Securitization and Heterogeneous-Belief Bubbles with Collateral Constraints

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Abstract

Miller(1977) or Harrison and Kreps(1978) show asset price is higher in heterogeneous model than common prior model. They assume no budget constraint or no limitation of financial market. Recent study explore the role of financial technology in heterogeneous belief model. In this paper, I show that some financial technology make the asset price as high as Harrison and Kreps. Key technology is securitization and especially loan backed security.

1 Introduction

There are many bubble theories in economics. For example, a fiat money in overlapping generation model.(Samuelson(1958))
In rational bubble models, investors are willing to hold a bubble asset because the price of the asset is expected to rise in the future. Bubbles can be sustained today because bubbles are expected to grow in the future, at least as long as bubbles do not burst.
However, a robust implication of rational bubble theory is that the price of the asset has to glow explosively. The bubble component has to glow in expectation at rate of r(interest rate). Rational bubbles can also ruled out by using a general equilibrium zero-sum argument.(Kreps(1977),Tirole(1982))

Another class of models rely on heterogeneous beliefs among investors to generate bubbles. Combining heterogeneous beliefs with short-sale constraints can result in overpricing. Optimists push up the asset prices, while pessimists cannot couterbalance it because of short-sell constraints. (Miller(1977)) Ofek and Richardson(2003) link this argument to the internet bubbles of the late 1990s.
In dynamic models, the asset price can even exceed the valuation of the most optimistic investors in the economy. The currently optimistic investors have the option to resell the asset in the future at a high price whenever they become less optimistic.(Harrison and Kreps(1978))
The existence of optimistic traders is the source of bubbles. Asset prices are affected by optimistic beliefs. Harrison and Kreps(1978) consider a dynamic asset pricing model with the heterogeneous beliefs. Investors whose beliefs are most optimistic in the period buy or hold the asset. Because they know that the asset will be able to be sold to other traders in the future, their estimations of holding the asset today is higher than their own value. The asset price can be higher than any trader's valuation.
However, in Harrison and Kreps(1978), traders have no budget constraints. All traders have enough cash or they can borrow cash from other traders. If financial markets are not advanced, the traders who have little cash may not participate in the trade. These traders can not affect asset pricing. Optimistic traders, who are considered to be some kind of irational trader, often lose cash in a long term trade. Suppose optimistic traders have less cash than others, they can not hold assets in equilibrium. As a result, asset prices get lower than Harrison and Kreps(1978).
Geanakoplos analyses heterogeneous belief model with collateral constraints. In their model, traders need some asset as collateral when they make loan contracts with other traders. Geanakoplos(1997) and Geanakoplos and Zame(1997) introduce endogenous collateral/margin constraints into a general equlibrium framework of Arrow-Debreu. No payments in future periods/states cannot be promised without duable assets as collateral. The margin/haircuts of collateralized borrowing are derived endogenously interaction with equilibrium prices. A key implication of collateral equlibrium is that the market will be endogenously incomplete. If collateral is scarece, only a small subset of contracts will

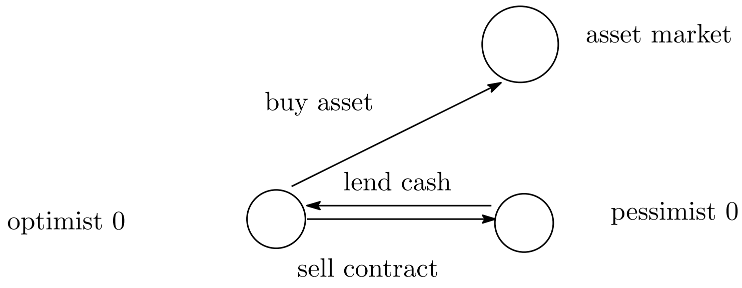
be traded in equlibrium. There are financial confriction in the market.
In Simsek(2013), he analyses the asset pricing in heterogeneous belief trader models with collateral constraints by using Genakoplos model. Optimistic traders evaluate a risky asset very high. They want to buy the asset but they have little cash. Pessimistic traders do not have incentive to buy the asset and they have a plenty of cash. Optimists will make loan contracts with pessimists. In Simsek(2013), optimists need to hold an asset as collateral like Geanakoplos model. So, optimists make the asset itself collateral and they borrow cash from pessimists.
If the asset return is high, the optimists can return cash to pessimists. But, If asset return is low, the asset will be held by pessimists because the asset is used as collateral. This collateral contracts make the asset price lower than Harrison and Kreps(1978).
Because pessimists have pessimistic beliefs about the asset return, pessimists hesitate to lend cash to optimists. As a result, the asset demand of optimists is low and the asset price is lower than Harrison and Kreps(1978).
I also assume the incomplete financial market in the model. There are heterogeneous beliefs, optimists and pessimists. Optimists have a little cash, so they must lend some cash from other investors. Traders must lend cash with collateral contract. But I asssume a securitization technology to the loan contract. Traders who hold the loan contract can make a new security, that is, loan backed security. They can receive loan payment at future date. They can sell the right to receive the payment to others.
In my paper, by introducing a new financial technology, a securitization, and a simple dynamic structure, the asset price is as high as HK(1978). Simsek model is static, one generation model. If there are many generations, many optimists and many pessimists, traders may have some speculative incentives to hold the asset. In HK(1978), traders hold the asset by strong speculative incentives and the asset price gets higher than any trader's expected return of the asset. Assume multi generation model, there must be some financial technology for participating in trading. Both in Simsek(2013) and my paper, a risky asset is only one type and asset supply is one. Optimists hold the asset and they do not have incentives to sell it. Then, other optimists have nothing to do for their profit. There is essentially inter-temporal frictions. Securitizations are financial technologies that allow many traders to participate in the market. Traders can sell the right to receive loan payments to other traders by the securitization. Pessimists who make loan contract with optimists can sell the security which have the return equal to loan payment to other optimists. New optimists also make loan contract with pessimists for buying this security. Pessimists can securitize this new loan contracts and sell them to other new optimists and so on. This security make the loan contract itself riskless. Because the lender of the loan contract can sell the security to some more optimistic one, the lender need not to hesitate to lend cash to optimistic traders. For pessimists, the security also raises their utilities. They find a new investment by securitization.
But, This security introduces bubbles to the asset. This system allows the optimistic belief to evaluate the asset. I will insist that the asset price may be

as high as Harrison and Kreps(1978). Pessimists have speculative incentives to lend cash to optimists and optimists buy the asset. These situation is repeated in each generation. Finally, optimistic traders can buy the asset with very high price.
At first, I will explain one generation model like Simsek(2013) at section 2. In equilibrium, the price formula is caluculated. The asset price lies between pessimistic expected return and optimistic expected return. This price is important for understanding multi generation model.
At section 3, the multi generation model (with single oprimist type) will be explained. In multi generation model, the asset price gets higher than Simsek(2013). At subsection 3-1, two generation model is analyzed. Two generation model is helpful to understand multi generation model. Securitizations are introduced to economy and new optimists and new pessimists come to market. The asset price is higher than one generation case. Multi generation model is dicussed at subsection 3-2. Two generation case is easily repeated and developed to general multi generation case. In this time, the asset price is equal to optimistic expected return.
At section 4, I will introduce various type optimists to multi generation model. In this model, there are many type of optimists who have different frequency about asset return state. At each generation, the security is sold to the optimist who has a highest value about it. Because this scheme allows speculative action, the asset price gets high. The asset price exceeds the optimistic expected return like Harrison and Kreps(1978).

In my paper, optimistic traders behave like "noise trader"(DeLong, Shleifer, Summers and Waldmann(1990)). Although there are many pessimistic traders, optimistic traders' influence is very large. Pessimists can get high return by exploiting optimistits by the a financiual technology.
My reserch is a part of theory that concerns the borrowing constraints on asset prices like Shleifer and Vishny(1992, 1997), Kiyotaki and Moore(1997),Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009). Financial technologies can eliminate the borrowing constraint by speculative incentive of traders. In heterogeneous belief model, borrowing constraints are important role for preventing bubble economies.
The optimist pushes up the asset price in heterogeneous belief model(Miller(1977),Harrison and Kreps(1978),Scheinkman and Xiong(2003)). Many papers by Genakoplos(1997,2003,2010), or Fostel and Genakoplos(2012) explore the role of ccredit in heterogeneous belief bubble. Simsek(2013) and this paper is also one of the study of the role of financial inovations.

2 One Generation Model:Simsek(2013)

Assume there are only one period, no trader comes at date 1 like Simsek(2013). There is a continuum of states at date 1, denoted by $s \in S = [0, s^{max}]$. The asset pays s dollars at state s. There are two type of traders, optimist and pessimist. Both type traders have risk neutral utility functions. But, they have



different beliefs about asset return. Trader type j have piror belief about state s, $F_j(s).(j = o, p)$ Note their expectation $E_o[.], E_p[.]$. Traders know each other's priors, that is, optimists and pessimists agree to disagree.
Optimists are optimistic about the asset return. Assume first order stochastic dominance. Optimist's distribution function is lower than pessimistic one for all state s.

Assumption 1 $F_p[s] \geq F_o[s]$ *for all* s

Assumption imply optimists are natural buyers of the asset.

Optimists are initially endowed with $n > 0$ dollars and zero unit of the asset. Because n is not larege, optimists might want to borrow cash from pessimists.

Assumption 2 $n < E_o[s]$

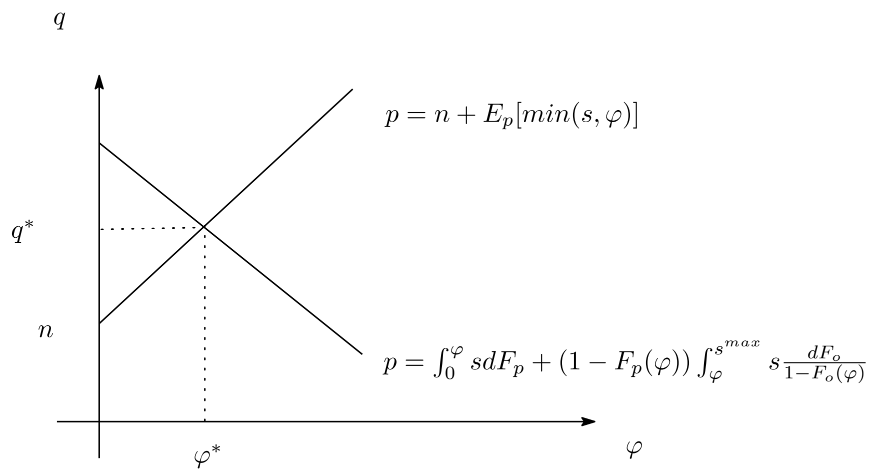
Pessimists have a plenty of cash. They are natural lenders of cash. As there are only one asset in this market, Lending contract is only way for pessimists to earn cash in the economy.
All borrowing contract in this economy is subject to a collateral constraint. That is, promise made by borrowers must be collateralized by the asset or the cash that they own. Traders choose their positions in all contracts taking the prices as given. Optimists make take-it-or-leave-it offer (φ, ϕ). Optimists promise the payment φ dollars at next period. Pessimists lend (ϕ) units of cash to optimists. In addition to the promise payment *varphi*, optimists must collateralized the asset itself. If low return state realize $(s < \varphi)$, optimists give the asset to pessimists. Then, for pessimists, this loan contract have payoff $E_p[min(s, \varphi)]$.

Let a be the optimist's demand of the asset and p the price of the asset. At date 0, the optimist buy the asset in the market and the pessimist lends cash to him. The optimist make the loan contract with collateral with the pessimist. So, optimist problem is:

$$\max_{a,\varphi} a(E_o[s] - p - E_o[min(s, \varphi)] - \phi)$$

$$s.t. ap = n + a\phi$$

Because there are many pessimists in the market and they have enough money, pessimists competition imply that the quantity of cash lending ϕ is equal to the contract's payoff.



$$\phi = E_p[min(s,\varphi)]$$

So, Optimist solve their problem:

$$\max_{a,\varphi} aE_o[s] - aE_o[min(s,\varphi)]$$

$$s.t. ap = n + aE_p[min(s,\varphi)]$$

By calculating, asset price p is caluculated.

$$p = \int_0^{\varphi} sdF_p + (1 - F_p(\varphi)) \int_{\varphi}^{s^{max}} s\frac{dF_o}{1 - F_o(\varphi)}$$

Or equivalentry,

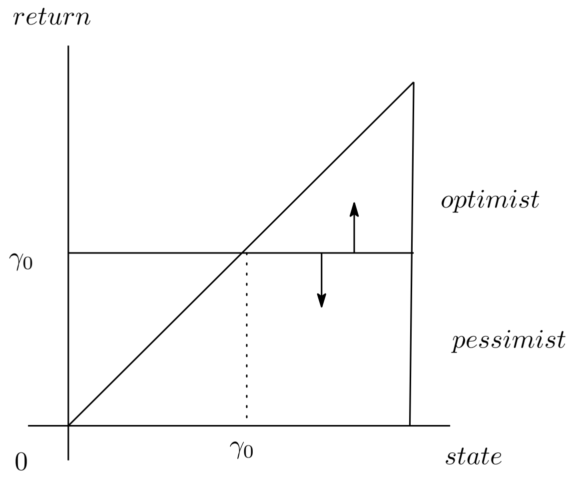
$$p = F_p(\varphi)E_p[s|s < \varphi] + (1 - F_p(\varphi))E_o[min(s,\varphi)|s \geqq \varphi]$$

Asset supplyn is one, so the budget constraint of the optimist characterizes his asset demand a.

$$p = n + E_p[min(s,\varphi)]$$

By these two equations, the equilibrium $((p,\varphi)$ are determinded.

the first equation has important implication about asset price determinance. If high return state is realized at period 1, the optimist hold the asset. If low return state is realized and the optimist's default occurs, the pessimist hold the asset. The asset return in low states is evaluated by pessimistic belief F_p and the return in high states is evaluated by optimistic belief F_o.



In Miller(1977) and Harrison and Kreps(1978), the asset prices are determinded by the optimistic beliefs. In this case, $E_o[s]$. Because the optimist faces the budget constraint, the asset price is influenced by his belief and by pessimistic belief. If the optimist has enough cash or he can borrow cash without limit, the price equal to $E_o[s]$.

For the pessimist, their payoff from the loan contract is $min(s, \varphi)$ at period 1. For assumption, first order stochastic dominance, this payoff is more valuable for optimists:

$$E_o[min(s, \varphi)] > E_p[min(s, \varphi)]$$

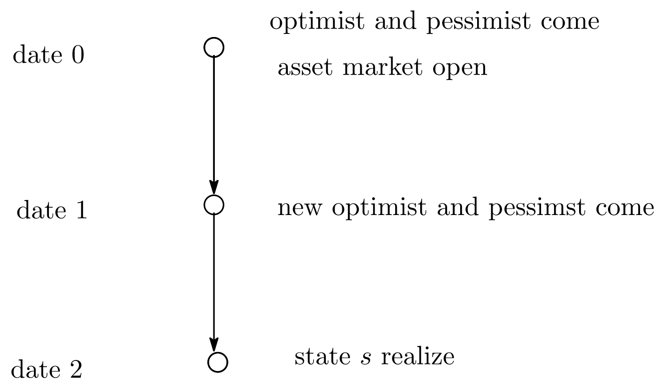
Then, pessimists have incentive to sell the right to receive the payoff at period 1 to some optimists if possible. At next section, multi generation model, i will analyze the case.

3 Multi Generation Model

In this section, i will introduce the new financial technology, "securitization" to Simsek(2013). Pessimists can sell securities, which payoff is backed by loan contracts. At first, i will explain the two generation model. In addition to Simsek's model, there is one more generation. The pessimist who lent cash to the optimist at date 0 can sell a security whose payoff is $min(s, \varphi)$ at the next date. In the two generation model, i show the asset price is raised by the securitization technology.

3.1 Two Generation Model

The asset pays s dollars at state s. $s \in S = [0, s^{max}]$. There are two type of traders, optimists and pessimists. Trader type j have piror belief about state



s, $F_j(s).(j = o, p)$ Both traders know each other's priors, that is, optimists and pessimists agree to disagree. Assume first order stochastic dominance as one generation model.
Time line is different from Simsek(2013). The asset return is realized at date 2.
At date 0, there exists one optimist and chosen pessimist. The asset is supplied to the economy.
At date 1, a new optimist comes and he choose a new pessimist.
At date 2, state s is revealed, the asset payoff is distributed a to traders as contracted.

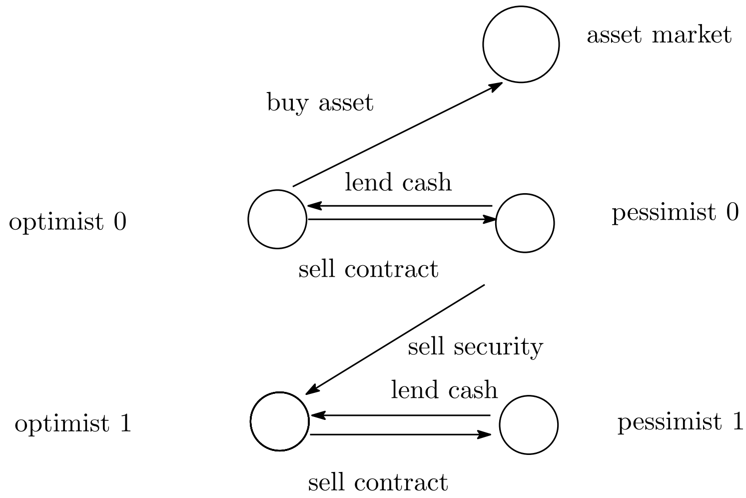
Optimists are initially endowed with $n > 0$ dollars and zero unit of the asset.

At date 0, the date 0 optimist make a loan contract with the pessimist. The date 0 optimist make take-it-or-leave-it loan contract offer. The loan contract is defined (φ_0, ϕ_0). φ_0 is the promise payment by the optimist and the pessimist lends cash ϕ_0 to the optimist.
At date 1, there is no asset supply. But One unit of a security is sold in the market.
The date 0 pessimist can securitize the right to receive payoff $min(s, \varphi)$ at date 1 with price q_1. Because $E_o[min(s, \varphi)] > E_p[min(s, \varphi)]$, the date 1 optimist has an incentive to buy this security from the date 0 pessimist. The optimist also has n units of cash.

Assumption 3 $n < E_o[min(s, \varphi_0)]$

Because $n < E_o[min(s, \varphi_0)]$, the optimist cannot buy the security by his own cash. The date 1 optimist can also borrow cash from the pessimist at date 1 with collateral constraint. Let φ_1 and ϕ_1 be the promise payment and borrowing cash in the loan contract at date 1.
At date 2, s is realized, each optimist receives the asset return and he pays the promised payment to each pessimist as contracted before.

It is natural that $\varphi_0 > \varphi_1$ in equilibrium.
The date 1 optimist borrow cash for buying the security whose payoff $min(s, \varphi_0)$.



This relation imply the asset return distribution in equilibrium.
If a high return state is realized, the date 0 optimist and the date 1 optimist can pay back the cash to pessimists.
If a medium return state is realized, the date 1 optimist can return cash but the date 0 optimist cannot.
If a low return state realize, both optimists cannot.
The security price q_1 is determinded by the security payment $min(s, \varphi_0)$ and the contract $min(s, \varphi_1)$ at date 1. The date 0 optimist chooses their asset position and outstanding debt to solve:

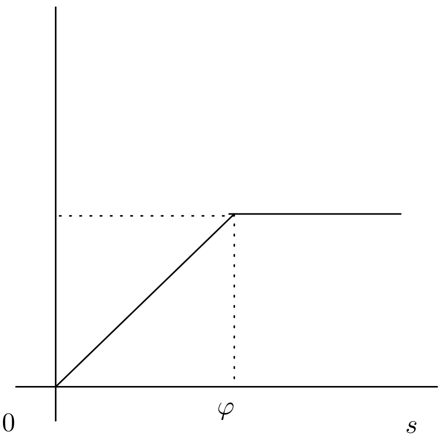
$$\max_{a_0, \varphi_0} a_0(E_o[s] - p - E_p[min(s, \varphi_0)] + \phi_0)$$
$$s.t. a_0 p = n + a_0 \phi_0$$

The date 0 pessimist lend cash to the date 0 optimist. The optimist pay $min(s, \varphi_0)$ to the pessimist if state s is realized.

The date 1 optimist's problem can be denoted like the date 0 optimist. The date 1 optimist's problem is:

$$\max_{a_1, \varphi_1} a_1(E_o[min(s, \varphi_0)] - q_1 - E_o[min(s, \varphi_1)] + \phi_1)$$
$$s.t. a_1 q_1 = n + a_1 \phi_1$$

The problem is similar to Simsek's model.
In one generation model, the asset return is s. In this time, security payoff is $min(s, \varphi_0)$. By substituting $min(s, \varphi_0)$ to s and q_1 to p, this is just the optimist's problem in Simsek(2013).



Because there are many pessimists in the market and they have enough money at date 1, pessimists competition imply that the quantity of cash lending ϕ_1 is equal to the contract's payoff.

$$\phi_1 = E_p[min(s, \varphi_1)]$$

At date 0, there are many pessimists. The pessimist who lend cash to the date 0 optimist can sell the security at date 1 with price q_1. Pessimists' competition imply no arbitrage condition.

$$\phi_0 = q_1(\varphi_0)$$

Then, the date 0 optimist problem is rewritten.

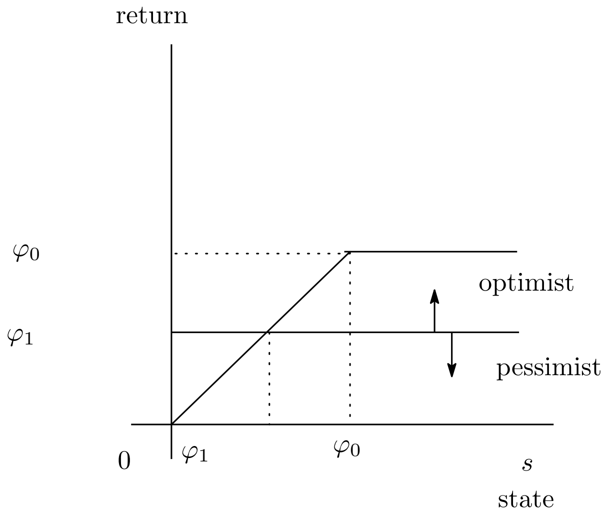
$$\max_{a_0, \varphi_0} a(E_o[s] - p - E_o[min(s, \varphi_0)] + q_1)$$
$$s.t. a_0 p = n + a_0 q_1(\varphi_0)$$

The date 1 optimist problem is rewritten:

$$\max_{a_1, \varphi_1} a_1(E_o[min(s, \varphi_0)] - q_1 - E_o[min(s, \varphi_1)] + E_p[min(s, \varphi_1)])$$
$$s.t. a_1 q_1 = n + a_1 E_p[min(s, \varphi_1)]$$

By solving the each date optimist problem, equilibrium $(\varphi_0, \varphi_1, q_1, p)$ are caluculated. From the date 1 optimist problem, given φ_0, the security price $q_1(\varphi_1)$ is:

$$q_1 = \int_0^{\varphi_1} s dF_p + (1 - F_p(\varphi_1)) \int_{\varphi_1}^{s^{max}} min(s, \varphi) \frac{dF_o}{1 - F_o(\varphi_1)}$$



Or equivalentry,

$$q_1 = F_p(\varphi_1)E_p[s|s < \varphi_1] + (1 - F_p(\varphi_1))E_o[min(s, \varphi_0)|s \geqq \varphi_1]$$

This formula shows that optimism is asymmetrically disciplined in equilibrium like Simsek(2013). Pessimistic beliefs assess the value of the asset conditional on default, while optimistic beliefs are used to assess the value of the asset conditonal on no default. By considering asset payoff $min(s, \varphi_1)$ instead of s, this is the same expression in Simsek(2013).
If asset return exceed φ_1, the date 1 optimist can repay the loan payment to the date 1 pessimsit. But if not, the date 1 optimist must give the security to the date 1 pessimist. So, this formula imply that low return state is evaluated by pessimist and high return state is evaluated by optimist just like asset price of one generation model.

The security supply is one, so the budget constraint of problem characterizes optimists' asset demand:

$$q_1 = n + E_p[min(s, \varphi_1)]$$

These two equations determine the equilibrium q_1 and φ_1.
By backward induction, the date 0 optimist problem can be solved.

$$p = q_1 + \frac{1 - F_p(\varphi_0)}{1 - F_o(\varphi_0)} \int_{\varphi_0}^{s^{max}} sdF_o$$

Because asset supply is one, supply function is written:

$$p = n + q_1(\varphi_0)$$

These two equation determine the equilibrium p and φ_0.

Let p^S be the asset price of one generation model. Next proposition show the asset price at two generation model gets higher than Simsek's price.

proposition 1 *In two generations case, the asset price exceeds the asset price of one generation case: $p > p^S$*

Proof 3.1 *From the date 0 optimist problem, the asset price at two generation model:*

$$p = q_1 + \frac{1 - F_p(\varphi_0)}{1 - F_o(\varphi_0)} \int_{\varphi_0}^{s^{max}} sdF_o$$

From the date 1 optimist problem, the security price q_1:

$$q_1 = \int_0^{\varphi_1} sdF_p + (1 - F_p(\varphi_1)) \int_{\varphi_1}^{s^{max}} min(s, \varphi_0) \frac{dF_o}{1 - F_o(\varphi_1)}$$

These two equations imply that the asset price p is rewritten:

$$p = \int_0^{\varphi_1} sdF_p + \frac{1 - F_p(\varphi_1)}{1 - F_o(\varphi_1)} \int_{\varphi_1}^{s^{max}} sdF_o$$

If $\varphi_1 < \varphi$ (φ is the loan promise from Simsek(2013)), the asset price is higher than Simsek(2013).

From the date 1 optimist's budget constraint:

$$q_1 = n + E_p[min(s, \varphi_1)]$$

From the date 0 optimist's budget constraint:

$$p = n + q_1$$

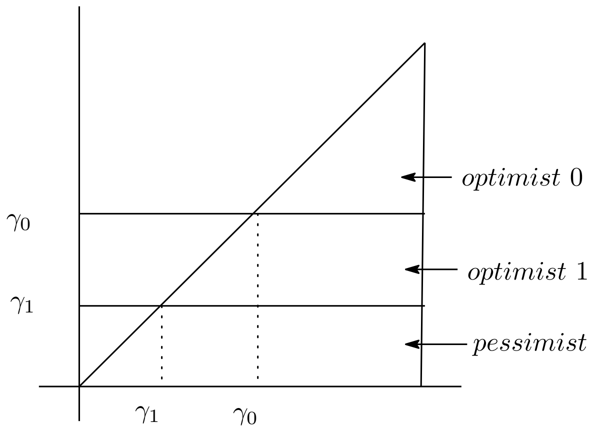
These two equations imply the asset price p:

$$p = 2n + E_p[min(s, \varphi_1)]$$

Assume $\varphi_1 \geq \varphi$. Two budget constraints imply:

$$p = 2n + E_p[min(s, \varphi_1)] \geq 2n + E_p[min(s, \varphi)] \geq n + E_p[min(s, \varphi)] = p^S$$

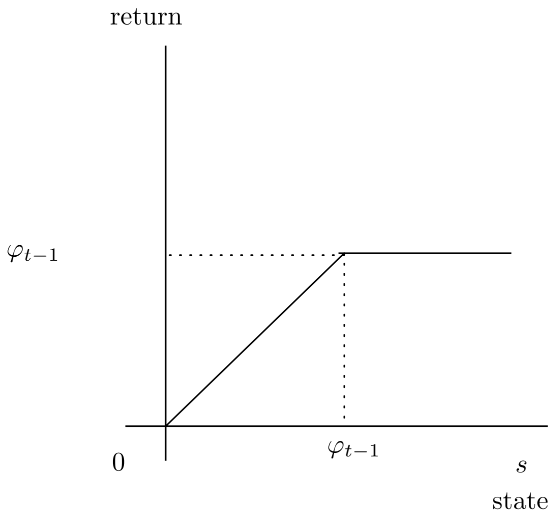
This is contradicted by the price formulation. Then, φ_1 is smaller than φ and $p > p^S$.



The proposition show the asset price at date 0 is larger than Simsek(2013). For the date 0 pessimist, lending cash to optimist is very profitable investment. By making loan contract, the pessimist can sell security at date 1. That is, pessimist has speculative incentive for loan contract.
In one generation model, high asset return states($s > \varphi$) are evaluated by optimistic belief and low return states ($s < \varphi$)are evaluated by pessimistic belief.
In two generation model, $\varphi_1 < \varphi$. High states upper than φ_1 are evaluated by optimists.
Optimistic belief dominates wider area of asset return than Simsek(2013).

3.2 Multi Generation Model

If security market is large and the security technology evolve, more investor participate this scheme. The scheme can be extended by a simple way. In this section, i will introduce multi generation model.
This is multi date or multi generation model. The asset state is realized at date T (T is some large natural number). At date 0, the date 0 optimist buys the asset and he borrows cash from the pessimist. The date 0 pessimist securitize the loan and make a new security that pay $min(s, \varphi_0)$. the date 1 optimst buy the security and he borrow cash from a new pessimist at period 1. The date 1 pessimist also securitizes the loan contract and he sells the new security to new optimist at period 2.
From date (t=1,2,3,..), the date t optimist borrows cash from the date t pessimist. He buys the security that is securitized by the date t-1 pessimist with price q_t. The date t pessimist also securitizes the loan contract that is made with the date t optimist and he sells it to the date t+1 optimist with price q_{t+1}.
The date t loan contract is defined by (φ_t, ϕ_t). The date t optimist promises the payment φ_t at date T and the date t pessimist lends cash ϕ_t to him.
Because the optimist collateralizes the security whose payoff is $min(s, \varphi_{t-1})$, the contract's payment is $min(s, \varphi_t)$.



At each date $T-1 > t \geq 0$, there are many pessimists and they compete with each other. The date t pessimist can sell the loan contract to the date t+1 optimist with price q_{t+1}. Then, no arbitrage condition determine the level of lending ϕ_t.

$$\phi_t = q_{t+1}(\varphi_t)$$

At date $T-1$, the pessimist cannot sell the loan contract at next date T. So, his no arbitrage condition:

$$\phi_{T-1} = E_p[min(s, \varphi_{T-1})]$$

The each date optimist problem is written like two generation model.
The date t optimist problem is written(t ≥ 1) :

$$\max_{a_t, \varphi_t} a_t[E_o[min(s, \varphi_{t-1})] - q_t - a_t E_o[min(s, \varphi_t)] + \phi]$$
$$s.t. a_t q_t = n + a_t \phi_t$$

The date 0 optimist problem:

$$\max_{a_0, \varphi_0} a_0[E_o[s] - p - E_o[min(s, \varphi_0)] + \phi_0]$$
$$s.t. a_0 p(\varphi_0) = n + a_0 \phi_0$$

By solving optimists problem, pessimists no-arbitrage condition, the equlibrium price is determined.

Definition 3.2 *Given n, F_o, F_p, equilbrium prices $(p^*, \{q_t^*\}_{t=1,2,..T-1})$ and $\{(\gamma_t^*, a_t^*, \phi_t^*)\}_{t=0,1,..T-1}$ satisfy the following conditions.*

- *Given* (p^*, q_t^*, ϕ_t^*)*, the optimist at each period solve the problem by* (a_t^*, γ_t^*)*.*

- *Given* (p^*, q_t^*, ϕ_t^*)*, the cometition among pessimists imply* $\phi_t^* = q_{t+1}^*(\gamma_t)$*.*

- *Market clearing condition at each date* $a_t^* = 1$ *for all* t*.*

Next proposition show the security price is evaluated by the optimistic belief and the next date security price.

Proposition 3.3 *In equlibrium, the security price at date* t*:*

$$q_t = q_{t+1} + \frac{1 - F_p(\varphi_{T-1})}{1 - F_o(\varphi_{T-1})} \int_{\varphi_t}^{s^{max}} min(s, \varphi_t) dF_o$$

The security price at date $T-1$*:*

$$q_{T-1} = \int_0^{\varphi_{T-1}} sdF_p + \frac{1 - F_p(\varphi_{T-1})}{1 - F_o(\varphi_{T-1})} \int_{\varphi_{T-1}}^{s^{max}} min(s, \varphi_{T-2}) dF_o \qquad (1)$$

The equlibrium date t *promise payment* φ_t*, the asset price* p *and the security price* q_t *are caluculated by the date* t *optimist problem, the date* t *pessimist competition and the market clearing condition* ($a_t = 1$)*.*

Proof 3.4 *From the date T-1 pessimist competition:*

$$\phi_{T-1} = E_p[min(s, \varphi_{T-1})]$$

The date T-1 optimist problem is rewritten:

$$\max_{a_{T-1}, \varphi_{T-1}} a_{T-1}[E_o[min(s, \varphi_{T-2})] - q_{T-1} - E_o[min(s, \varphi_{T-1})] + E_p[min(s, \varphi_{T-1})]]$$

$$s.t. a_{T-1} q_{T-1} = n + a_{T-1} E_p[min(s, \varphi_{T-1})]$$

By solving the problem, the date T-1 security price is caluculated:

$$q_{T-1} = \int_0^{\varphi_{T-1}} sdF_p + \frac{1 - F_p(\varphi_{T-1})}{1 - F_o(\varphi_{T-1})} \int_{\varphi_{T-1}}^{s^{max}} min(s, \varphi_{T-2}) dF_o$$

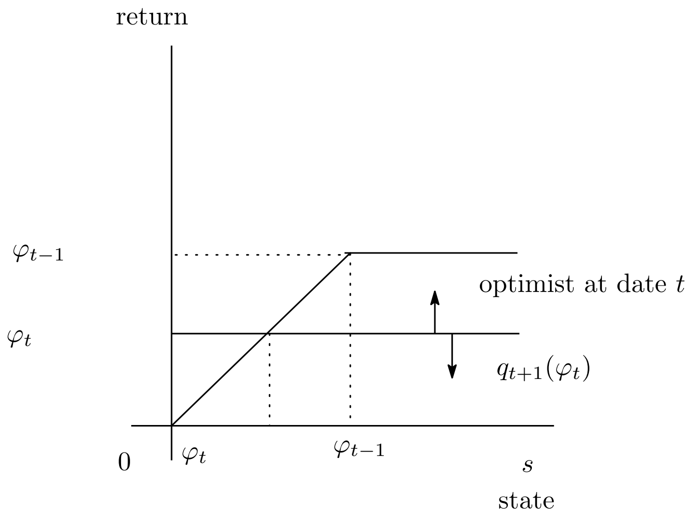
From the date t pessimist competition:

$$\phi_t = q_{t+1}(\varphi_t)$$

The date t optimist problem is rewritten:

$$\max_{a_t, \varphi_t} a_t[E_o[min(s, \varphi_{t-1})] - q_t - E_o[min(s, \varphi_t)] + q_{t+1}]$$

$$s.t. a_t q_t = n + a_t q_{t+1}(\varphi_t)$$



By solving the problem, date t price is calculated:

$$q_t = q_{t+1} + \frac{1-F_p(\varphi_{T-1})}{1-F_o(\varphi_{T-1})} \int_{\varphi_t}^{s^{max}} min(s,\varphi_t)dF_o$$

The security price equation imply that the security return area is divided into two partitions. The return area above φ_t is evaluated by the date t optimist and the area under φ_t is evaluated by the next date security price q_{t+1}.

The asset price is caluculated by solving the date 0 optimist problem. From the date 0 pessimists competition:

quation $\phi_0 = q_1(\varphi_0)$quation

The date 0 optimist problem is rewritten:

$$\max_{a_0,\varphi_0} a_0[E_o[min(s,\varphi_0)] - p - E_o[min(s,\varphi_0)] - q_1]$$
$$s.t. a_0 p = n + a_0 q_1(\varphi_0)$$

This problem and market clearing condition at date 0 ($a_0 = 1$) determine the equlibrium asset price p.

In equlibrium, the asset price p:

$$p = \int_0^{\varphi_{T-1}} sdF_p + \frac{1-F_p(\varphi_{T-1})}{1-F_o(\varphi_{T-1})} \int_{\varphi_{T-1}}^{s^{max}} sdF_o$$

Proof 3.5 *From the date 0 optimist problem:*

$$p = q_1 + \frac{1-F_p(\varphi_0)}{1-F_o(\varphi_0)} \int_{\varphi_0}^{s^{max}} sdF_o$$

From the security price at date 1:

$$q_1 = q_2 + \frac{1-F_p(\varphi_1)}{1-F_o(\varphi_1)} \int_{\varphi_1}^{s^{max}} min(s, \varphi_0) dF_o$$

By repeating the recursive calculation:

$$q_1 = \int_0^{\varphi_{T-1}} s dF_p + \frac{1-F_p(\varphi_{T-1})}{1-F_o(\varphi_{T-1})} \int_{\varphi_{T-1}}^{s^{max}} min(s, \varphi_0) dF_o$$

By substituting the equation to q_1, the asset price p:

$$p = \int_0^{\varphi_{T-1}} s dF_p + \frac{1-F_p(\varphi_{T-1})}{1-F_o(\varphi_{T-1})} \int_{\varphi_{T-1}}^{s^{max}} s dF_o$$

The asset price p depends on the promise payment at date $T-1$, φ_{T-1}. The asset return areas upper than φ_{T-1} are evaluated by optimistic belief. If φ_{T-1} is low, the asset price is high.
The security payoff at date t is $min(s, \varphi_{t-1})$. The date t optimist has cash n. If $E_o[min(s, \varphi_{t-1})] \leq n$, the optimist can buy the security without the loan contract. Assume there is the date t when $E_o[min(s, \varphi_{t-1})] \leq n$ in equilibrium. Let t' be the first date when n exceeds $E_o[min(s, \varphi_{t'-1})]$. After $t'+1$, there is no security supply. The optimists need not to make loan contract with the pessimists. At date t'. the sequlity price is simply equal to the optimistic expected return:

$$q_{t'} = E_o[min(s, \varphi_{t'-1})]$$

Because the optimist does not need to make the loan contract, the promise payment $\varphi_{t'} = 0$. After date t', there is no security supply in the market. Then, the security price $t > t'$ is equal to zero. The date t ($t > t'$) optimist does not make the loan contract. The promise payment at date $t \geq t'$:

$$\varphi_t = 0$$

If the date t' exists, the maximum asset price is determinded. If T is large, the assset price is as high as the asset price of Harrison and Kreps(1978).

Proposition 3.6 *If T is large enough($T-1 \geq t'$), the equlibrium asset price is equal to Harrison and Kreps(1978):*

$$p = E_o[s]$$

Proof 3.7 *At date t', the optimist buy the security by his cash. The security price is equal to the optimistic expectation.*

$$q_{t'} = E_o[min(s, \varphi_{t'-1})]$$

The date $t'-1$ optimist problem:

$$\max_{a_{t'-1},\varphi_{t'-1}} a_{t'-1}[E_o[min(s,\varphi_{t'-2})] - q_{t'-1} - E_o[min(s,\varphi_{t'-1})] + E_o[min(s,\varphi_{t'-1})]$$
$$s.t. a_{t'-1}q_{t'-1}(\varphi_{t'-2}) = n + a_t E_o[min(s,\varphi_{t'-1})]$$

By solving the problem:

$$\begin{aligned} q_{t'-1} &= \int_0^{\varphi_{t'-1}} sdF_o + (1-F_o(\varphi_{t'-1})) \int_{\varphi_{t'-1}}^{s^{max}} min(s,\varphi_{t'-2}) \frac{dF_o}{1-F_o(\varphi_{t'-1})} \\ &= E_o[min(s,\varphi_{t'-1})] \end{aligned}$$

By repeating backward induction, the security price at date $t' > t > 0$:

$$q_t = E_o[min(s,\varphi_{t-1})]$$

The date 0 optimist problem:

$$\max_{a_0,\varphi_0} a_0[E_o[s] - p - E_o[min(s,\varphi_0)] + E_o[min(s,\varphi_0)]]$$
$$s.t. a_0 p(\varphi_0) = n + a_t E_o[min(s,\varphi_0)]$$

By solving the problem, the asset price p:

$$\begin{aligned} p &= E_o[min(s,\varphi_0)] + \frac{1-F_o(\varphi_0)}{1-F_o(\varphi_0)} \int_{\varphi_0}^{s^{max}} sdF_o \\ &= E_o[s] \end{aligned}$$

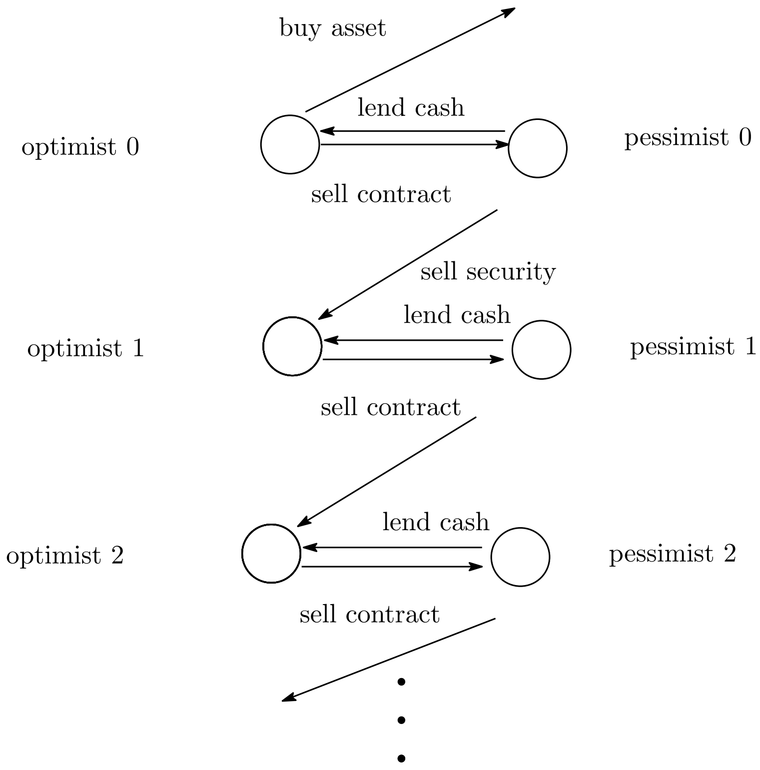
From theorem 1, the asset price p:

$$p = \int_0^{\varphi_{T-1}} sdF_p + \frac{1-F_p(\varphi_{T-1})}{1-F_o(\varphi_{T-1})} \int_{\varphi_{T-1}}^{s^{max}} sdF_o$$

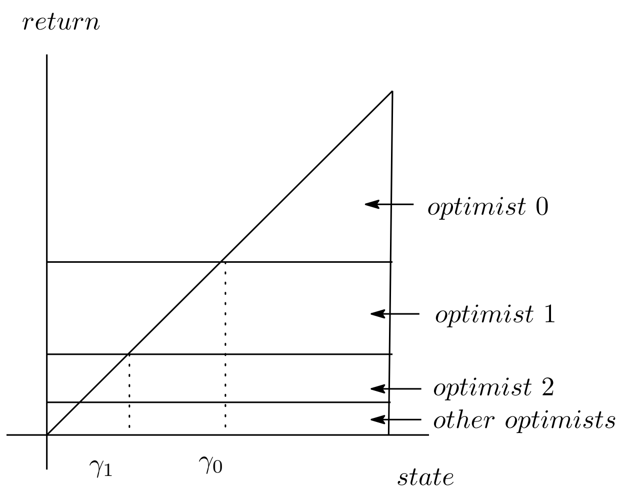
By substituting $\varphi_{T-1} = 0$ to this equation, the asset price is equal to $E_o[s]$. The asset price is valued by the optimistic belief.

At time T, the state s is realized. $s^{max} > \varphi_1 > \varphi_2 > .... > \varphi_{t'-1} > s^{min}$. Let t^* satisfying $\varphi_{t^*-1} > s > \varphi^{t^*}$. The realized payoff for the date t optimist is:

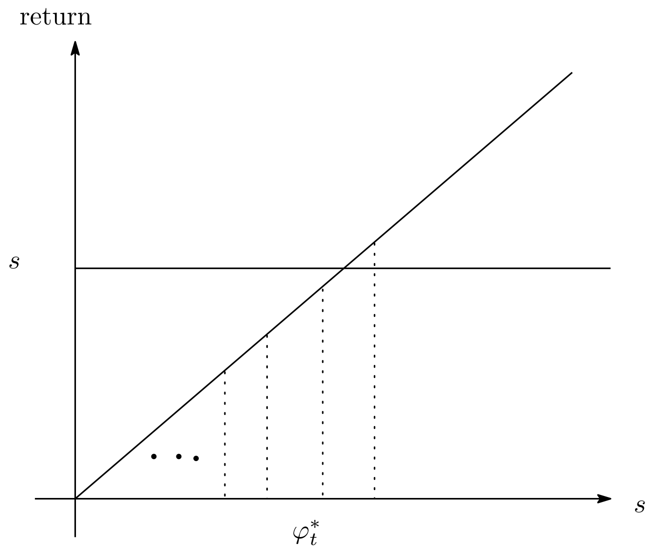
1. $t < t^*$: 0
2. $t = t^*$: $s - \varphi_{t^*}$
3. $t' \geq t > t^*$: $\varphi_t^* - \varphi_{t^*+1}$
4. $t > t'$: 0



buy asset
lend cash
optimist 0
pessimist 0
sell contract
sell security
lend cash
optimist 1
pessimist 1
sell contract
lend cash
optimist 2
pessimist 2
sell contract



return
optimist 0
optimist 1
optimist 2
other optimists
γ_1
γ_0
state



The asset is evaluated by the each date optimist. As noted in two generation case, the area which are evaluated by optimist gets wider in multi generation model.
The asset price is the same as Harrison and Kreps(1978). In Harrison and Kreps(1978), the asset price may be higher than any trader's expectation. Because the asset is hold by the most optimistic investor in each period, the asset price is higher than anyone's expectation. In this paper, the asset price is shared among optimistic traders. Harrison and Kreps assume complete market. So, any trader have enough cash or rent cash in each period.

They can hold asset when their expectaion is highest in the market.
In Simsek(2013), the optimist does not have enough cash, he must borrow cash from pessimist. Because loan contract is limited, he must collateralize the asset itself. So the asset price must be influenced by pessimistic belief.
In my paper, the optimists are distributed among generations. They have incentive to buy the asset, but they does not have enough cash. Without the security market, they have no way to cooperate with each other. Security and loan contract play role for helping their cooperation.
The loan contract causes the security and it causes the loan contract. The scheme allows the each optimist to participate in the market.

Pessimistic belief vanishes in the asset price equation. However, if there is no pessimist, the date 0 optimist have no way to bring enough cash to buy the asset. There are many pessimists in the security market.
The pessimists know that the new optimists will come to the market at the next date. So, they have strong incentive to lend their cash to the optimists. Lending cash is speculative action for the pessimist. The pessimist can sell the risk of the optimist's default to the other optimist by securitization. It is one

of risk-shifting problem.(Shleifer and Vishny(1992))
As a result, the asset return is shared by the optimists and the asset price rises. This security raise the utility for both optimists and pessimists. The scurity technology compensate for incomplete financial market.
The asset price is influenced by optimistic belief. High asset price imply much optimist participate in the markets. From budget constraint of the date 0 optimist:

$$ap = n + aq_1$$

Asset supply is one $a = 1$:

$$p = n + aq_1$$

From the budget constraint of optimist at date 1 and security supply is one:

$$q_1 = n + q_2$$

q_2 is security price at date 2. Then, budget constraint at date 0 imply:

$$p = n + n + q_2$$

By repeating caluculation, budget constraint imply asset price is sum of optimist cash n.

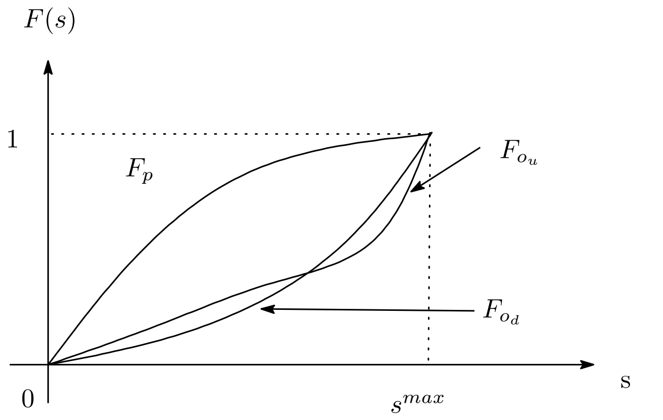
p = n + n + n + ... + n + $E_o[min(s, \varphi_{t'})] = t'n + E_o[min(s, \varphi_{t'})]$

In equilibrium, asset price is $E_o[s]$. Then, the number of optimists who buy asset or securities (that is t') is:

$$t' = \frac{E_o[s] - E_o[min(s, \varphi_{t'})]}{n}$$

t' optimists buy each area of asset return in equilibrium. If there is no security market, only one optimist and one pessimist participate in the market. Many optimists can participate in economy by these securitization market.

In general, if there are heterogeneous beliefs exists, completeness of security market make economy riskier. The advanced security market allow heterogeneous investor to act freely, so their action cause variety effect. Optimist can act more optimistically, and pessimist can also act more pessimistically.
In the next section, I will show that asset price exceeds any traders' expectation in various optimist setting. The intution is very simple. In multi generation case, the each date optimist buys the security and he evaluates each area of the asset return. If various optimists buy the each security, asset return is evaluated by many optimistic belief. Because the each date security is bought by a trader who has the most optimistic belief about the security return, the security price is higher than single type optimist case.
In this section, asset price is $E_o[s]$. This is one case of Harrison and Kreps(1978), one type optimist case. In the next section, asset price exceed optimist's expectation:



$$p \geq E_o[s]$$

Like general case of Harrion and Kreps(1978), this is the bubble caused by heterogeneous belief.

4 Multi Generation with Various Optimists

In this section, I will show that the asset prices exceed any traders' expectstions in various optimistitic types case. Model settings are almost same as multi generations case.
For simplicity, I will give two type optimists case. It is helpful to understand multi generation with various optimists model.

4.1 Example:Two Type Optimist Case

In addition to multi generation case, there are two type optimists in this model, upside optimist and downside optimist. Optimists type j have optimistic belief $F_j(s).(j = O_u, O_d)$(Note expectation E_{O_u},E_{O_d}) Optimists have same expectation about the return of the asset ($E_{O_u}[s] = E_{O_d}[s] = E_o[s]$). Upside optimists thinks bad event is unlikely and downside optimists think good event is unlikely. (see figure.)

Both optimistic beliefs first order stochastic dominates pessimistic belief.

Assumption 4 *$F_p[s] \geq F_j[s]$ for all s and $j = O_u, O_d$*

The asset state is realized at date T (T is some large natural number). At date 0, there exists two optimists and one pesimist. The pessimist is chosen by optimist from many pessimists. There are three type of traders at each date.. The upside optimist, the downside optimist and the pessimist. Both optimists are initially endowed with $n > 0$ dollars and zero unit of the asset.

All borrowing in this economy is subject to a collateral constraint. The loan contract made by optimists must be collateralized by the asset or the security. Because the asset or the security supply is one, only one optimist contracts with the pessimist and he buys the asset.
This scheme ends at the date when some optimist buys the security by his own cash. At period t, an optimist type j borrow cash and he buys the security. The pessimist who lend cash to optimist at period t can securitize the loan contract and sell the security to optimist at date $t+1$ with price $q_{t+1}(\varphi_t)$. The pessimists competition imply:

$$\phi_t = q_{t+1}\phi_{T-1} = E_p[min(s, \varphi_{T-1})]$$

Then, the optimist problem who buys the security at date t:

$$\max_{a_t, \varphi_t} a_t[E_{O_j}[min(\varphi_{t-1}, \varphi_t)] - q_t - E_{O_j}[min(s, \varphi_t)] + q_{t+1}(\varphi_t)$$
$$s.t. a_t q_t(\varphi_{t-1}, \varphi_t) = n + a_t q_{t+1}(\varphi_t)$$

The security supply is one. By substituting $a_t = 1$ to budget constraint:

$$q_t = n + q_{t+1}(\varphi_t)$$

The optimist problem and this equation solve the sequirty price.

Because asset prices are very heavily influenced by the belief of the asset holder, who buys the asset or the securities at each date is very important problem. As seen the multi generation case, at some period t', the optimist buys the security by his cash. This security's payoff is $min(s, \varphi_{t'})$.
From the optimist type definition, $E_{O_d}[min(s, \varphi_{t'})] > E_{O_u}[min(s, \varphi_{t'})]$.
The downside optimist have higher evaluation about $min(s, \varphi_{t'})$. Then, the date t' security must be bought by the downside optimist. For the same reason, we can guess that the asset is bought by the upside optimist at date 0.
The upside optimist thinks upper return state is likely and downside optimist thinks opposite.
Then some period $\bar{t}(0 < \bar{t} < t')$ exists. For $t < \bar{t}$, the asset and securities are bought by upside optimists. For $t \geq \bar{t}$, the securities are bought by downside optimists.

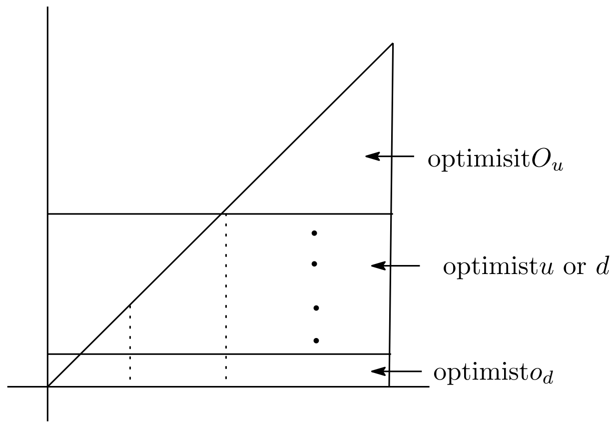
By backward induction, the original asset price is calculated. In this time, the asset price is higher than the multi-generation case with single optimist type.

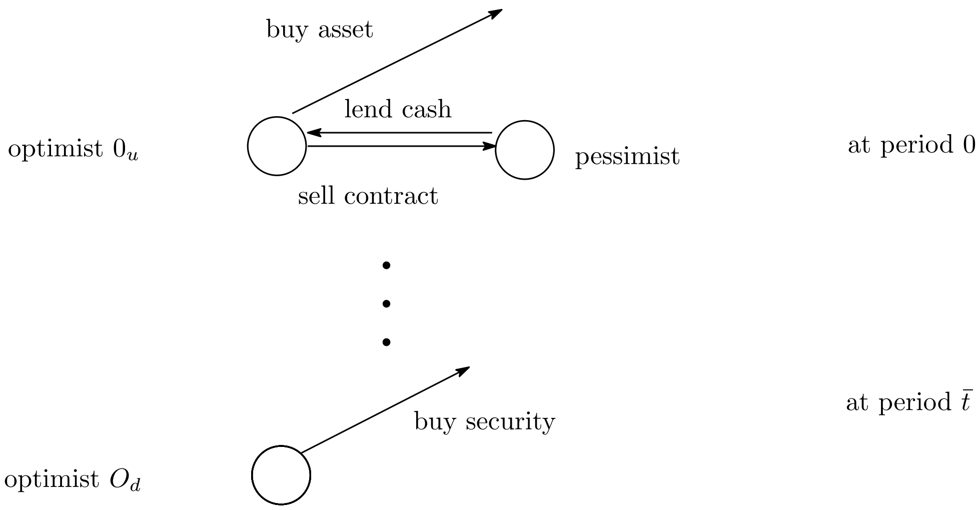
proposition 2 *In multi-generation with two type optimist model, the asset price exceeds any optimistic trader's expecatation of asset return:*

$$p > E_o[s]$$

Proof 4.1 *As noted above, the security price at date t' is $E_{o_d}[min(s, \varphi_{t'-1})]$.*

Assume optimist type j buy the security at date $t'-1$. Optimist problem at date $t'-1$ is:


optimisitO_u
optimist$u$ or d
optimisto_d


buy asset
lend cash
optimist 0_u
pessimist
at period 0
sell contract
at period $\bar{t}$
buy security
optimist O_d

$$\max_{a_{t'-1},\varphi_{t'-1}} a_{t'-1}[E_{O_j}[min(s,\varphi_{t'-2})] - q_{t'-1} - E_{O_j}[min(s,\varphi_{t'-1})] + E_{O_d}[min(s,\varphi_{t'-1})]]$$
$$s.t. a_{t'-1}q_{t'-1} = n + a_{t'-1}E_{O_d}[min(s,\varphi_{t'-1})]$$

Like muti-generation case with single optimist type, if downside optimist buy the security at date $t'-1$, the security price $q_{t'-1}(\varphi_{t'-2},\varphi_{t'-1})$ is simply $E_{O_d}[min(s,\varphi_{t'-2})]$. By simple backward induction, the security price at date $\bar{t}$ is downside optimist's expected return.

$$q_{\bar{t}}(\varphi_{\bar{t}-1},\varphi_{\bar{t}}) = E_{O_d}[min(s,\varphi_{\bar{t}-1})]$$

The date $\bar{t}$ optimist problem:

$$\max_{a_{\bar{t}-1},\varphi_{\bar{t}-1}} a_{\bar{t}-1}[E_{O_u}[min(s,\varphi_{\bar{t}-2})] - q_{\bar{t}-1} - E_{O_u}[min(s,\varphi_{\bar{t}-1})] + E_{O_d}[min(s,\varphi_{\bar{t}-1})]]$$
$$s.t. a_{\bar{t}-1}q_{\bar{t}-1} = n + a_{\bar{t}-1}E_{O_d}[min(s,\varphi_{\bar{t}-1})]$$

This equation is very similar to the problem of the single optimist type case at period $\bar{t}-1$. If both types' expectations about $min(s,\varphi_{\bar{t}-1})$ are the same value, this problem has very simple solution:$q_{\bar{t}-1}(\varphi_{\bar{t}-2},\varphi_t - 1) = E_{O_u}[min(s,\varphi_{\bar{t}-2})]$ By solving the problem, the security price at date $\bar{t}-1$ is calculated.

$$q_{\bar{t}-1} = \int_0^{\varphi_{\bar{t}-1}} sdF_{O_d} + (1-F_{O_d}(\varphi_{\bar{t}-1}))\int_{\varphi_{\bar{t}-1}}^{s^{max}} min(s,\varphi_{\bar{t}-2})\frac{dF_{O_u}}{1-F_{O_u}(\varphi_{\bar{t}-1})}$$

The security supply is one. Substitute $a=1$ to budget constraint, the supply function is calculated.

$$q_{\bar{t}-1}(\varphi_{\bar{t}-2}) = n + E_{O_d}[min(s,\varphi_{\bar{t}-1})]$$

These two equations determine the security price at date $\bar{t}-1$. From assumption, downside optimists evaluate $min(s,\varphi_{\bar{t}-2})$ higher than upside optimists. That is:

$$E_{O_d}[min(s,\varphi_{\bar{t}-1})] > E_{O_u}[min(s,\varphi_{\bar{t}-1})]$$

By comaring to the single optimist type case, upside optimist at date $\bar{t}-1$ have bigger budget. Then, the security price $q_{\bar{t}-1}(\varphi_{\bar{t}-2})$ is higher than single optimist type case:

$$q_{\bar{t}-1} > E_{o_u}[min(s,\varphi_{\bar{t}-2})]$$

At date $\bar{t}-2$, the upside optimist buys the security. The date $\bar{t}-2$ optimist problem:

$$\max_{a_{\bar{t}-2},\varphi_{\bar{t}-2}} a_{\bar{t}-2}[E_{O_u}[min(s,\varphi_{\bar{t}-3})] - q_{\bar{t}-2}(\varphi_{\bar{t}-3},\varphi_{\bar{t}-2}) - E_{o_u}[min(s,\varphi_{\bar{t}-2})] + q_{\bar{t}-1}(\varphi_{\bar{t}-2},\varphi_{\bar{t}-1})]$$

$$s.t. a_{\bar{t}-2}q_{\bar{t}-2} =$$

By solving this problem, the security price at period $\bar{t}-2$ is calculated. Because $q_{\bar{t}-1}(\varphi_{\bar{t}-2}) > E_{o_u}[min(s,\varphi_{\bar{t}-2})]$, the security price at period $\bar{t}-2$ is higher than single optimist type case:

$$q_{\bar{t}-2}(\varphi_{\bar{t}-2}) > E_{o_u}[min(s,\varphi_{\bar{t}-3})]$$

By repeating caluculation, the date 1 sequrity price:

$$q_1(\varphi_0) > E_{o_u}[min(s,\varphi_0)]$$

Upside optimist problem at period 0 is:

$$\max_{a_0,\varphi_0} a_0[E_{o_u}[s] - p - E_{o_u}[min(s,\varphi_0)] + q_1(\varphi_0)]$$
$$s.t. a_0 p = n + a_0 q_1(\varphi_0)$$

Because $q_1(\varphi_0) > E_{O_u}[min(s,\varphi_0)]$, The upside optimist have bigger budget than single type optimist case. In single type optimist case, the asset price is $E_{O_u}[s] = E_o[s]$. So, in multi generation with two type optimist case, price exceed the optimistic expectation:

$$p > E_o[s]$$

In multi-generation with single optimist type case, asset price is equal to the optimistic expectation. The optimists holds the asset and the securities. Because there are two type optimists, upside and downside, high return states are evaluated by upside optimist and low rturn states are evaluated by downside optimist.

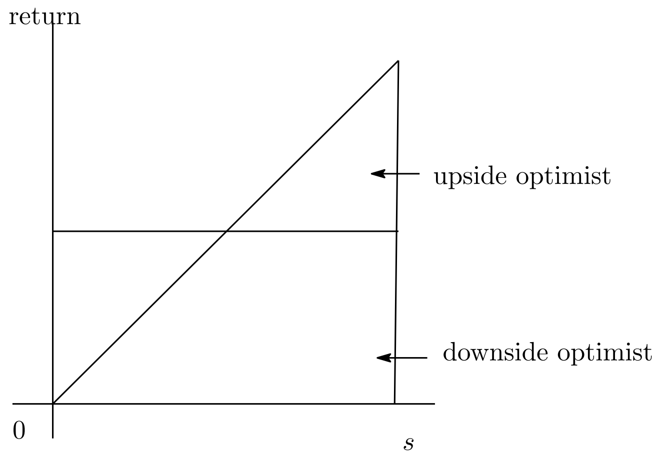
4.2 Various Optimist Types Case:Setting

Asset price exceeds both optimists' expectation in two type optimists case. I analyse the general case, multi generation with various optimists model.

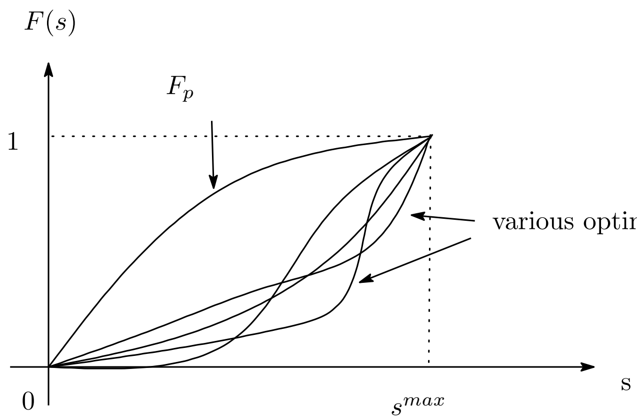
There are $J+1$ type of traders in this economy. J types of optimists and one type of pessimist. The optimist's type j have optimistic belief $F_{O_j}(s)$. All types of optimists have same expectation about the return of the asset $(E_{O_j}[s] = E_o[s] \forall j \in J)$.

Each optimistic belief F_j first order stochastically dominates pessimistic belief F_p.

Assumption 5 *$F_{O_j}[s] \geq F_p[s]$ $\forall s$ and $\forall j \in J$*



return
upside optimist
downside optimist
0
s



F(s)
F_p
1
various optimists
0
s^max
s

The asset, the securities and the loan contracts is the same as multi generation model. At date 0, there exists J optimists and one pessimist. The pessimist is chosen by the optimist from many pessimists. Each optimist is initially endowed with $n > 0$ dollars and zero unit of the asset. Assume the pessimists have enough cash to lend the optimists. All borrowing in this economy is subject to a collateral constraint. Because asset supply is one, only one optimist contracts with the pessimist and buys the asset.
So, optimist type j problem at date 0 is:

$$\max_{a_0,\varphi_0} a_0[E_{O_j}[s] - p - E_{O_j}[min(s,\varphi_0)] - \phi_0]$$
$$s.t. a_0 p = n + a_0\phi_0$$

The optimist who bids the highest price to the asset buys it. Let p_j be the bidding price of type j optimist. Note j_0^* be the optimist type who have the highest p_j. That is:

$$j_0^* = argmax_j \ \ p_j$$

At date t$(t = 1, 2...)$, the optimist buys the security with the loan contract. The type j optimist problem:

$$\max_{a_t,\varphi_t} a_t[E_{O_j}[min(s,\varphi_{t-1})] - q_t - E_{O_j}[min(s,\varphi_t)] - \phi_t]$$
$$s.t. a_t q_t(\varphi_{t-1},\varphi_t) = n + a_t\phi_t$$

From each date t and $T-1$ pessimists' competition:

$$\phi_t = q_{t+1}(\varphi_t)\phi_{T-1} = E_p[min(s,\varphi_{T-1})]$$

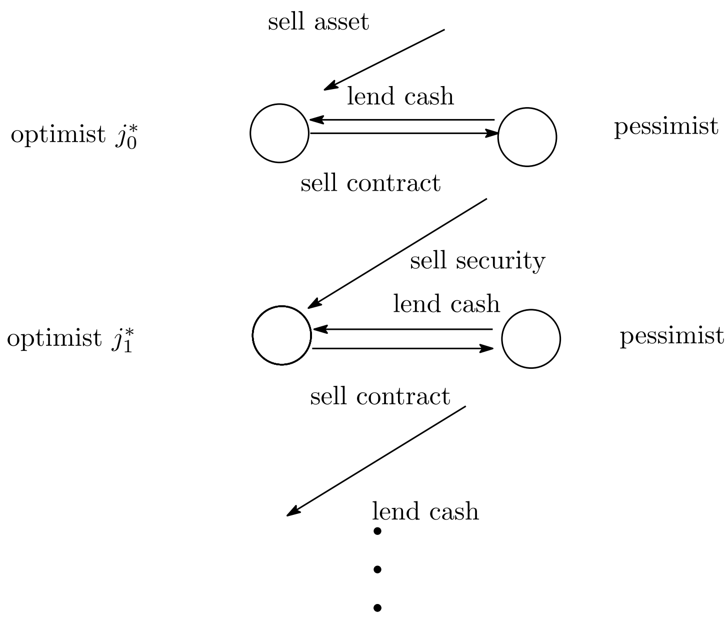
By solving the each date problem, p, q_t and φ_t for all t are calculated. Like single type optimist model, the scheme ends at the time some optimist buy the security by his own cash. Let this date be t'. At date t', the security price is simply $q(\varphi_{t'-1}) = E_{O_{j^*_{t'}}}[min(s,\varphi_{t'-1})]$. $j^*_{t'}$ is the type who has highest security price in equilibrium.

The optimist who bid highest price to the security buys it. Let the highest type optimist be j_t^*.

$$j_t^* = argmax_j \ \ q_t(\varphi_{t-1},\varphi_t)$$

By solving the problem, the asset price p, the each security prices q_t and the each promise payment φ_t are calculated.

In this model, asset price is solvede by backward induction like multi generation case. But in this case, we need to know what type of optimist get each security or the asset.
From budget constraint of optimists' problem at date t:



$$aq_t = n + aq_{t+1}(\varphi_t)$$

In equlibrium, asset supply is one:

$$q_t = n + q_{t+1}(\varphi_t)$$

At date t', security price is expected return of $j^*_{t'}$.

$$q(\varphi_{t'-1}) = E_{O_{j^*_{t'}}}[min(s, \varphi_{t'-1})]$$

Then, security price at date $t' - 1$ is:

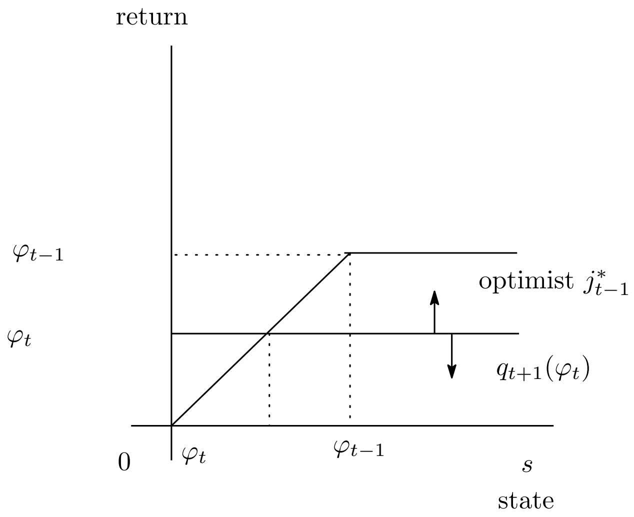
$$q_{t'-1} = n + q_{t'}(\varphi_{t'-1}) = n + E_{O_{j^*_{t'}}}[min(s, \varphi_{t'-1})]$$

Then, $q_{t'-1}(\varphi_{t'-1}, \varphi_{t'})$ is increasing at $\varphi_{t'}$. By repeating this calcultation, q_t is increasing at φ_t.
Then, type j^*_t, which has higest q_t in equilibrium, has the highest $\varphi_{t'}$.
From type j optimist's problem at date t,

$$q_t = q_{t+1} + (1 - F_p(\varphi_t)) \int_{\varphi_t}^{s^{max}} min(s, \varphi_{t-1}) \frac{dF_{o_j}}{1 - F_{o_j}(\varphi_t)}$$

Asset supply is one:

$$q_t = n + q_{t+1}(\varphi_t)$$



This two equations determine equilibrium promise payment φ_t and security price q_t. First equation depends on optimist type j.(see figure.) Pessimist sell the security who has highest φ_t in equilibrium.

Type j^* has a belief that the state around φ_{t-1} occurs with high probability. Then he has high valuation about security at date t.
By backward inductions, the asset price is caluculated. In this case, the asset price exceeds any trader's expectation about the asset return.

proposition 3 *In multi-generation with various type optimist model, the asset price exceeds optimistic trader's expecatation of asset return:*

$$p \geq E_o[s]$$

Proof 4.2 *As noted above, the security price at date t' is $E_{o^*_j t}[min(s, \varphi_{\bar{t'}-1})]$.*

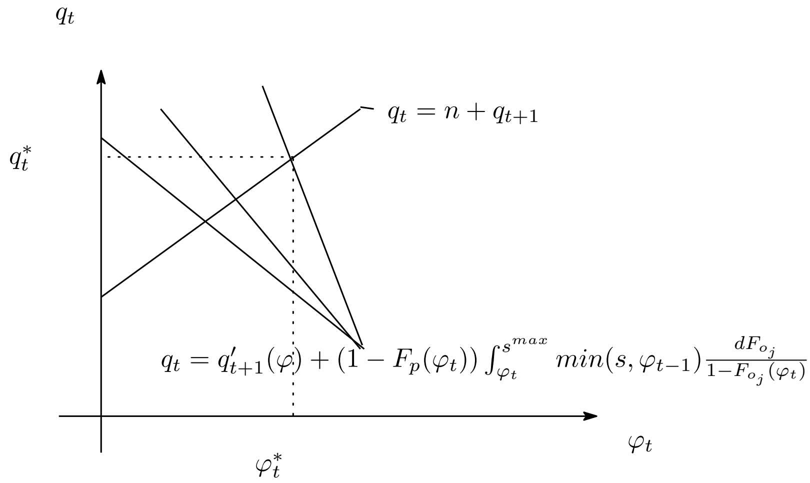
The optimist type $j^*_{t'-1}$ buys the date $t'-1$ security. The optimist problem at date $t'-1$ is:

$$\max_{a_{t'-1}, \varphi_{t'-1}} a_{t'-1}[E_{O_{j^*_{t'-1}}}[min(s, \varphi_{t'-2})] - q_{t'-1} - a_{t'-1}E_{O_{j^*_{t'-1}}}[min(s, \varphi_{t'-1})] + E_{O_{j^*_{t'}}}[min(s, \varphi_{t'-1})]$$
$$s.t. a_{t'-1}q_{t'-1} = n + a_{t'-1}E_{O_{j^*_{t'}}}[min(s, \varphi_{t'-1})]$$

By solving this problem,

$$q_{t'-1}(\varphi_{t'-2}, \varphi_{t'-1}) = \int_0^{\varphi_{t'-1}} sdF_{O^*_j t'} + (1 - F_{O^*_j t'}(\varphi_{t'-1})) \int_{\varphi_{t'-1}}^{s^{max}} min(s, \varphi_{t'-2}) \frac{dF_{O_{j^*_{t'-1}}}}{1 - F_{O_{j^*_{t'-1}}}(\varphi_{t'-1})}$$

The security supply is one:



$$q_{t-1} = n + q_t(\varphi_{t'-1})$$

If $j^*_{t'-1} = j^*_{t'}$, that is, the same type optimists buy the securities at both dates, the date $t'-1$ security price is simply $E_{O^*_{j\,t'-1}}[min(s, \varphi_{t'-1})]$. In the various optimists model, the optimist type who buys the security can be different at each date. As noted above, the security is bought by optimist who have the highest value. If the different type optimist buys the security at date $t'-1$, the date $t'-1$ optimist can borrow $E_{O_{j^*\,t'}}[min(s, \varphi_{t'-1})]$ units of cash. Because $E_{O_{j^*\,t'}}[min(s, \varphi_{t'-1})] \geq E_{O^*_{j\,t'-1}}[min(s, \varphi_{t'-1}]$, the date $t'-1$ optimist increase his security demand by comparing with the sigle type optimist model. Then, the security price is higher than the single optimist model. Let the security price of single optimist type model be q_t^{sin}

$$q_{t'-1} \geq E_{O_{j^*\,t'-1}}[min(s, \varphi_{t'-2})] = q^{sin}_{t'-1}$$

The security price can be exceed the optimistic expected return. The same caluculation imply the each date security price exceeds the single optimist type model.

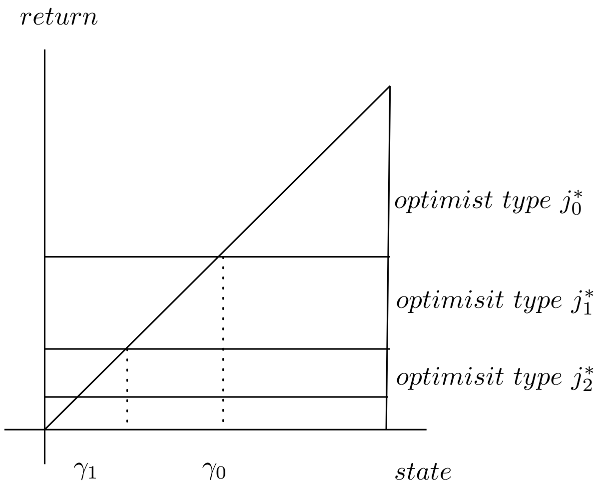
$$q_t \geq q_t^{sin} = E_o[min(s, \varphi_t)] \forall t, \varphi_t$$

The type j_0^* optimist problem at date 0 is:

$$\max_{a_0, \varphi_0} a_0[E_o[s] - p - E_{O_{j_0^*}}[min(s, \varphi_0)] - q_1(\varphi_0)]$$

$$s.t. a_0 p = n + a_0 q_1(\varphi_0)$$

Because $q_1(\varphi_0) \geq E_o[min(s, \varphi_0)]$, the type j_0^* optimist have bigger budget than single optimist type model. So, asset demand a_0 increase and asset supply is one, the asset price p exceeds the single type price $E_o[s]$.



$$p \geq E_o[s]$$

The existance of various optimist leads to the high asset price. This price is similar to Harriso and Kreps(1978). In HK, there are various type traders. They have heterogenous beliefs about asset. At some point, trader x is optimist and trader y is pessimist. But at different point, x is pessimist and y is optimist. The holder of asset changes at each date and the most optimistic trader buys it. The asset holder knows that he can sell it to some other optimist at some future date. Then, asset price is higher than the asset holders' expectations.

In my paper, the optimistic belief about asset return is same:$E_o[s]$. The securitization technology divides the asset return and the each optimist evaluate the each partition.

The each area are evaluated by the most optimistic trader. In Simsek(2013), asset return is evaluated by the optimist and the pessimist. In equilibrium, as noted section 2, asset price:

$$p = \int_0^{\varphi} s dF_p + (1 - F_p(\varphi)) \int_{\varphi}^{s^{max}} s \frac{dF_o}{1 - F_o(\varphi)}$$

This price equation imply that optimist evaluate upper return area and pessimist evaluate lower area. Simsek insists that only high state part of optimistic belief is used for determining the asset price. This is the reason why asset price is lower than Harrison and Kreps(1978). In my paper, the trader's belief is used for the area which he evaluate higher than any other traders. Only his optimistic part of belief is used for evaluation of asset return. Because each optimist have his optimistic part of asset return, the asset price is much more

optimitic than anyone's expectation.
Tranching and asset pricing are analized by Fostel and Geanakoplos(2012) and they insist that tranching technology raise asset price in recent housing bubble. In my model, each state of asset are distributed to most optimistic trader. Then, equlibrium distribution is similar to some equlibrium with tranching in FG(2012).
From budget constraint of optimist problem:

$$p = n + n + ...n + E_{O^{j*}_{t}{}'}[min(s, \varphi_{t'-1})]$$

Asset price is higher than expected return $E_o[s]$ and $n > E_{O^{j*}_{t'}}[min(s, \varphi_{t'-1})]$. The number of optimists is:

$$t' = \frac{p - E_{O^{j*}_{t}{}'}[min(s, \varphi_{t'-1})]}{n}$$

So, the number of optimist is larger than multi generation with single type optimist. Many optimists participate in the market and they buy the securities at each date.

5 conclusions

In my paper, by introducing the securitization and the simple dynamic setting to Simsek(2013), the asset price will be as high as Harrison and Kreps.
In homogeneous belief model, securitizations improve market efficiency. On the other hand, in heterogeneous belief model, securitizations allow many optimists to participate in markets and asset prices will be raised.
In heterogeneous belief model, financial frictions can make the asset price lower level. Like Harrison and Kreps(1978) or Miller(1977), optimists have heavy influence on pricing. Financial frictions like borrowing constraints can prevent these trader to participate in market. In these situations, financial technologies, like securitizations, loose this friction and the asset price is raised by noisy trader who have littele cash.
In my model, securitizations are very important. By securitizations, loan contracts are appealing for cash lender. Because of speculative reason, pessimists lend cash to optimists very easily.
By the dynamic securitization scheme, the asset return is distributed to many partitions. Each partition is very small, but this samllness allow various optimist to receive payoff. Most optimistic traders evaluate their partitions and the total asset evaluation is higher than any trader's expected return of asset. Heterogeneous Belief Bubbles occur in this model.
This is very similar to Bubble in HK(1978). In their model, traders' beliefs change in each date and the most optimistic trader get the asset at each date. Then, the asset price is raised. In my paper, traders' beliefs are different about the frequency of state and expected return is same among optimists. But only by the heterogeneity about frequency, the asset price exceeds any traders' expectation in my model.

Heterogeneous beliefs are said to be eliminated in markets in economics. Noisy traders may lose cash and they may stay out from markets in future.
Financial technologies allow many traders to participate in market before the asset return realizes. Before the asset return realizes, there are littele incentive to correct their beliefs. If there are heterogeneity in markets, financial technologies can amplify it.

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