Endogenous Entry To Security-Bid Auctions*

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Abstract

We endogenize entry to a security-bid auction, where participation is costly, and bidders must decide given their private valuations whether to participate. We first suppose that the minimum reserve security-bid yields the seller an expected revenue equal to the asset's stand-alone value to the seller. Demarzo et al. (2005) establish that with a fixed number of bidders, auctions with steeper securities yield the seller more revenues. Counterintuitively, we find that auctions with steeper securities also attract more entry, further enhancing the revenues from such auctions. We then establish that with optimal reserve securities, auctions with steeper securities always yield higher expected revenues.

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1 Introduction

DeMarzo, Kremer, and Skrzypacz (2005) (hereafter, DKS) characterize expected seller revenues for general classes of security bid auctions—auctions whose payouts depend on both the security that is bid by the winner of the auction, and the ultimate (stochastic) payoff of the asset won by the bidder. DKS consider a setting where ex-ante symmetric bidders receive i.i.d. signals of the private value of the asset to them if they win the auction. DKS establish that auctions using steeper securities—those whose payments to the seller are more tightly tied to the private valuation of the winning bidder—provide the seller with greater expected revenues. Thus, call options provide greater expected revenues than equity, which, in turn, provides greater expected revenues than debt.

We extend their analysis to a setting where it is costly for a potential bidder to participate in the security-bid auction. Potential bidders know their private valuations when deciding whether to enter. A potential bidder will only participate if the expected payoffs from winning given its signal cover its participation costs; and those expected payoffs will depend on the class of securities used. A natural conjecture is that since auctions that use steeper securities for payments provide the seller greater expected revenues, it must be that using steeper securities attracts fewer bidders—as more revenues for the seller would seemingly imply less for the winning bidder. Indeed, Gorbenko and Malenko (2011) show that this is what happens when bidders choose which of many ex ante identical auctions to enter before learning about their valuations of the goods being auctioned. Our paper shows that the opposite is true when bidders know their valuations before deciding whether to participate in a single auction: not only do steeper securities extract more revenues from any given set of bidders, but they also attract more bidders, and this increased entry enhances revenue extraction from bidders.

We consider a seller seeking to sell an asset in an open outcry security-bid auction
design. Potential bidders receive their signals, and must then weigh whether it is worthwhile to participate in the auction given its security-bid design. Participation demands nontrivial resources—bid preparation costs, time costs, and so on. In addition, we allow for the possibility that the expected net value of the asset with the potential bidder could be less than the stand-alone value to the seller; i.e., “synergies” could be negative.

Because potential bidders with low signals will choose not to participate—and a seller regrets an outcome in which a bidder with negative private valuations wins—the seller must specify a reserve security that a bidder pays with when it is the sole auction participant. This reserve security is the minimum security-bid accepted by the seller. As a benchmark, we first assume that the reserve security is set so that the seller’s expected revenues given a single bidder equal its stand-alone value. A natural interpretation of this reserve security is that the asset is being sold as a result of bankruptcy. The seller, perhaps the firm’s trustee, cannot reject a bid that is expected to have a higher value than the asset’s stand-alone value—and this pins down the reserve security.

Potential bidders enter the auction if and only if their private signals are sufficiently high. The marginal auction participant knows that it will win the auction only if it is the sole entrant. Thus, the expected payoffs that it retains when it pays with the reserve security just compensate for its participation costs.

Our central result is that a security-bid design that features steeper securities attracts more entry. The reasoning behind the seemingly paradoxical result that steeper securities both extract more from winning bidders and attract more entry is as follows. The seller sets the reserve security to break even (relative to its stand-alone value) conditional on the information that only one bidder participates in the auction—that is, conditional on the winner’s type being better than the marginal type who is indifferent

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1 We pose our analysis in an open outcry auction rather than a second-price auction largely to deal with semantics following a single entrant. With a single entrant, the winning bid in an open outcry auction will be the reserve security. Modulo this distinction, our analysis extends to a second-price auction in which bidders know how many others participate in the auction.
to entry. Thus, conditional on a single bidder, the seller expects the same revenues regardless of the security design.

However, precisely because payments of steeper securities are more strongly linked to a private valuation of the winning bidder, extracting more revenues from bidders with higher valuations, when the reserve security breaks even, the seller expects to extract less revenues from bidders with lower valuations. Thus, the steeper is the security design, the less the marginal auction participant expects to pay with the reserve security. Hence, the steeper is the security design, the more willing are bidders with lower valuations to participate.

One might then conjecture that because the marginal entering bidder might have a negative private valuation, this extra entry could harm the seller; i.e., steeper security designs could result in lower expected revenues. This conjecture is also false. Once entry occurs, participation costs are sunk, and a bidder has a weakly dominant strategy to drop out when the expected payoffs that it would retain just equal its stand-alone value (now reduced by the participation costs). Moreover, because each auction participant expects to make enough payoffs to cover participation costs, it must be willing to pay above the reserve security; otherwise, it would be better off not participating. Hence, with multiple entrants, all losing bidders drop out at security bids above the reserve security. Thus, the greater entry with steeper securities increases the expected revenues that the seller extracts whenever there are multiple bidders. It follows that the greater auction revenues generated by steeper securities are further enhanced when auction participation is endogenized in this way.

This logic is only reinforced when we compare auctions ordered by steepness that employ reserve securities of a fixed given expected value that exceed the seller’s stand-alone value: steeper securities always attract more low-valuation bidders, and this greater participation yields the seller higher expected revenues. Indeed, when the value of reserve securities exceeds the seller’s stand-alone value, rather than being indifferent between
auction designs conditional on attracting zero vs. one bidder, in expectation, the seller strictly prefers to attract one bidder rather than none, and steeper securities raise this probability. It follows directly that an auction using steeper securities and associated optimal reserve security always yields the seller higher expected payoffs than an auction using less steep securities and its associated optimal reserve security.

Our paper contributes to the security-bid auction literature (see a review by Skrzypacz (2013)). Hansen (1985) is the first to show that equity auctions yield higher expected revenues than standard cash auctions. DKS extend this result to a general class of security-bid auctions, establishing that the greater is the linkage between a bidder’s private information and the expected payment he would make upon winning, the higher is the expected revenue that a seller receives (Milgrom and Weber (1982)). Other papers that study security-bid auctions include Rhodes-Kropf and Viswanathan (2000), Che and Kim (2010), Kogan and Morgan (2010), Abhishek, Hajek, and Williams (2015) and Liu (2015).

Our paper is most closely related in spirit to Gorbenko and Malenko (2011). They endogenize competition in auction design of simultaneous second-price security-bid auctions between a finite number of sellers. Potential bidders make entry decisions based on the auction designs, but not knowing their private valuations of the stochastically identical objects being auctioned. The opportunity cost to a bidder of participating in one auction is not participating in another. In this setting, steeper security designs extract more from a given number of bidders, but flatter security designs draw more bidders precisely because steeper securities extract more from bidders. The equilibrium security design typically trades off between these two considerations.

We consider a seemingly similar notion of endogenous entry to a security-bid auction. However, our findings are diametrically opposed. Our setting features a single auction—rather than entry being endogenous due to the opportunity cost of participating in one auction rather than another, entry is endogenous because participation itself is costly.
Quite crucially and differently from Gorbenko and Malenko (2011), bidders know their private valuations before deciding whether to participate. We show that with this structure, when the reserve security has a fixed expected value, the steeper is the security design, the greater is entry—a seller does not need to tradeoff between extracting more from a winning bidder and attracting more entry—and the greater are expected revenues.

Fishman (1989) builds a sequential entry model in which, in equilibrium, low-valuation bidders use securities and high-valuation bidders use cash. Bidders signal a high value by bidding with cash to preempt competition. Although he too considers entry, his focus is on preemptive bids. In contrast, we focus on the relationship between rent extraction and entry, and revenue comparisons across security-bid auctions.

We next present the model. A brief conclusion follows.

2 Model

We modify the framework of DKS by introducing an entry decision by risk-neutral bidders to an open outcry security-bid auction held by a risk-neutral seller. There are \( n \) ex-ante identical potential bidders. A bidder incurs cost \( \phi > 0 \) if it participates in the auction. Each bidder has a stand-alone value of \( v_B > \phi \), which means that a bidder has enough resources to participate in the auction. The asset being auctioned has a stand-alone value to the seller of \( v \). If bidder \( i \) acquires the asset, it will yield a stochastic payoff of \( Z_i \) at date 2.

At date 0, each potential bidder \( i \) receives a private signal \( \Theta_i \) of the incremental value of the asset to the bidder. Conditional on the asset being acquired by bidder \( i \) of type \( \Theta_i = \theta \), the expected value of \( Z_i \) is normalized to

\[
E(Z_i|\Theta_i = \theta) = v_B + v + \theta - \phi,
\]

where \( Z_i \) is i.i.d. according to a density \( h(\cdot|\theta) \) with full support on \([0, \infty)\). We assume that the family \( \{h(\cdot|\theta)\} \) has the strict monotone likelihood ratio property (sMLRP):
\(h(z|\theta)/h(z|\theta')\) is increasing in \(z\) for \(\theta > \theta'\). That is, higher signals are good news.

The signals \(\Theta_i\) are distributed i.i.d. according to a distribution \(F(\cdot)\) with full support over \([\hat{\theta}, \overline{\theta}]\). We assume that this support satisfies \(\hat{\theta} < \phi < \overline{\theta}\). Thus, the net value of the asset to the bidder may or may not exceed its stand-alone value to the seller. In particular, \(\overline{\theta} > \phi\) means that it is efficient to allocate the asset to a potential bidder with a high private valuation. One can interpret an acquisition by type \(\theta > \phi\) as generating a value-enhancing synergy with an expected value of \(\theta - \phi\). Conversely, it is not efficient for potential bidders with low valuations \(\theta < \phi\) to participate in the auction—it would be better for the seller to retain the asset. Note that our formulation allows for the possibility that \(\theta < 0\). That is, not only may “synergies” fail to cover auction participation costs, but they may be negative in nature.

After receiving their private signals, potential bidders simultaneously decide at date 1 whether to participate in a security-bid auction \((S, \underline{s}(S))\) for the asset. \((S, \underline{s}(S))\) specifies a set of feasible bids \(S\) and a reserve security \(\underline{s}(S)\). Bids are made in the form of securities that are contingent on the stochastic payoff \(Z_i\), which is realized at date 2. The reserve security \(\underline{s}(S)\) is the minimum bid accepted under \(S\); this security is pinned down via a break-even condition that we describe below. We slightly modify DKS’s notion of ordered securities. Let \(S(s, z)\) denote the payment to the seller when \(Z_i = z\) is the payoff realized at date 2 for security \(s\). Bids are restricted to an ordered set of feasible securities \(S = \{S(s, \cdot) : s \in [\underline{s}(S), \bar{s}]\}\) such that (i) for all \(s\), \(S(s, z)\) and \(z - S(s, z)\) are weakly increasing in \(z\), satisfying \(0 \leq S(s, z) \leq z\), and (ii) \(\partial ES(s, \theta)/\partial s > 0\) for all \(\theta\), and \(ES(\bar{s}, \overline{\theta}) \geq v + \overline{\theta}\), where \(ES(s, \theta) \equiv E(S(s, Z_i) | \Theta_i = \theta)\) is the expected value of security \(S(s, \cdot)\) conditional on \(\Theta_i = \theta\).

At date 2, the asset payoff \(Z_i = z\) is realized and payments are made as follows: when bidder \(i\) is the sole entrant, \(i\) wins the auction if and only if it submits a feasible bid; i.e., its bid \(s_i\) weakly exceeds the reserve security \(\underline{s}(S)\), paying \(S(s_i, z)\). When two or more bidders submit feasible bids, the winning bidder \(i\) pays with the security bid \(s^2\).
of the last bidder to drop out of the auction, paying $S(s^2, z)$.

We use the notion of steepness introduced in DKS: an ordered set of securities $S_A$ is steeper than $S_B$ if for all $S_A \in S_A$, $S_B \in S_B$, $s_A \in [s(S_A), \bar{s}_A]$, and $s_B \in [s(S_B), \bar{s}_B]$, $ES_A(s_A, \theta^*) = ES_B(s_B, \theta^*)$ implies $\partial ES_A(s_A, \theta^*) / \partial \theta > \partial ES_B(s_B, \theta^*) / \partial \theta$. Steeper securities imply that if a bidder with a private valuation $\theta^*$ expects to pay the same amount with securities $s_A$ and $s_B$, then a bidder with a higher private valuation $\theta > \theta^*$ expects to pay strictly more with the steeper security $s_A$ than with $s_B$. Thus, the payment of the steeper security is tied more tightly to the winning bidder’s private valuation.

We first consider bidding decisions conditional on entry, i.e., on paying the participation cost $\phi$. The logic in Proposition 1 of DKS yields the following results.  

- If a bidder $i$ with type $\Theta_i = \theta$ is the sole entrant, it has a dominant strategy to bid $s(S)$ if $ES(s(S), \theta) \leq v + \theta$; and not to bid if $ES(s(S), \theta) > v + \theta$.

- With multiple bidders, it is a weakly dominant strategy for a bidder $i$ of type $\Theta_i = \theta$ to drop out at the bid $s^*(\theta)$ such that $ES(s^*(\theta), \theta) = v + \theta$. Further, $s^*(\cdot)$ increases in $\theta$.

- If the ordered set of securities $S_A$ is steeper than $S_B$, then conditional on the entry of the highest and second-highest types, the expected equilibrium revenue to the seller is greater under $S_A$ than under $S_B$.

Next we consider the entry decisions of bidders à la Samuelson (1985). Due to the participation costs $\phi$, not all potential bidders may enter. Since the equilibrium expected payoff upon entry is increasing in $\theta$ for a given auction $(S, s(S))$, there must be some cutoff $\theta(S)$ such that only bidders with $\theta \geq \theta(S)$ enter the auction. The marginal

\footnote{Identical characterizations obtain for second-price auctions where auction participants know how many bidders participate, and a single bidder pays the reserve.}
bidder with type $\theta(S)$ is indifferent between participating or not. Moreover, bidder $\theta(S)$ wins only if all other bidders are of type $\theta < \theta(S)$; that is, bidder $\theta(S)$ wins only if no one else enters. Therefore, the cutoff $\theta(S)$ solves:

$$[E(Z_i|\Theta_i = \theta(S)) - ES(\underline{s}(S), \theta(S))] F^{n-1}(\theta(S)) + (v_B - \phi) (1 - F^{n-1}(\theta(S))) = v_B.$$  

(1)

**Lemma 1** $s^*(\theta(S)) > \underline{s}(S)$.

**Proof.** The left-hand side of (1) is decreasing in $\underline{s}(S)$ and would become $v_B - \phi < v_B$ if we replaced $\underline{s}(S)$ with $s^*(\theta(S))$. □

This result implies that on the equilibrium path, if multiple bidders enter the auction, then the winning bidder will pay with a security bid that is at least $s^*(\theta(S))$, which is strictly higher than $\underline{s}(S)$.

The seller accepts any bid that yields higher expected revenue than its stand-alone value $v$. This premise fits well with bankruptcy auctions. When a bankruptcy trustee’s valuation of the asset is $v$, the trustee cannot reject a bid that is thought to yield more than $v$. In equilibrium, when there is a single bidder, that bidder bids $\underline{s}(S)$, and the seller only learns that the winning bidder’s type $\theta$ is at least $\theta(S)$. Thus, $\underline{s}(S)$ solves:

$$\int_{\theta(S)}^{\bar{\theta}} ES(\underline{s}(S), \theta) F(d\theta|\theta \geq \theta(S)) = v.$$  

(2)

In an open outcry auction, when there is only one entrant, the seller learns only that the sole entrant’s type is at least $\theta(S)$. These would also be the inferences that a seller would draw in a second-price auction where a bidder knows how many others are participating, because a single bidder knows it will win and pay the reserve regardless of what bid (above the reserve) he submits. In particular, these would be the inferences drawn if a single bidder always bids the reserve.

A bidder’s break-even condition (1) can be rewritten as

$$\phi = [v + \theta(S) - ES(\underline{s}(S), \theta(S))] F^{n-1}(\theta(S)).$$  

(3)
That is, the expected payoff from participation must at least compensate for participation costs. The right-hand side is increasing in $\theta(S)$, and from (2) it would become $\bar{\theta}$ by substituting $\bar{\theta}$ for $\theta(S)$. Therefore, the assumption that entry costs are not prohibitive, i.e., $\phi < \bar{\theta}$, ensures that $\theta(S) < \bar{\theta}$ holds for all $S$, i.e., a bidder enters with strictly positive probability.\footnote{Note that $\theta(S)$ may be negative—a bidder with negative synergies may enter when the surplus associated with high-valuation bidders is sufficiently large.}

We now establish that more entry occurs when bids are paid with steeper securities.

**Proposition 2** Suppose the ordered set of securities $S_A$ is steeper than $S_B$. Then auction $(S_A, \underline{s}(S_A))$ attracts more entry than auction $(S_B, \underline{s}(S_B))$: $\theta(S_A) < \theta(S_B)$.

**Proof.** Let $\underline{s}(S_A)$ and $\underline{s}(S_B)$ be the reserve securities under $S_A$ and $S_B$, respectively. By way of contradiction, first suppose $\theta(S_A) = \theta(S_B) = \tilde{\theta}$. Then,

$$\int_{\tilde{\theta}}^{\bar{\theta}} ES_A(\underline{s}(S_A), \theta) F \left( d\theta | \theta \geq \tilde{\theta} \right) = \int_{\tilde{\theta}}^{\bar{\theta}} ES_B(\underline{s}(S_B), \theta) F \left( d\theta | \theta \geq \tilde{\theta} \right) \quad (4)$$

must hold to satisfy (2). Also, using (1) yields $ES_A(\underline{s}(S_A), \tilde{\theta}) = ES_B(\underline{s}(S_B), \tilde{\theta})$, which, together with the definition of steepness, implies that

$$\int_{\tilde{\theta}}^{\bar{\theta}} ES_A(\underline{s}(S_A), \theta) F \left( d\theta | \theta \geq \tilde{\theta} \right) > \int_{\tilde{\theta}}^{\bar{\theta}} ES_B(\underline{s}(S_B), \theta) F \left( d\theta | \theta \geq \tilde{\theta} \right),$$

a contradiction to (4). Next suppose $\theta(S_A) > \theta(S_B)$. Then, it follows that

$$\int_{\theta(S_A)}^{\bar{\theta}} ES_B(\underline{s}(S_B), \theta) F \left( d\theta | \theta \geq \theta(S_A) \right) > \int_{\theta(S_B)}^{\bar{\theta}} ES_B(\underline{s}(S_B), \theta) F \left( d\theta | \theta \geq \theta(S_B) \right)$$

$$= \int_{\theta(S_A)}^{\bar{\theta}} ES_A(\underline{s}(S_A), \theta) F \left( d\theta | \theta \geq \theta(S_A) \right), \quad (5)$$

where the equality holds by (2). This, together with the definition of steepness, implies $ES_B(\underline{s}(S_B), \theta(S_A)) > ES_A(\underline{s}(S_A), \theta(S_A))$; otherwise, the left-hand side of (5) would
become smaller, a contradiction. Let \( U(\xi_j, \theta) \) denote a type \( \theta \)'s expected payoff when the reserve security is \( \xi_j \) for \( j = A, B \). Then, since \( ES_B(\xi_B, \theta_j) > ES_A(\xi_A, \theta_A) \),

\[
U(\xi_A, \theta_A) > U(\xi_B, \theta_A) > U(\xi_B, \theta_B) = U(\xi_A, \theta_A),
\]

a contradiction, where the last inequality holds by \( \theta_A > \theta_B \) and the equality holds by (1). Therefore, \( \theta_A < \theta_B \).

To establish existence and uniqueness of equilibrium outcomes, we show that there is a unique \( \xi \) and \( \theta \) that solve the system of equations, (2) and (3). Let \( \xi(\theta) \) be the reserve security associated with a cutoff type \( \theta \); i.e.,

\[
\int_{\theta}^{\overline{\theta}} ES(\xi(\theta), \theta') F(d\theta' | \theta' \geq \theta) = v,
\]

for those \( \theta \) large enough that such a security exists. Then, \( \xi(\theta) \) is decreasing and continuous in \( \theta \). Similarly, let \( \hat{\theta}(\xi(\theta)) \) be the cutoff type for reserve security \( \xi(\theta) \) who is indifferent about participation; i.e.,

\[
\phi = \left[ v + \hat{\theta}(\xi(\theta)) - ES(\xi(\theta), \hat{\theta}(\xi(\theta))) \right] F^{n-1}\left( \hat{\theta}(\xi(\theta)) \right).
\]

Then, \( \hat{\theta}(\cdot) \) is increasing and continuous in its argument. Therefore, \( \hat{\theta}(\xi(\theta)) \) is decreasing and continuous in \( \theta \). We have already established that \( \hat{\theta}(\xi(\theta)) < \overline{\theta} \), and \( \phi > \overline{\theta} \) implies that \( \hat{\theta}(\xi(\theta)) > \theta \), so that we have \( \theta < \hat{\theta}(\xi(\theta)) < \overline{\theta} \). Therefore, there exists a unique fixed point that characterizes the equilibrium. □

To see the intuition for this result, observe that regardless of the class of securities, the reserve security breaks even for the seller—the seller’s expected revenues when there is a single bidder are the same regardless of the class of securities. However, steeper securities extract a greater share of its revenues from bidders with higher valuations. It
follows that steeper securities extract less from bidders with lower valuations. In particular, the steeper is the security design, the less the marginal auction participant expects to pay with the reserve security. Hence, the steeper is the security design, the more willing bidders with lower valuations are to participate—for any realization of bidder signals, auction \((S_A, \underline{\theta}(S_A))\) attracts at least as many entrants as auction \((S_B, \underline{\theta}(S_B))\).

Note that the seller would prefer to retain the asset (in expectation) whenever the marginal auction participant is the sole bidder; and, indeed, the marginal participant’s private valuation could even be negative. One might therefore think that because a steeper security design draws more participants with low valuations, it might reduce expected seller revenues. Proposition 3 shows that this is not so:

**Proposition 3** If the ordered set of securities \(S_A\) is steeper than \(S_B\), then for any set of signal realizations, \(\{\theta_i\}_{i=1}^n\), auction \((S_A, \underline{\theta}(S_A))\) has at least as much entry as \((S_B, \underline{\theta}(S_B))\). Moreover,

- If auction \((S_A, \underline{\theta}(S_A))\) attracts multiple entrants, then it yields the seller higher expected revenue than \((S_B, \underline{\theta}(S_B))\).

- If auction \((S_A, \underline{\theta}(S_A))\) attracts zero or one entrant, then it yields the seller the same expected revenue as \((S_B, \underline{\theta}(S_B))\).

**Proof.** Let \(\theta^2\) be the second-highest type. By Proposition 2, there are three cases: \(\theta^2 \leq \theta(S_A), \theta^2 \in (\theta(S_A), \theta(S_B)]\) and \(\theta^2 > \theta(S_B)\). When \(\theta^2 \leq \theta(S_A)\), at most one bidder enters for both auctions. Then, the seller expects to receive \(v\) in both auctions. When \(\theta^2 \in (\theta(S_A), \theta(S_B)]\), at least two bidders enter for \((S_A, \underline{\theta}(S_A))\) yielding a strictly higher revenue than \(v\) by Lemma 1, while at most one bidder enters for \((S_B, \underline{\theta}(S_B))\) yielding a revenue of \(v\). When \(\theta^2 > \theta(S_B)\), at least two bidders enter both auctions. Then, it follows from DKS that the seller receives higher revenue in the auction with \((S_A, \underline{\theta}(S_A))\). \(\square\)

Thus, the greater entry to auctions with steeper securities reinforces their revenue-enhancing advantages.
More generally, the logic underlying Propositions 2 and 3 carries over for any reserve security of a fixed given value that exceeds the seller’s value as a stand-alone entity. Suppose that the reserve security $\hat{s}(S)$ solves:

$$
\int_{\theta(S)}^{\bar{\theta}} ES(\hat{s}(S), \theta) F(d\theta|\theta \geq \theta(S)) = \hat{v} \in (v, v + \bar{\theta} - \phi).
$$

(6)

We now establish that more entry occurs when bids are paid with steeper securities.

**Proposition 4** If the ordered set of securities $S_A$ is steeper than $S_B$, then auction $(S_A, \hat{s}(S_A))$ attracts more entry than $(S_B, \hat{s}(S_B))$: $\theta(S_A) < \theta(S_B)$. Moreover,

- If auction $(S_A, \hat{s}(S_A))$ either attracts multiple entrants or more entrants than $(S_B, \hat{s}(S_B))$, then it yields the seller higher expected revenue.

- If auctions $(S_A, \hat{s}(S_A))$ and $(S_B, \hat{s}(S_B))$ both attract zero entrants or both attract one entrant, then they yield the seller the same expected revenue.

We omit the proof because it mirrors those for Propositions 2 and 3. Once more, for any reserve security of a fixed value $\hat{v} \in (v, v + \bar{\theta} - \phi)$, the steeper is the security, the greater is the expected portion of that fixed value that comes from high-valuation bidders. That is, the steeper is the security, the smaller is the expected portion of that fixed value that comes from low-valuation bidders. Therefore, steeper securities attract more low-valuation bidders. In turn, this increased participation generates higher expected payoffs for the seller. Indeed, a stronger result obtains, because when the steeper security design attracts a single entrant but no entry occurs with the less steep security design, with the steeper security design, the seller now expects revenues that exceed its stand-alone value since $\hat{v} > v$. We now establish the following proposition.

**Proposition 5** When the seller faces no constraints on the reserve security that it sets for a given class of securities, with the optimal reserve security, the steeper is the security-bid design, the greater are the seller’s expected revenues.
Proof. This follows directly from the fact that for a given value of the reserve (including the optimal reserve for the less steep security design), expected revenues are higher with the steeper security design (even though the value of that reserve security need not be the optimal one for the steeper security design. □

3 Conclusion

We endogenize entry to security-bid auctions, by introducing a cost to participation. We first consider a scenario where, with a single entrant, the minimum security bid accepted is pinned down by a break-even (indifference) condition for the seller. Counterintuitively, we establish that security-bid auctions that use steeper securities for payment, which generate greater expected revenues for the seller for a fixed number of bidders, also make bidders with worse signals more willing to participate. We show that even when the marginal participant has a negative private valuation, this increased participation reinforces the revenue superiority of such auctions.

We extend this logic to any reserve security of a fixed given value that exceeds the seller’s stand-alone value: steeper securities attract more low-valuation bidders, and increased participation by low-valuation bidders yields the seller higher expected revenues. Furthermore, an auction using steeper securities that sets its optimal reserve security yields higher expected revenues than the one using less steep securities that sets its optimal reserve security.

References


