

An Entry Game with Learning and Market Competition

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- With strategic interdependence, firms' entry decisions can be inefficient from the social point of view.
 - Too many or too few, e.g., excess entry theorem
 - Too early or too late
- This paper focuses (mainly) on the second issue when potential entrants interact on two different levels (learning and market competition).

- We focus on two factors: learning and market competition.
- Consider a setup where potential entrants privately gain information about the market condition over time.
- Since entry is observable, it serves as a signal (of good market condition).
- Given the market condition, the profit level is decreasing in the number of firms in the market (market competition).

- Intuition suggests:
 - The possibility of learning from the rival firm suggests gains from waiting (the second-mover advantage).
 - Market competition implies that it pays to enter earlier than the rival (the first-mover advantage).
- Each firm trades off the benefit of collecting more information against the cost of losing (potential) monopoly profits.
- This paper analyzes: with learning and market competition,
 - when is the first-mover (or second-mover) advantage likely to prevail?
 - how does market competition affect the entry timing?

- Numerous works in traditional IO literature analyze entry decisions with subsequent market competition.
- The usual framework goes as follows:
 - Each firm (simultaneously or sequentially) decides whether to enter the market.
 - The firms compete with each other (typically via Cournot or Bertrand).
- The possibility of learning is typically assumed away.

- On the other hand, few learning models consider market competition.
- Chamley and Gale (1994) consider a setup where agents may have an option to exercise while the benefit of exercising an option *increases* with the number of agents who do so.
- There is also a related class of games, called exit games, where agents decide when to exit from the game with some element of learning: Horner (2002), Bar-Isaac (2003), Murto and Valimaki (2011), Daley and Green (2012), Atkeson et al. (2015).
- Market competition is less of an issue in exit games: if you exit, the game ends.

- There are very few works, if any, which consider both learning and competition.
- A work that is most closely related is Decamps and Mariotti (2004).
 - They consider a duopoly model where each player learns the quality of a common value project and decides when to invest.
 - Project value is common but investment costs are not.
 - Each player can learn from a public signal and the experiences of the other player.
 - They briefly analyze the case where the first one to invest can earn higher payoff.

- Some differences from DM:
 - The first one to invest earn higher payoff regardless of what the follower does: the first mover's payoff is determined when he invests.
 - Background signal is public, so they hold the same belief about the project quality.
 - Above all, they only discussed this case as an extension and do not give any characterization.
- More on these points later.

- A dynamic game of market entry with two potential entrants, called firms 1 and 2.
- The firms contemplate to enter the market of unknown size, e.g., foreign market.
- The market condition, which is initially unknown to both firms, is either good or bad.
- Each firm collects information before it makes an irreversible entry decision.
- The entry cost is c and common for both firms.

- Two sources of information:
 - Each firm may observe a signal of the market condition with some probability.
 - The entry decision of each firm is publicly observable and hence serves as an additional signal.
- The fact that a firm can observe the other's entry implies a benefit of "waiting," giving rise to the second-mover advantage.

- Consider discrete time although we will focus on continuous-time limit in the end.
- We say that a firm is *active* if it has entered the market and *inactive* otherwise.
- If a firm is inactive, it may observe a signal of the market condition.
- The signal is of the bad-news type and arrives only if the market is bad.
- Conditional on the market being bad, a firm observes a signal with probability λdt for $[t, t + dt)$.
- The prior probability that the market is good is p_0 .

- The firms face a tradeoff because the profitability of each firm is decreasing in the number of firms in the market.
- When the market is good, the instantaneous profit is π^m if there is only one firm in the market and π^d if there are two.
- Define $\rho := \pi^m - \pi^d > 0$ as the monopoly premium.
- The instantaneous profit is always zero if the market is bad.
- The common discount rate is r .

Some words on our setup

- Unlike an exit game, the game does not end when a firm enters.
- With market competition, the continuation payoff of entering the market depends on the other firm's entry decision, especially the reaction to its own entry.
- Learning is private, so the firms may hold divergent beliefs.

Benchmark: single firm

- Suppose that there is only one firm (or else, the other firm is already in the market with no information spillover).
- The firm earns π^d if it is active and the market is good.
- If the firm observes a signal, it knows for sure that the market is bad, and it is clearly optimal to stay inactive indefinitely.
- We only need to consider the case where the firm has observed no signal.
- Let p_t denote the belief at t , i.e., the probability that the market is good, conditional on that the firm is inactive and has observed no signal.
- No news is good news: the belief gradually goes up as long as no signal is observed.

Benchmark: single firm

- If the firm enters now: $p_t \frac{\pi^d}{r} - c$.
- If the firm waits and enters tomorrow:
 $(1 - r\Delta) \left(p_t \frac{\pi^d}{r} - (p_t + (1 - p_t)(1 - \lambda\Delta))c \right)$.
- The benefit of waiting: gain more accurate information to avoid wrong entry.
- The cost of waiting: the foregone profit.

Proposition

In the single firm case, the firm enters the market once and for all when p_t reaches $\bar{p} := \frac{(r+\lambda)c}{\pi^d + \lambda c}$.

- The situation is much more complicated because each firm now has private information which can only be revealed through its entry decisions.
- The amount of information revealed depends on the entry strategy.
- Assume $p_0 < \bar{p}$, so that the firms are initially skeptical enough about the prospect of the market.
- Focus on a symmetric Markov PBE.

- There are three possible states:
 - The market is good (and the other firm is uninformed by assumption): p_t .
 - The market is bad, and the other firm is uninformed: q_t
 - The market is bad, and the other firm is informed: $1 - p_t - q_t$.
- The belief is denoted as (p_t, q_t) .
- All probabilities are conditional on the firm being inactive and uninformed.

Second mover

- The second mover's optimal strategy is straightforward.
- The problem is essentially the same as the single-firm benchmark: enter if and only if the belief exceeds \bar{p} .
- If a firm enters at time t , the other firm's belief jumps up to

$$\lim_{\Delta \rightarrow 0} p_{t+\Delta} = \phi_t := \frac{p_t}{p_t + q_t}.$$

- If the updated belief exceeds \bar{p} , the firm will follow immediately, so that the first mover can appropriate almost no monopoly rent.

Lemma

Conditional on having observed no signal, a firm enters immediately after observing the other firm's entry at t if and only if $\phi_t \geq \bar{p}$.

- Let $\sigma_t = \sigma(p_t, q_t)$ be a firm's entry strategy given p_t and q_t ; i.e., a firm enters with probability $\sigma_t \Delta$ for $[t, t + \Delta)$.
- The evolution of the belief depends on σ : conditional on no entry observed, the next-period belief can be written as $p_{t+\Delta} = f_p(p_t, q_t)$ and $q_{t+\Delta} = f_q(p_t, q_t)$ where

$$f_p(p_t, q_t) = \frac{(1 - \sigma_t \Delta)p_t}{(1 - \sigma_t \Delta)(p_t + q_t e^{-\lambda \Delta}) + (1 - p_t - q_t)e^{-\lambda \Delta}},$$

$$f_q(p_t, q_t) = \frac{(1 - \sigma_t \Delta)e^{-2\lambda \Delta} q_t}{(1 - \sigma_t \Delta)(p_t + q_t e^{-\lambda \Delta}) + (1 - p_t - q_t)e^{-\lambda \Delta}},$$

with the initial prior $q_0 = 1 - p_0$.

- What matters for the first mover is ϕ_t , the second mover's belief if observing the first mover's entry
- Fortunately, although p_t and q_t may follow quite complicated paths, it is easy to compute ϕ_t as it is independent of the entry strategy:

$$\phi_{t+\Delta} = \frac{p_t}{p_t + q_t e^{-2\lambda\Delta}} > \phi_t.$$

- ϕ_t is monotonically increasing in t for any given σ .
- Two stages for the first mover:
 - Waiting game: $\phi_t \geq \bar{p}$
 - Preemption game: $\phi_t < \bar{p}_t$

Waiting game

- This is a phase where if a firm enters, the other firm immediately follows, thereby quickly dissipating the monopoly rent.
- No pure-strategy equilibrium exists. The firms enter the market gradually over time.

Proposition

For $\phi_t \geq \bar{p} > p_t$, there exists a unique continuation equilibrium in which:

- 1 *Neither firm enters until p_t reaches \bar{p} ;*
- 2 *When p_t reaches \bar{p} , the two firms start entering at a rate to keep $p_t = \bar{p}$ while q_t is increasing;*
- 3 *Once a firm enters, the other firm immediately follows at the next instant.*

- The strategic nature of the problem flips once ϕ_t reaches \bar{p} .
- Let τ denote the time of the earliest possible entry:
$$\tau := \inf\{t : \sigma_t > 0\}.$$
- We say that *preemptive entry occurs* if $\sigma_t > 0$ for any $\phi_t < \bar{p}$ or equivalently $\phi_\tau < \bar{p}$.

Proposition

Suppose that preemptive entry occurs. Then, in any symmetric equilibrium, there exist $\underline{\tau}$ and $\bar{\tau} \in (\underline{\tau}, \infty)$ such that

$$\sigma_t \begin{cases} \in (0, \infty) & \text{for } t \in [\max\{\tau, \varepsilon\}, \underline{\tau}) \cup (\bar{\tau}, \infty), \\ = 0 & \text{for } t \in (\underline{\tau}, \bar{\tau}), \end{cases}$$

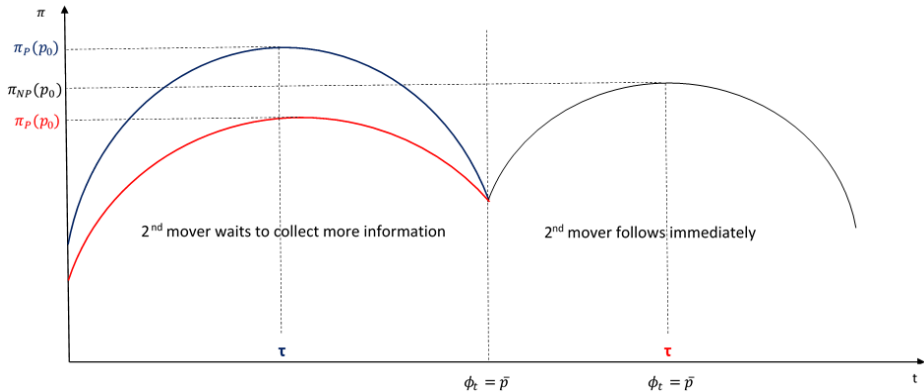
where ε is an arbitrarily small number. Moreover, $\phi_{\underline{\tau}} < \bar{p}$.

- If preemptive entry occurs, it should occur earlier than later.
- This is because entry reveals less information at earlier stages (with less information asymmetry).
- The proposition suggests that entry may occur in waves, with a period of no entry in-between.

- To analyze the possibility of preemptive entry, consider a hypothetical case where firm 2 can enter only after firm 1 does so, i.e., firm 2 is restricted to be the second mover.
- This case is easier to analyze as it excludes the possibility of entry competition.
- The problem faced by firm 1 is much simpler: it decides when to enter conditional on having observed no bad signal.
- Still, this case is instrumental in illustrating when preemptive entry occurs in equilibrium.

- If no preemptive entry, i.e., firm 1 enters when $\phi_t \geq \bar{p}$, the earliest possible entry occurs when p_t reaches \bar{p} .
- Let $\Pi_{NP}(p_0)$ denote the expected profit without preemptive entry.
- If preemptive entry occurs, i.e., firm 1 enters when $\phi_t < \bar{p}$, firm 2's belief p_t jumps up to ϕ_t , but firm 2 still needs to wait until p_t reaches \bar{p} , which allows firm 1 to monopolize the market for a duration δ_t .
- Let $\hat{\tau}$ denote the optimal timing of entry under this scenario (which is uniquely pinned down).
- Let $\Pi_P(p_0)$ denote the expected profit when firm 1 adopts this strategy.

No Entry Competition



Emergence of preemptive entry

- Under the restriction that firm 2 must be the second mover, it is optimal for firm 1 to enter once and for all at \hat{t} if $\Pi_P(p_0) > \Pi_{NP}(p_0)$.
- Clearly, this is not an equilibrium because firm 2 is also an active player who can enter at any time.
- In fact, any symmetric equilibrium must involve randomization.
- Even then we can show that the two payoffs Π_{NP} and Π_P provide a necessary and sufficient condition for preemptive entry.

Emergence of preemptive entry

Proposition

Preemptive entry occurs in equilibrium if and only if $\Pi_P(p_0) > \Pi_{NP}(p_0)$.

Proposition

There exist ρ^ and p_0^* such that preemptive entry occurs if $\rho > \rho^*$ or if $p_0 > p_0^*$.*

Timing of preemptive entry

- If firm 2 must be the second mover, then $\tau = \hat{\tau}$ if $\Pi_P > \Pi_{NP}$.
- When firm 2 is also an active player, this cannot occur in a symmetric equilibrium.
- The second-mover (first-mover) advantage prevails in the waiting (preemption) game.
- Therefore, the entry competition does not affect the timing of entry in the waiting game, but shifts the timing of entry forward in the preemption game (inducing the firms to enter before the optimal timing).

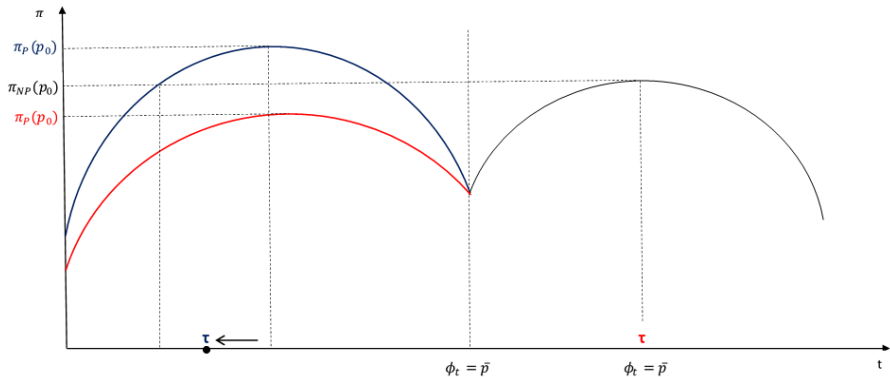
Proposition

Suppose that $\hat{\tau}(p_0) > 0$. If preemptive entry occurs in equilibrium, then $\tau < \hat{\tau}(p_0)$.

Proposition

If preemptive entry occurs in equilibrium, then each firm's equilibrium payoff falls in $(\Pi_{NP}(p_0), \Pi_P(p_0)]$.

With Entry Competition



- This paper presents a model of market entry which captures both the first-mover and second-mover advantages.
- The game is divided into two phases:
 - Preemption game: first-mover advantage prevails; firms enter before the optimal timing
 - Waiting game: second-mover advantage prevails.
- We derive a necessary and sufficient condition for when the first-mover advantage dominates and preemptive entry occurs.