates to $a_j = E$. There are two cases to be considered.

Case 1: $j = 2$**.** In this scenario, $CW = 3$. Given history $h_{CW} = \{1, 2\}$, citizen CW exits since she never wins. As a result, citizen j's payoff is $v^j(GW) - d$, and this is a profitable deviation for citizen j, which is a contradiction.

Case 2: $j = 3$**.** Note that $\hat{h}_j(E, \sigma^*_{-1}) = \{1, 2(= CW)\}$. Hence, this deviation strictly improves citizen j's payoff since it changes the winner from citizen CW to citizen GW, which is a contradiction.

Therefore, for any subgame perfect equilibrium σ^*, $\sigma^*_{CW}(\hat{h}_{CW}(E, \sigma^*_{-1})) = N$ should hold in this scenario. ■

A.3.1 Proof of Proposition 4

When $CW = GW$, Lemma 2 implies the result. Then, consider the case of $CW \neq GW$.

Sufficiency Note that if $C(\hat{S}(\sigma^*)) = M$, then $\hat{S}(\sigma^*) = \{M\}$ should hold. However, by Lemma 3, it is impossible. Hence, either $C(\hat{S}(\sigma^*)) = CW$ or GW. Suppose, in contrast, that there exists a subgame perfect equilibrium σ^* such that $C(\hat{S}(\sigma^*)) = GW$ when $v^M(CW) > v^M(GW) - d$. By Lemmas 3 and 5, $\hat{S}(\sigma^*) = \{GW\}$ must hold.

Note that $\sigma^*_{CW}(\hat{h}_{CW}(\sigma^*)) = N$, and citizen CW's utility is $v^{CW}(GW)$. To support this behavior, $C(\hat{S}(\tilde{\sigma}_{CW}, \sigma^*_{-CW})) = GW$ must hold where:

$$\tilde{\sigma}_{CW}(h_{CW}) \equiv \begin{cases} E & \text{if } h_{CW} = \hat{h}_{CW}(\sigma^*) \\ \sigma^*_{CW}(h_{CW}) & \text{otherwise.} \end{cases}$$

Otherwise, citizen CW has an incentive to enter. There are two cases to be considered.

Case 1: citizen M **is followed by citizen** CW**.** Note that $\sigma^*_M(\hat{h}_M(\sigma^*)) = N$, so $M \notin \hat{h}_{CW}(\sigma^*)$. In this scenario, citizen CW has an incentive to enter since she wins for certain by entering. That is, $C(\hat{S}(\tilde{\sigma}_{CW}, \sigma^*_{-CW})) = CW$, which is a contradiction.

Case 2: citizen M follows citizen CW. Note that $CW \in \tilde{h}_M \equiv \hat{h}_M(\tilde{\sigma}_{CW}, \sigma^*_{-CW})$. Consider citizen M's decision-making under history $\tilde{h}_M$. If $a_M = E$, then citizen M's payoff is either $v^M(CW) - d$ or $v^M(GW) - d$. If $a_M = N$, then citizen M's payoff is $v^M(CW)$. Thus, $\sigma^*_M(\tilde{h}_M) = N$ since $v^M(CW) > v^M(GW) - d$. However, this implies that $C(\hat{S}(\tilde{\sigma}_{CW}, \sigma^*_{-CW})) = CW$, which is a contradiction.

Therefore, if $v^M(CW) > v^M(GW) - d$, then $C(\hat{S}(\sigma^*)) = CW$ must hold for any subgame perfect equilibrium σ^*.

Necessity Suppose, in contrast, that there exists a subgame perfect equilibrium σ^* such that $C(\hat{S}(\sigma^*)) = CW$ when $v^M(CW) < v^M(GW) - d$. By Lemmas 3, 4, and 5, $\hat{S}(\sigma^*) = \{CW\}$ must be satisfied. Note that $\sigma^*_{CW}(\hat{h}_{CW}(\sigma^*)) = E$. There are the following two subcases to be considered.

(i) $CW = 1$**.** Note that $\sigma^*_2(\hat{h}_2(\sigma^*)) = N$, and citizen 2's payoff is $v^2(CW)$. Now, suppose that citizen 2 deviates to $a_2 = E$ under history $\hat{h}_2(\sigma^*)$. Since $v^M(CW) < v^M(GW) - d$, citizens 2 and 3 prefer citizen GW being the winner with paying the entry cost to citizen CW being the winner with exit. That is, $\sigma^*_3(\{1, 2\}) = E$. As a result, this deviation gives citizen 2 payoff $v^2(GW) - d$. Hence, citizen 2 has an incentive to enter, which is a contradiction.

(ii) $CW \neq 1$**.** Note that $\sigma^*_1 = N$, and citizen 1's payoff is $v^1(CW)$. Now, suppose that citizen 1 deviates to $a_1 = E$. Then, by Lemma 6, citizen CW exits in the subgame after $a_1 = E$. As a result, citizen GW becomes the winner, and the citizen 1 obtains payoff $v^1(GW) - d$. Since $v^M(CW) < v^M(GW) - d$, this is a profitable deviation to citizen 1, which is a contradiction.

Therefore, if $C(\hat{S}(\sigma^*)) = CW$ for any subgame perfect equilibrium σ^*, then $v^M(CW) > v^M(GW) - d$ must hold. ■

A.3.2 Proof of Proposition 2

Necessity Let σ^* be a subgame perfect equilibrium where strategic candidacy occurs. By Lemmas 4 and 5, condition 2 should hold. Hence, it is sufficient to show condition 1. Suppose, in contrast, that there exists a subgame perfect equilibrium σ^* in the game where citizen M is a follower of citizen CW. Note that $\sigma^*_M(\hat{h}_M(\sigma^*)) = E$. There are two cases to be considered.

Case 1: $\sigma^*_{CW}(\hat{h}_{CW}(\sigma^*)) = N$**.** Since strategic candidacy occurs in subgame perfect equilibrium σ^*, $\hat{S}(\sigma^*) = \{M, GW\}$ must hold. Hence, citizen M's equilibrium payoff is $v^M(GW) - d$. Now suppose that citizen M deviates to $a_M = N$ under history $\hat{h}_M(\sigma^*)$. However, in this scenario, citizen GW is still the winner, so citizen M's payoff is $v^M(GW)$. Hence, citizen M has an incentive to exit for saving the entry cost, which is a contradiction.

Case 2: $\sigma^*_{CW}(\hat{h}_{CW}(\sigma^*)) = E$**.** By condition 2 and Proposition 4, $C(\hat{S}(\sigma^*)) = GW$. Hence, $\hat{S}(\sigma^*) = \mathcal{N}$ should hold. However, it is impossible by Lemma 3, which is a contradiction.

Thus, if strategic candidacy occurs, then condition 1 must hold.

Sufficiency Suppose that conditions 1 and 2 hold. By Zermelo's theorem, there exists a subgame perfect equilibrium σ^*. Lemma 3 implies $\hat{S}(\sigma^*) \neq \emptyset$, $\hat{S}(\sigma^*) \neq \mathcal{N}$, and $\hat{S}(\sigma^*) \neq \{M\}$ and Proposition 4 implies $C(\hat{S}(\sigma^*)) \neq CW$. Hence, either $\hat{S}(\sigma^*) = \{GW\}$ or $\{M, GW\}$ must hold. Suppose, in contrast, that $\hat{S}(\sigma^*) = \{GW\}$. Note that condition 1 yields $M \notin \hat{h}_{CW}(\sigma^*)$, implying $\sigma^*_{CW}(\hat{h}_{CW}(\sigma^*)) = E$ since she wins for certain by entering, which is a contradiction to $\hat{S}(\sigma^*) = \{GW\}$. Thus, $\hat{S}(\sigma^*) = \{M, GW\}$ must hold. It is obvious that citizen M attempts strategic candidacy in subgame perfect equilibrium σ^*.

■

A.3.3 Proof of Proposition 3

1. Suppose that citizen M is a follower of citizen CW, and $v^M(CW) < v^M(GW) - d$. By Zermelo's theorem, there exists a subgame perfect equilibrium σ^*. By Proposition 4, $C(\hat{S}(\sigma^*)) = GW$. By Lemma 3, either $\hat{S}(\sigma^*) = \{GW\}$ or $\{GW, M\}$ should hold. Suppose, in contrast, that $\hat{S}(\sigma^*) = \{GW, M\}$ holds. It is obvious that citizen M attempts strategic candidacy in equilibrium σ^*. However, citizen M never attempts strategic candidacy in equilibrium σ^* by Proposition 2 since citizen M is a follower of citizen M, which is a contradiction. Thus, $\hat{S}(\sigma^*) = \{GW\}$ holds.

2. Suppose that citizen M is a follower of citizen CW, and $v^M(CW) > v^M(GW) - d$. By Zermelo's theorem, there exists a subgame perfect equilibrium σ^*. By Proposition 4, $C(\hat{S}(\sigma^*)) = CW$. By Lemma 3, either $\hat{S}(\sigma^*) = \{CW\}$ or $\{CW, GW\}$ should hold. Suppose, in contrast, that $\hat{S}(\sigma^*) = \{CW, GW\}$ holds. It is obvious that citizen GM attempts strategic candidacy in equilibrium σ^*. However, by

Lemma 4, citizen GW never attempts strategic candidacy in equilibrium σ^*, which is a contradiction. Thus, $\hat{S}(\sigma^*) = \{CW\}$ holds. ■

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