A Simple Economics of Inequality
-Market Design Approach-

Yosuke YASUDA | Osaka University
yosuke.yasuda@gmail.com

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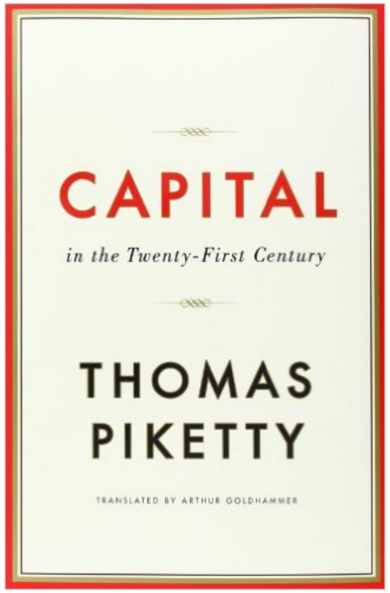
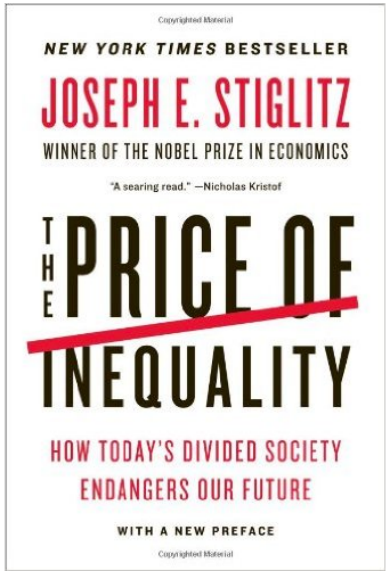
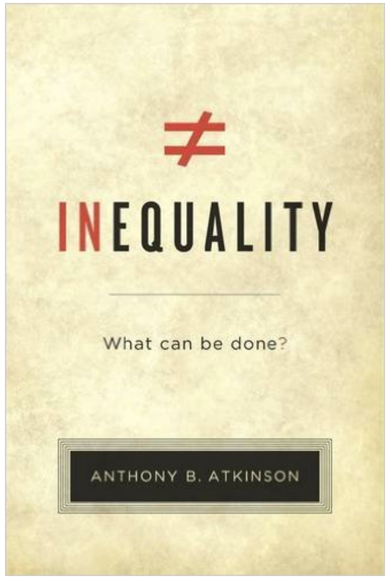
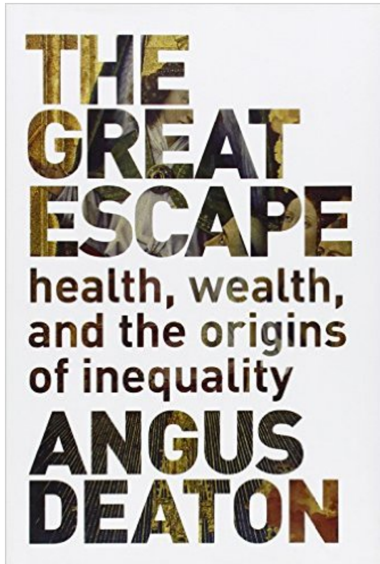
Motivation

- Inequality at the forefront of public debate!



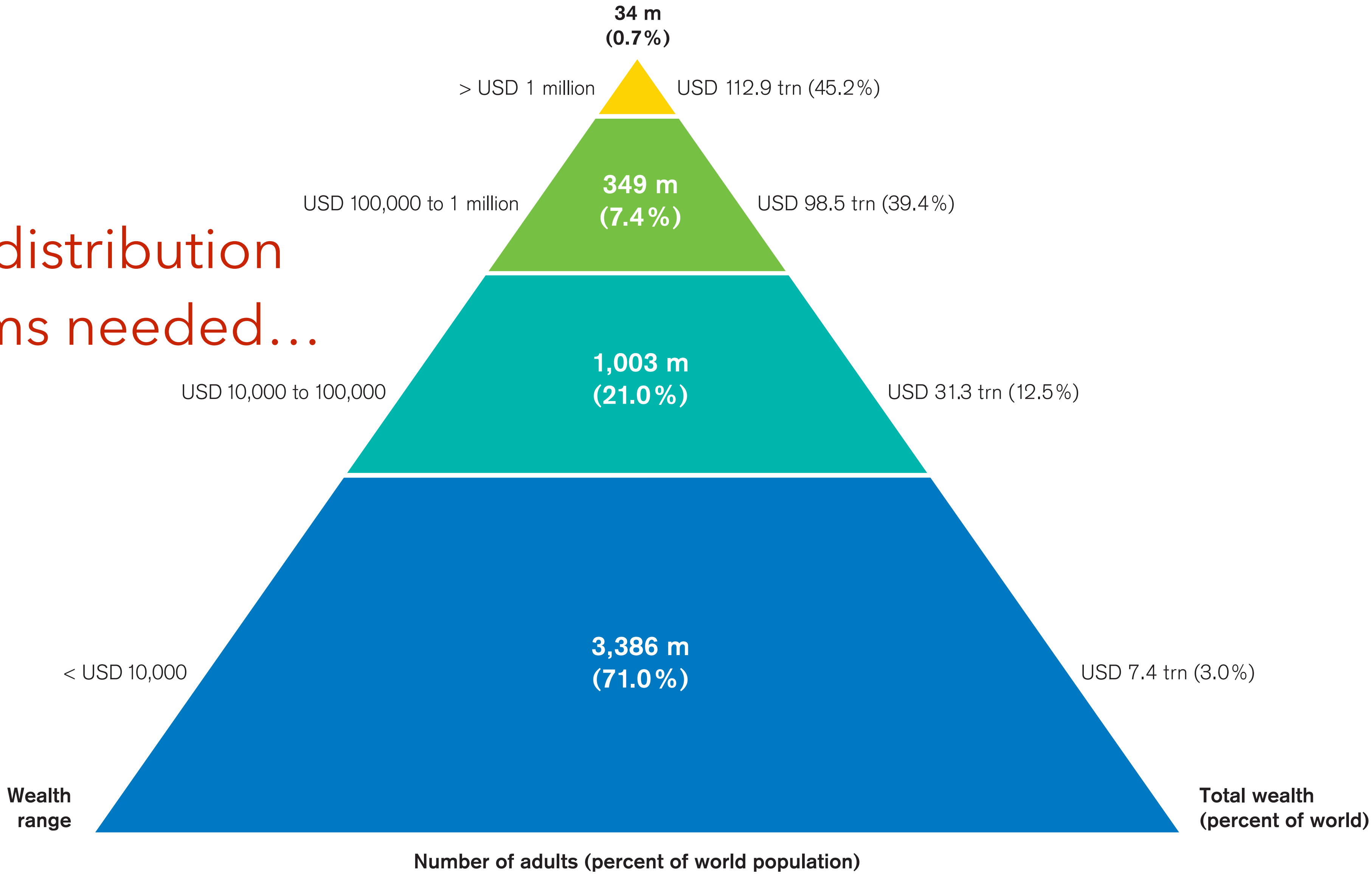






Global Wealth: Top 1% > Bottom 99 %

Redistribution seems needed…



Source: James Davies, Rodrigo Lluberas and Anthony Shorrocks, Credit Suisse Global Wealth Databook 2015

Redistribution

- Transfer from (super) rich to poor seems not work.
- Why is redistribution difficult?
  - Efficiency loss: distortion on incentives
  - Not so effective: capital gains, tax haven
  - Difficult to enforce: lobbying by rich

Our Approach

- Observation: Redistribution is difficult.
- Our Model: Redistribution is **impossible**.
    - Feasible allocation / welfare evaluation change.
    - Better understand **limitation** of market economy.
- Q: Does market economy accelerate concentration?
- A: Yes (!?): Market tends to reduce trading volume.

Summary

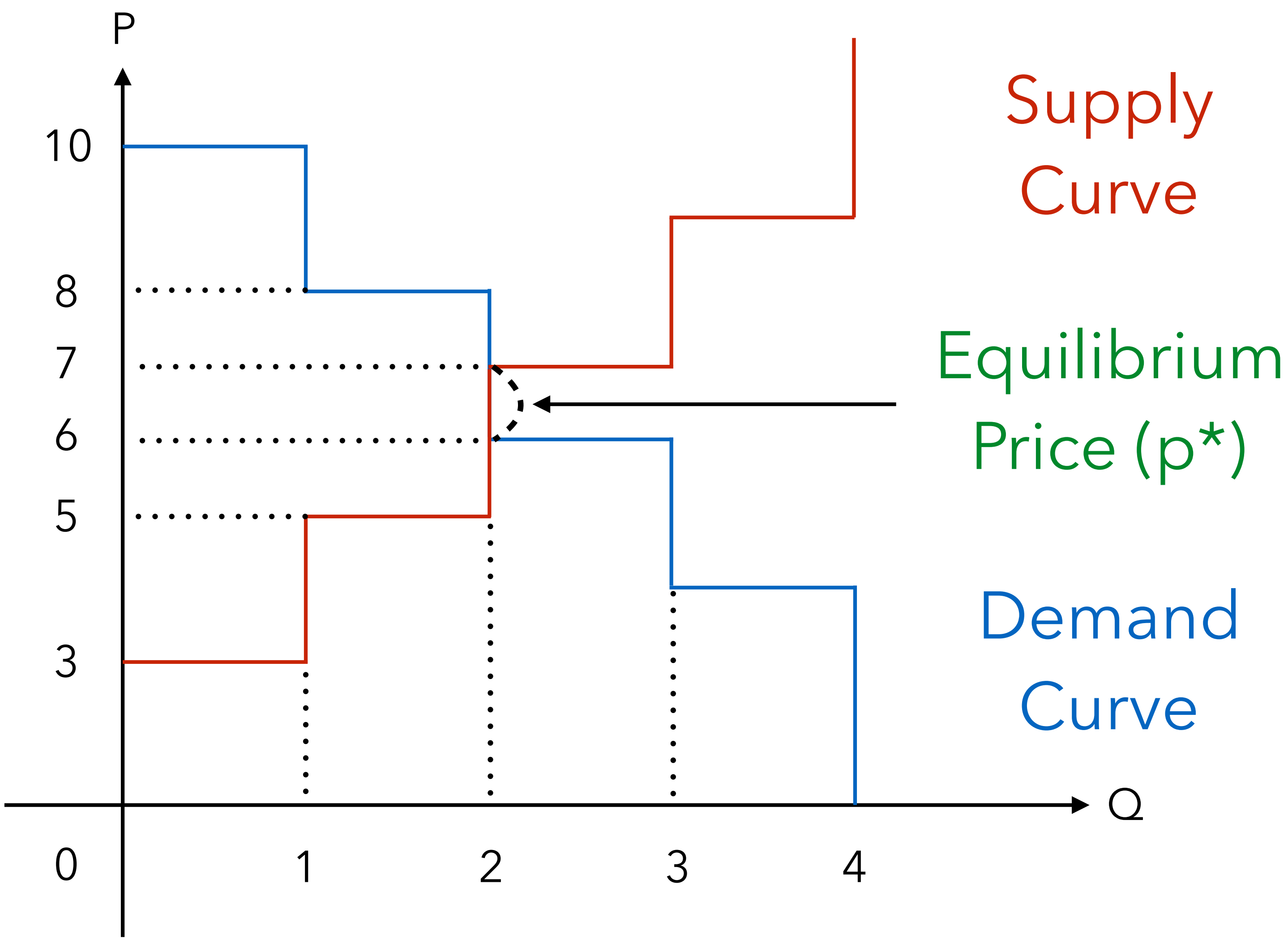
- We consider the relationship between total surplus (efficiency) and trade volume (quantity) for **homogenous good markets**, assuming that
  - (i) each buyer/seller has a **unit demand/supply**
  - (ii) redistribution (by the third party) is **infeasible**.
    - Pareto Efficiency with No Side-payment: **PENS**
- Show that competitive market **minimizes** # of trades.

Example 1

- 4 buyers, 4 sellers, unit demand/supply

Buyer	B1	B2	B3	B4
Value ($)	10	8	6	4
Seller	S1	S2	S3	S4
Cost ($)	3	5	7	9

Supply-Demand

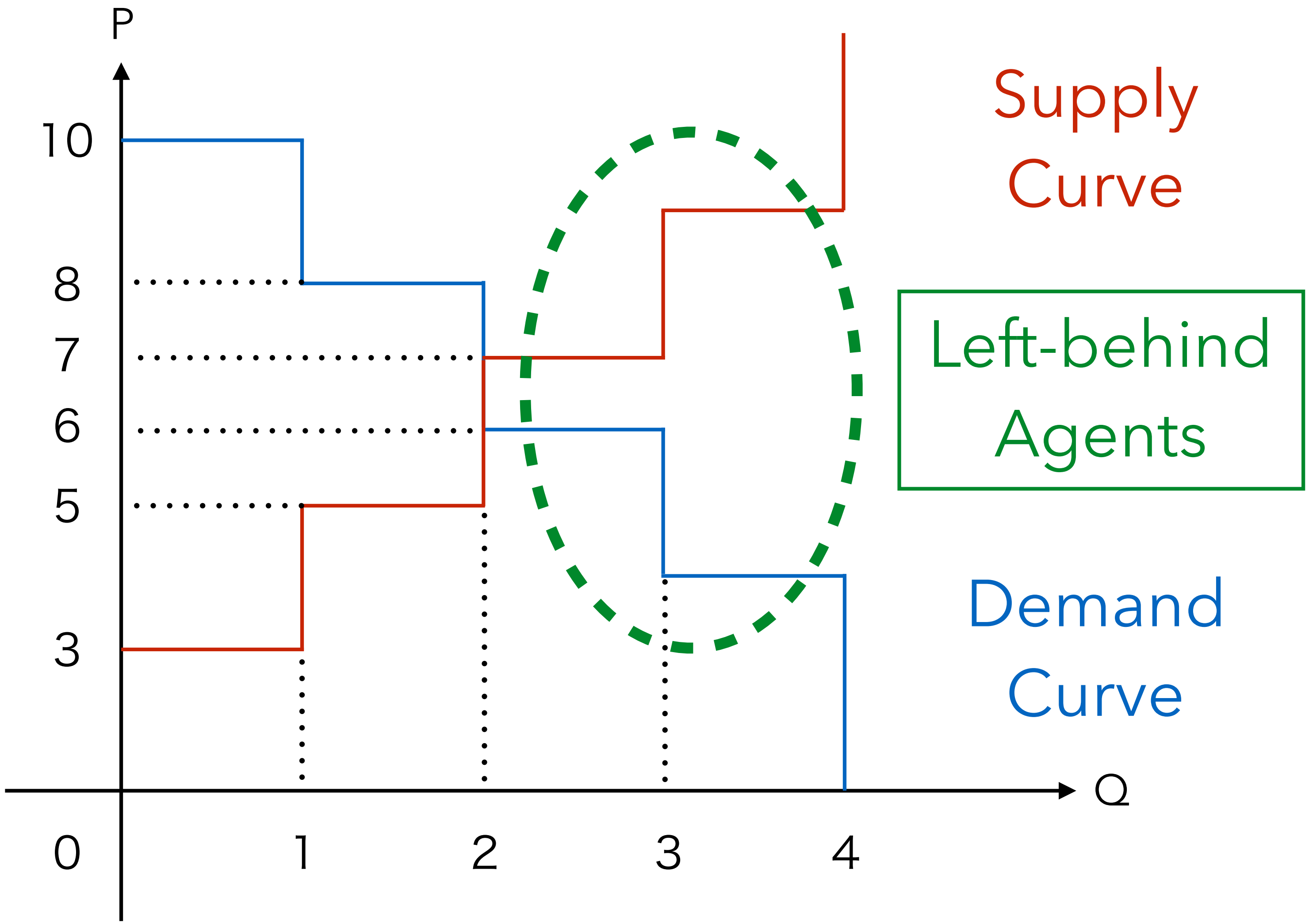


Competitive Eqm. (CE)

- Maximizes total surplus, $10: assume p* = 6.5

Buyer	B1	B2	B3	B4
Surplus ($)	**3.5**	**1.5**	0	0
Seller	S1	S2	S3	S4
Surplus ($)	**3.5**	**1.5**	0	0

CE Maximizes Surplus, but…



Alternative: X

- Trade pairs: B1-S3, B2-S2, B3-S1: $p = (V+C)/2$

Buyer	B1	B2	B3	B4
Surplus ($)	**1.5**	**1.5**	**1.5**	0
Seller	S1	S2	S3	S4
Surplus ($)	**1.5**	**1.5**	**1.5**	0

Alternative: Y

- Trade pairs: B1-S4, B2-S3, B3-S2, B4-S1

Buyer	B1	B2	B3	B4
Surplus ($)	**0.5**	**0.5**	**0.5**	**0.5**
Seller	S1	S2	S3	S4
Surplus ($)	**0.5**	**0.5**	**0.5**	**0.5**

Comparison

- Trade-off: efficiency vs. quantity

Allocation	CE	X	Y
Total Surplus	**10**	9	4
# of Trading Agents	4 (50%)	6 (75%)	**8 (100%)**
PENS & IR	**Yes**	**Yes**	**Yes**
Unique Price	**Yes**	No	No

Efficiency vs. Quantity

Total Surplus

Trading Volume

Competitive market maximizes surplus
at the expense of trading volume…

Market Economy

- Homogenous good market
- Finitely many buyers and sellers (**n** total agents)
- Each has unit demand/supply
- Other simplifying assumptions:
  - A. 0 utility for non-trading agents
  - B. No buyer-seller pair generates 0 surplus

Pareto Efficiency

- Allocation z is **Pareto efficient** if and only if there exists NO other feasible allocation z′, which makes
  - every one weakly better off, and
  - someone strictly better off.
- **Feasibility**: allocation must be achieved through bilateral transactions (buyer-seller pairs).
- **Preferences**: larger surplus is better (unit demand).

Definition of PENS

- Consider $Z = \{x^1, x^2, \ldots, x^n\}$ (bilaterally achievable (**BA**) allocations):
  - $x^1 + x^2 + \cdots + x^n = e^1 + e^2 + \cdots + e^n$ (resource constraint), and
  - for each agent **i**, $x^i = e^i$ (no trade), or
  - there exist agent **j** such that $x^i + x^j = e^i + e^j$ (bilateral trade).
- Allocation z is called **PENS** if there exists no allocation z′ in Z such that z′ Pareto dominates z.
- PE allocation (in Z) is always PENS, but NOT vice versa.
  - **PENS** is weaker than standard **PE**.

Why are X and Y PENS?

- CE allocation Pareto dominates neither X nor Y.

Buyer	B1	B2	B3	B4
Surplus ($)	**3.5**	**1.5**	0	0
Seller	S1	S2	S3	S4
Surplus ($)	**3.5**	**1.5**	0	0

If Side-Payment Possible

- Transfer from B1 to B3, B4 and S1 to S3, S4.

Buyer	B1	B2	B3	B4
Surplus ($)	**3.5**	**1.5**	0	0
Seller	S1	S2	S3	S4
Surplus ($)	**3.5**	**1.5**	0	0

$0.5

$1.5

$1.5

$0.5

X and Y are Not PE

- CE + **side-payment** Pareto dominates X & Y.

Buyer	B1	B2	B3	B4
Surplus ($)	**1.5**	**1.5**	**1.5**	**0.5**
Seller	S1	S2	S3	S4
Surplus ($)	**1.5**	**1.5**	**1.5**	**0.5**

Main Theorem

- **Lemma 1**
  Any CE allocation is BA and PENS.

- **Theorem 1**
  The number of trading agents (trading volume) under a CE allocation is minimum among all BA allocations that are PENS.

Proof of Theorem 1

1. Suppose not. Then, there must exist a PENS allocation, say z, which has strictly fewer (trading) buyer-seller pairs than the competitive equilibrium.

2. There are at least a buyer, say B*, and a seller, S*, who would receive non-negative surplus in CE but cannot engage in any trade, i.e., receive zero surplus, in the alternative allocation z.

3. V_{B^*} is (weakly) larger than p* which is also larger than C_{S^*}.

4. B*-S* pair generates positive surplus. <= $V_{B^*} > C_{S^*}$

5. Contradicts to our presumption that z is PENS.

Converse

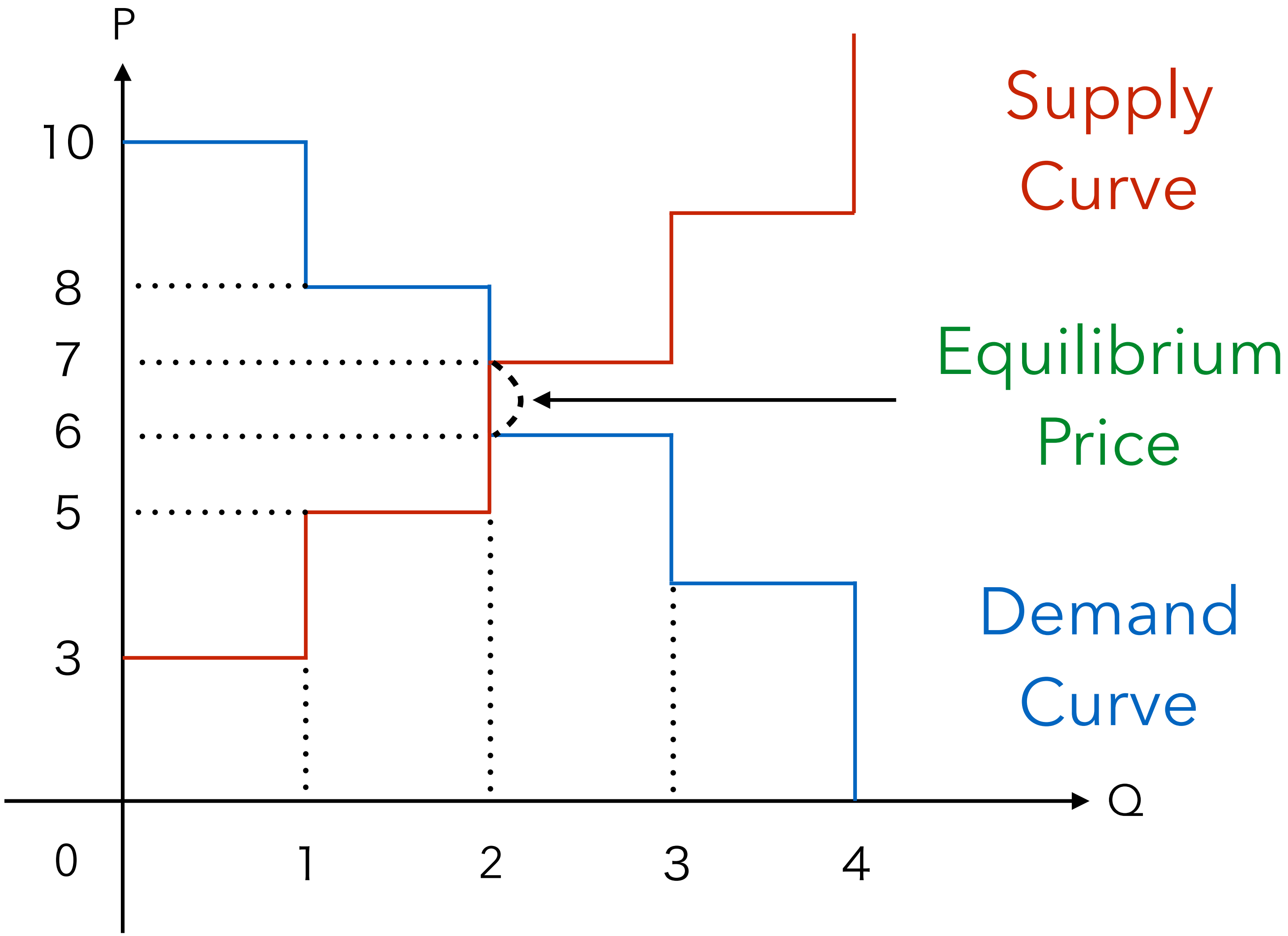
- **Theorem 2**
  Let **k** be the trading volume under a CE. Then, there exists a BA, PENS and IR allocation that entails strictly larger number of trades than **k** if and only if

  - (i) value of B_1 exceeds the cost of S_{k+1}, and

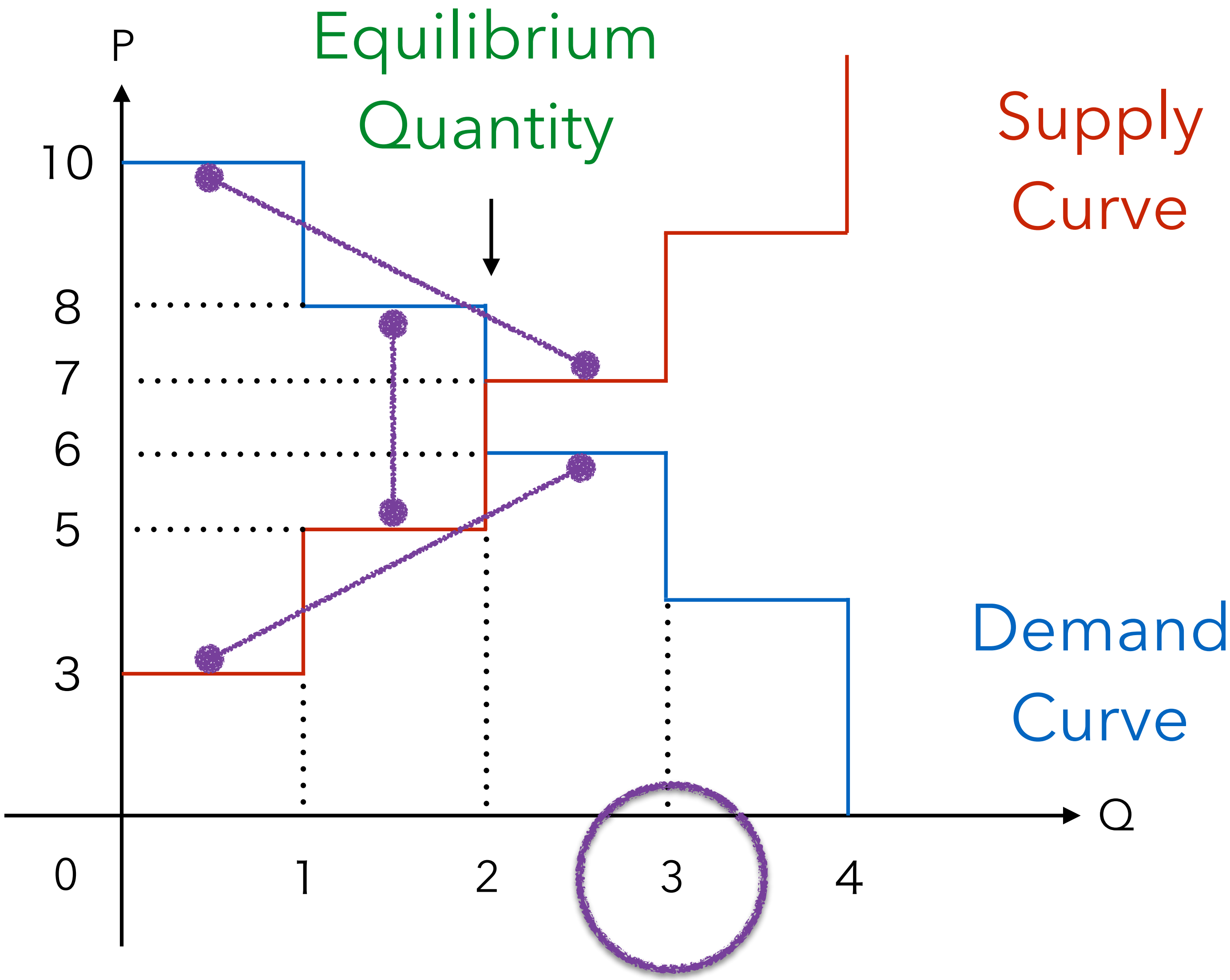
  - (ii) value of B_{k+1} exceeds the cost of S_1,

  where buyer/seller with smaller number has higher value/lower cost.

Equilibrium (k = 2)



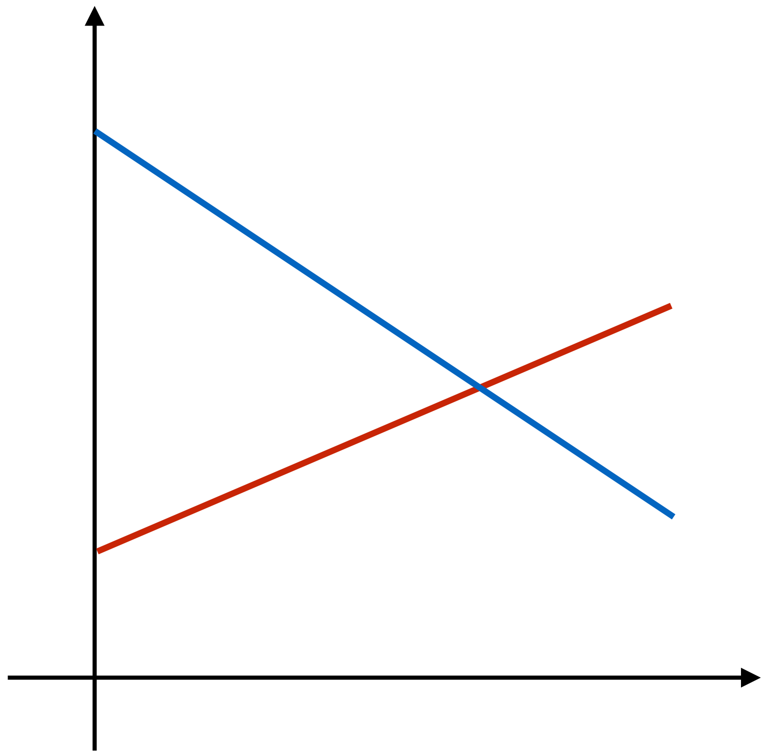
Illustration



Proof of Theorem 2

- If part (<=)
  - B_1-S_{k+1} and B_{k+1}-S_1 pairs generate positive surplus.
  - Let B_2, …, B_k trade with S_2, …, S_k.
  - This is a PENS and IR allocation with k+1 trades.
- Only if part (=>)
  - If (i) is not satisfied, S_{k+1} cannot engage in any profitable trade.
  - If (ii) is not satisfied, B_{k+1} cannot engage in any profitable trade.
  - Impossible to make k+1 (or more) profitable trading pairs.

Graphical Intuition



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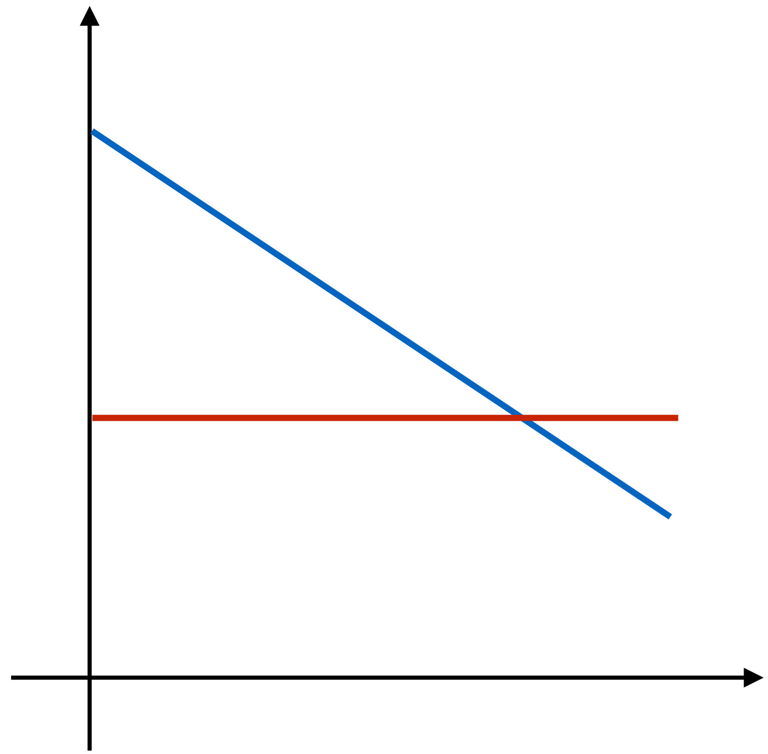

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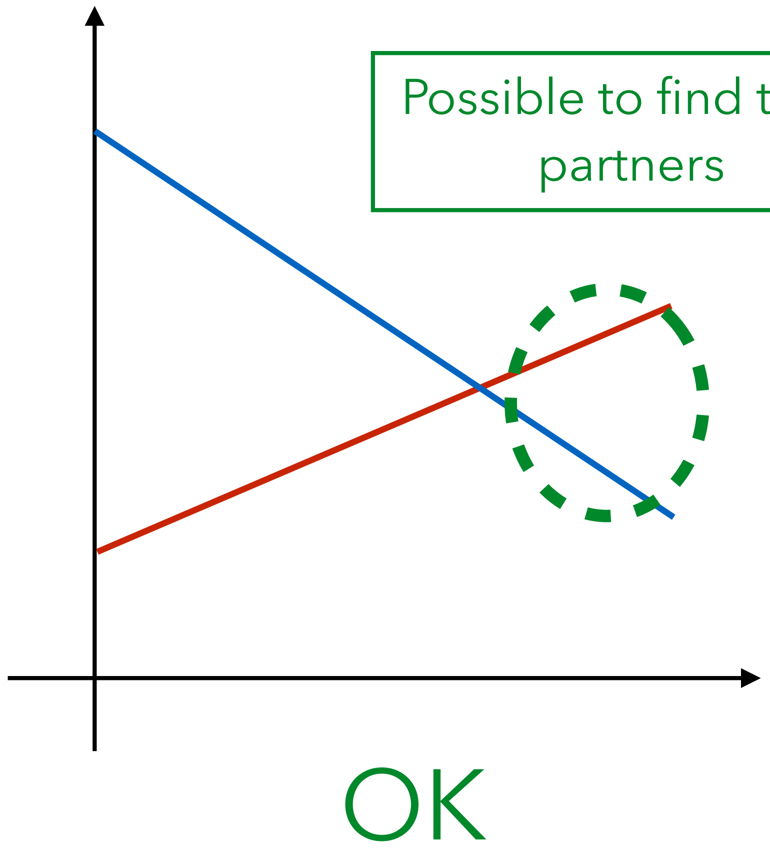
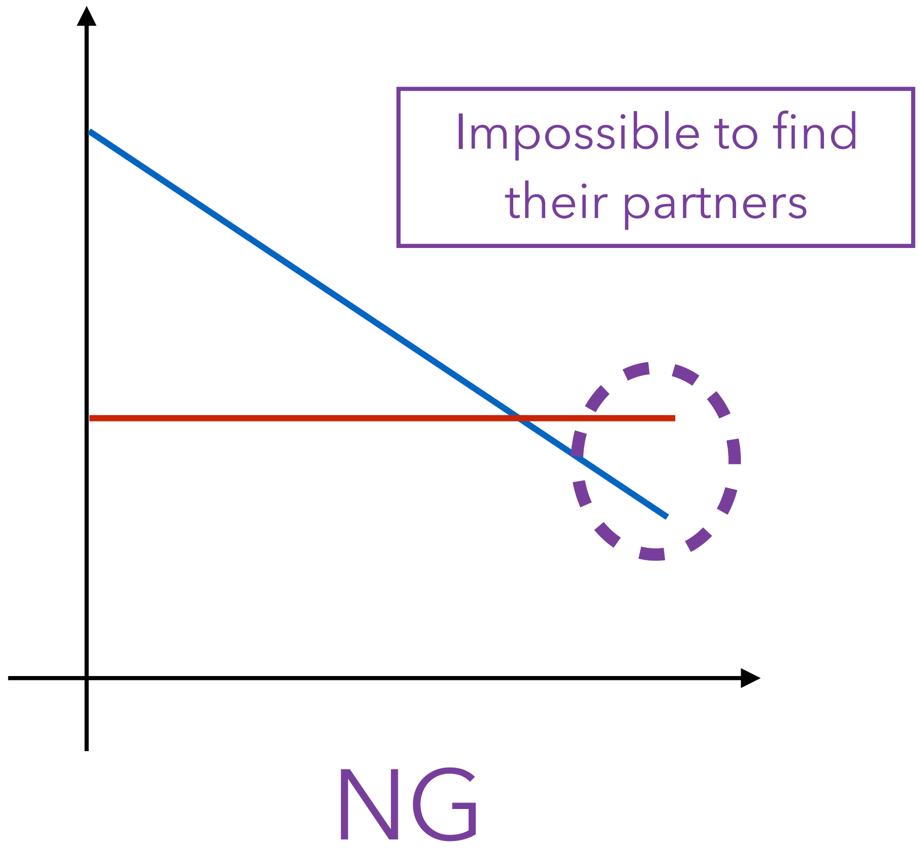
Graphical Intuition





Pioneering Experiments

- Connection to the experimental studies:
- **Chamberlin (1948)** vs. **Smith (1962)**
  - Chamberlin, E. H. (1948). “An experimental imperfect market.”
    - The Journal of Political Economy, 95-108.
  - Smith, V. L. (1962). “An experimental study of competitive market behavior.”
    - The Journal of Political Economy, 111-137.

Pioneering Experiments

- **Chamberlin (1948)** vs. **Smith (1962)**
  - In Chamberlin, buyers and sellers engage in bilateral bargaining, transaction price is recorded on the blackboard as contracts made; single period.
    => Imperfect market: **Excess quantities**
  - In Smith's **double auctions**, each trader's quotation is addressed to the entire trading group one quotation at a time; multiple periods (learning).
    => Converge to perfectly competitive market

Chamberlin (1948)

Transactions B	Transactions S	Transactions P
56	18	55
54	26	40
72	30	50
84	34	45
44	44	44
102	42	42
80	20	40
60	28	55
48	40	45
76	36	45
94	52	55
68	58	62
66	46	55
82	32	58
90	72	72
104	54	54
52	50	50
86	64	64
74	62	69

Left Over

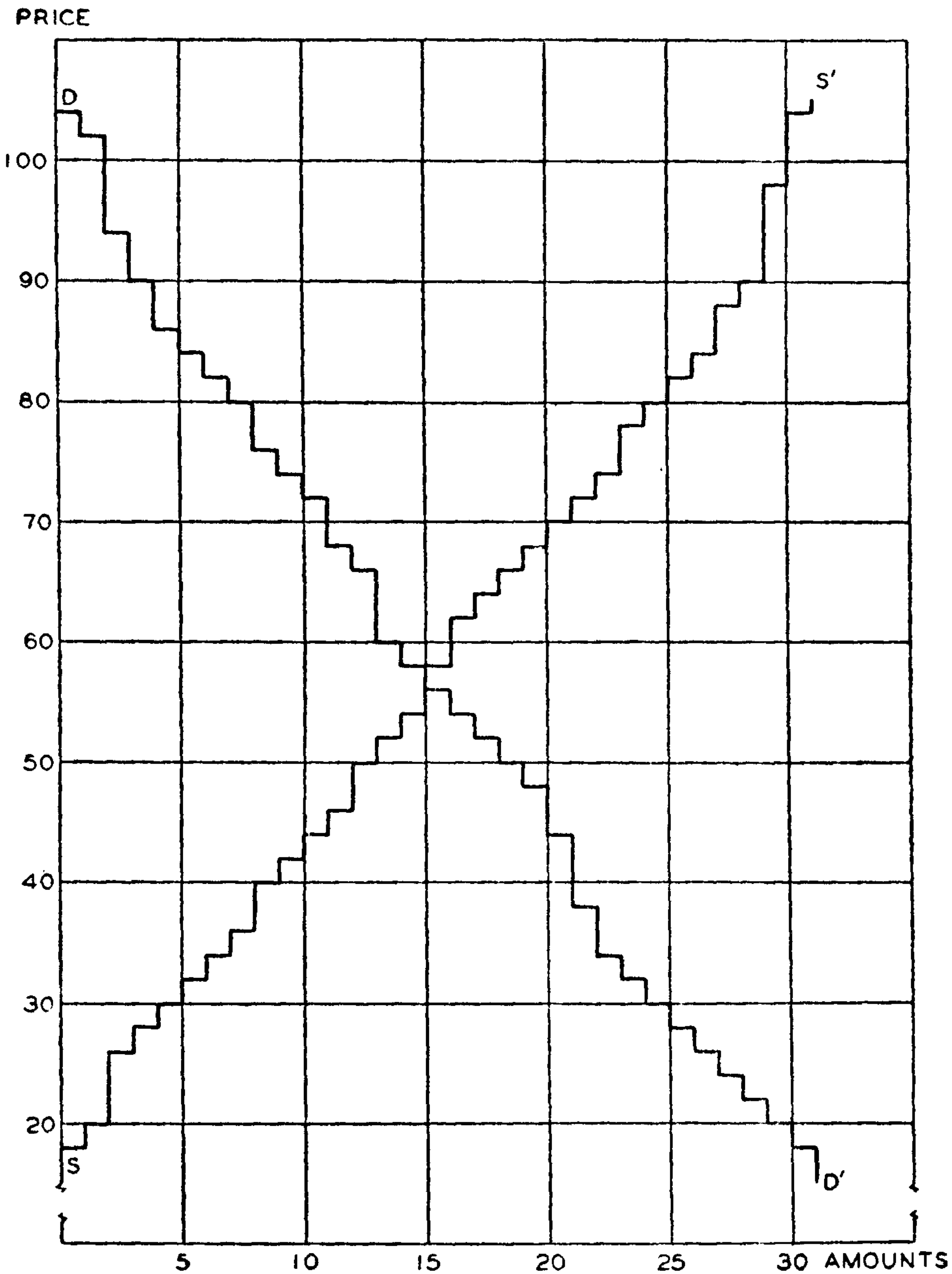
B	S
38	68
50	66
28	82
32	88
18	90
26	84
22	104
24	78
30	80
20	98
34	74
58	70

Market Schedules

B	S
104	18
102	20
94	26
90	28
86	30
84	32
82	34
80	36
76	40
74	42
72	44
68	46
66	50
60	52
58	54
56	58
54	62
52	64
50	66
48	68
44	70
38	72
34	74
32	78
30	80
28	82
26	84
24	88
22	90
20	98
18	104

Equilibrium sales 15
Actual sales 19

Equilibrium price 57 (56–58)
Average of actual prices . . . 52.63



Excess Quantity

- Chamberlin's **excess quantity** puzzle:
  - Sales volume > equilibrium quantity => 42/46
  - Sales volume = equilibrium quantity => 4/46
  - Sales volume < equilibrium quantity => 0/46
- "price fluctuation render the volume of sales **normally greater than the equilibrium amount** which is indicated by supply and demand curves"
- Our results may account for Chamberlin's puzzle.

Extension: Matching

- Stable matching (Core) induce minimum pairs.
  => Examples 2a, 3, 4a

- # of Stable matching pairs not always minimum.
  => Examples 2b, 4b

- NTU – Anything can happen. (PE = PENS)

- TU – Assortative stable matching is minimum.

NTU: Example 2a

- 2 doctors, 2 hospitals

Agent	D1	D2	H1	H2
1st	H1	H1	D1	D1
2nd	H2	-	D2	D2

- Unique Stable Matching: D1-H1 (D2, H2 single)

- An Alternative: D1-H2, D2-H1 <= PE and IR

=> All agents find their mates under non-stable outcome.

NTU: Example 2a

- 2 doctors, 2 hospitals (H2: rural hospital)

Agent	D1	D2	H1	H2
1st	**H1**	**H1**	**D1**	**D1**
2nd	**H2**	-	**D2**	D2

- Unique Stable Matching: D1-H1 (D2, H2 single)
- An Alternative: D1-H2, D2-H1 <= PE and IR

=> All agents find their mates under non-stable outcome.

NTU: Example 2b

- 2 doctors, 2 hospitals

Agent	D1	D2	H1	H2
1st	H1	H1	D2	D1
2nd	H2	-	D1	D2

- Unique Stable Matching: D1-H2, D2-H1
- An Alternative: D1-H1 (D2, H2 single) <= PE and IR

=> All agents find their mates under stable outcome.

NTU: Example 2b

- 2 doctors, 2 hospitals

Agent	D1	D2	H1	H2
1st	**H1**	**H1**	**D2**	**D1**
2nd	**H2**	-	**D1**	D2

- Unique Stable Matching: D1-H2, D2-H1
- An Alternative: D1-H1 (D2, H2 single) <= PE and IR

=> All agents find their mates under stable outcome.

TU: Assignment Game

- Finitely many workers and firms
- Each matched with at most one agent
  - Receive 0 utility if unmatched.
  - Each pair yields surplus by production.
- Monetary transfers allowed (UT: Transferable Utility)
  - Paris arbitrarily divide production surplus.
- No side-payment beyond each worker-firm pair

Result in TU Case

- **Theorem 3**
  The number of worker-firm pairs under the assortative stable matching is minimum among all BA outcomes that are PENS and IR.

- **Def. of assortative stable matching (ASM)**

  - Agents in both sides are linearly ordered.
    (Surplus $\mathbf{A_{ij}}$ is weakly decreasing in $\mathbf{i}$ and $\mathbf{j}$.)

  - Matching results in 1st-1st, 2nd-2nd, and so on.

Proof (Theorem 3)

1. Suppose not. Then, there must exist a PENS and individually rational outcome, say T, which has strictly fewer worker-firm pairs than ASM.

2. There are at least a worker, say W*, and a firm, F*, that would receive non-negative surplus in ASM but cannot engage in any trade, i.e., receive zero surplus, in the alternative outcome T.

3. Production surplus between W* and F* must be positive.

    1. Both W* and F* are (weakly) smaller than k. <= (2)

    2. $A_{W^*F^*}$ must be (weakly) larger than A_{kk}, a positive surplus. <= (1)

4. Contradicts to the presumption that T is PENS.

Application: Example 3

- Revisit (reformulate) Example 1 <= $A_{ij} := V_i - C_j$

	S1	S2	S3	S4
B1	7	5	3	1
B2	5	3	1	-1
B3	3	1	-1	-3
B4	1	-1	-3	-5

- Core: B1-S1, B2-S2 or B1-S2, B2-S1
- X: B1-S3, B2-S2, B3-S1   Y: B1-S4, B2-S3, B3-S2. B4-S1

Application: Example 3

- Revisit (reformulate) Example 1

	S1	S2	S3	S4
B1	7	5	3	1
B2	5	3	1	-1
B3	3	1	-1	-3
B4	1	-1	-3	-5

- Core: B1-S1, B2-S2 or B1-S2, B2-S1

- X: B1-S3, B2-S2, B3-S1   Y: B1-S4, B2-S3, B3-S2. B4-S1

TU: Example 4a

- 2 workers, 2 firms

	F1	F2
W1	10	4
W2	4	-5

- Unique Core: W1-F1 (W2, F2 single)

- Alternative: W1-F2, W2-F1 <= PE and IR

TU: Example 4a

- 2 workers, 2 firms

	F1	F2
W1	**10**	**4**
W2	**4**	-5

- Unique Core: W1-F1 (W2, F2 single)

  (5 - 5)

- Alternative: W1-F2, W2-F1 <= PE and IR

  (2 - 2) (2 - 2)

TU: Example 4b

- 2 workers, 2 firms

	F1	F2
W1	10	8
W2	4	-5

- Unique Core: W1-F2, W2-F1
- Alternative: W1-F1 (W2, F2 single) <= PE and IR

TU: Example 4b

- 2 workers, 2 firms

	F1	F2
W1	**10**	**8**
W2	**4**	-5

- Unique Core: W1-F2, W2-F1
  (7 - 1) (1 - 3)
- Alternative: W1-F1 (W2, F2 single) <= PE and IR
  (5 - 5)

Summary: Main Results

- Equilibrium allocation may be seen **most unequal**:
  - **The quantity of good traded under the competitive market equilibrium is minimum among all feasible allocations that are PENS.**
- The converse result also holds:
  - **Unless a demand or supply curve is completely flat, there always exists a feasible allocation that is PENS , IR and entailing strictly larger number of trades than that of the equilibrium quantity.**

Heterogenous Goods

- Generalization to **assignment games** (TU game in one-to-one **matching markets**).

- **Theorem 3**
  The number of buyer-seller pairs under the assortative stable matching is minimum among all BA outcomes that are PENS.

- The assortative matching assumption is often imposed in labor markets or marriage markets.

Last Remarks

- Should we aim to design/achieve "competitive" market?
  - YES: Efficiency – the greatest happiness
  - NO: Equality – of the minimum number
  - **Trade-off:** efficiency vs. equality **New!**
- Better understand why market accelerates concentration.
- Redistribution is crucial when market is competitive.

=> May better consider **equitable market design.**

Many Thanks :)

Yosuke YASUDA | Osaka University
yosuke.yasuda@gmail.com

Any comments and questions are appreciated.