The chicken type and incompatible demands in bargaining

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July 2013

Abstract

We study whether the possibility of the chicken type in bargaining causes incompatible demands and also investigate who benefits from such a possibility. The game starts with two players making demands. If the demands are not compatible, the chicken type of a player yields to the demand of the opponent immediately unless it gives her less than her reservation utility. Neither when the initial demands are compatible nor when there is no chicken type, the game moves to costly bargaining, in which the shares at the settlement is (approximately) determined by the lower demand. Our analysis shows that the ordering at the demand stage matters substantially. When the players make their demands sequentially and only the second mover may be the chicken type, the modest possibility of the chicken type does not affect the players' strategies at the demand stage and they make just compatible demands. Either when they make their demands sequentially and the first mover may be the chicken type or when they make their demands simultaneously, their demands become incompatible. In those cases, the player whose opponent may be the chicken type benefits from such possibility and if the player with the possibility of the chicken type turns out not to be, even he may obtain more than what he receives in the case without the possibility of the chicken type.

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** This version is prepared for the 2013 Autumn Meeting of the Japanese Economic Association held at Kanagawa University.

1. Introduction

In the real bargaining, some people behave in a way which cannot be rationalized on the basis of the materialistic objective. These players are often referred to as behavior types and the previous analyses have shown that the possibility of them has a strong effect on the bargaining outcome. For example, Myerson (1991), Abreu and Gul (2000) and Kambe (1999) have shown that, when a player is considered to be a stubborn type with a higher probability, she tends to obtains a higher payoff. In this paper, we study another behavior type, who is prone to yield to the demand of the other at the early stage of the negotiation. We call it *the chicken type*. This type is often observed in the reality and is thought as the cause for high demands that some players make initially. This paper studies the effect of the chicken type on bargaining and, in particular, investigates whether the possibility of the chicken type causes incompatible demands and also who benefits from such a possibility.

The mere presence of the chicken type may not cause inefficiency. For example, if we introduce the possibility of the chicken type into the alternating offer model by Rubinstein (1982), the players may make high demands in the first two rounds in order to exploit the chicken type but, afterward, the continuation equilibrium becomes exactly what is predicted in the original model without such possibility. Namely, if the initial demands do not have any lasting effect, the possibility of the chicken type does not cause meaningful inefficiency. On the other hand, when the initial demand is irrevocable, introducing the possibility of the chicken type may not affect the outcome unless its probability is considerably large. Consider the demand game by Nash (1953) and suppose that one player, say player 2, is the chicken type with the probability γ. (We assume that the other player, player 1, is known not to be the chicken type.) Then, there are equilibria in which player 2 obtains a positive share and, in any pure equilibrium of such kind, the players make compatible demands. In any situation in which the incompatibility of initial demands causes the complete loss of the joint surplus, the players will have a strong incentive to avoid it in equilibrium. Thus, in order to study the inefficiency that is caused by the possibility of the chicken type, this paper considers the situation in which initial demands have some effect on their payoffs even in the later stage of the game and also its incompatibility does not cause the total loss of payoffs.

Specifically, we consider the following model. The two players negotiate over the partition of one dollar. We assume that there are three types for each player: the rational type, the chicken type and the stubborn type. The rational type tries to maximize the discounted value from the settlement while the other types behave according to their specific concerns. The chicken type yields to the demand of the other as soon as she discovers the initial demands are not compatible. (In the analysis, we assume that she has some reservation utility and leaves the negotiation if the offer of the other falls short of it.) The stubborn type never accepts the demand of the other when the negotiation comes to the bargaining which is conducted as the war-of-attrition. We assume that a player learns her own type just after she makes her demand and that, during the negotiation, she is never able to tell the type of the other. The negotiation starts with the players making demands. We consider two scenarios. In *the sequential demand game*, we suppose that player 1 makes her demand first and then player 2 makes his demand. In *the simultaneous demand game*, we suppose that two players announce their demands simultaneously. If the demands are compatible, they split one dollar accordingly. If not, the chicken type immediately yields to the demand of the other unless it gives her less than her reservation utility. If no player yields, then the players pay some cost and initiate a formal bargaining. We study two situations with respect to how the players negotiate there. In both Section 3 and Section 5, we suppose that the more modest demand is used to determine the players' shares at the settlement. We call this way of partition the equitable rule. In Section 4, we suppose that the players engage themselves in the so-called war-of-attrition, in which a player chooses the time to accept the demand of the other.

We have two key findings from the analysis. First, whether the player with the possibility of the chicken type makes the first demand or not is critical in causing incompatible demands. In the sequential demand game, when player 1 does not have the possibility of the chicken type but player 2 does, the players do not make incompatible demands as long as the probability of the chicken type is not so large. This is because player 1 fears that a high demand from her is going to be exploited by player 2 when player 2 turns out to be the rational type. Because the probability of the chicken type is not so large, it deters player 1 from making a high demand. On the other hand, when player 1 has the possibility of the chicken type, either if she makes her demand first or if the players make their demands simultaneously, player 2 wants to exploit its possibility. In the sequential demand game, player 2 makes the highest demand that the chicken type accepts. Knowing that he makes a high demand, player 1 also demands a moderately high demand, which player 2 will accept if player 1 turns out not to be the chicken type. In the simultaneous demand game, in addition to these considerations, the players have an incentive to underbid the other. Thus, they randomize their demands between the highest demand that the chicken type accepts and some lower demands. In both cases, the demands become incompatible. An interesting point in those cases is that even the player whose opponent is known not to be the chicken type also makes a high demand. Because of it, in the sequential demand game, both players may gain by the possibility that one player may be the chicken type.

We describe the basic model in the second section. The following two sections both assume that the demands are made sequentially but have different formulations for the ensuing bargaining stage. In Section 3, we study the situation where the equitable rule is used to determine the bargaining outcome once the bargaining stage is reached. Section 4 supposes that the players play the non-cooperative game at the bargaining stage in which a player chooses the time to accept the demand of the other. In Section 5, we study the situation where the demands are made simultaneously and the equitable rule is used to determine the bargaining outcome once the bargaining stage is reached. All the proofs are relegated to the appendix.

2. Basic Model

Two players, player i and player j ($i, j \in \{1, 2\}$ and $i \neq j$), bargain over the partition of one dollar. (To clarify the identities of players, a generic player, player 1, and player i take the female identity while her opponent, player 2, and player j take the male identity.) We assume that each player is one of three types: a rational type, a chicken type, and a stubborn type. We explain the detail about the types shortly.

The negotiation is comprised of three stages. *The demand stage* occurs at the beginning. In the following analysis, we study two different timings in terms of making demands. In *the sequential demand game*, the two players sequentially announce their demands with player 1 as the first proposer. Both Section 3 and Section 4 suppose the sequential demand game. In Section 5, we study *the simultaneous demand game*, where the two players simultaneously announce their demands. Player i makes the demand x_i, which indicates the offer of $1 - x_i$ to player j. We say that the demands are *compatible* when $x_1 + x_2 \leq 1$ and that they are *incompatible* when $x_1 + x_2 > 1$. When $x_1 + x_2 = 1$, we say that the demands are *just compatible*. We say that player 2 makes the just compatible demand when player 2 makes the demand of $1 - x_1$ given player 1's demand x_1. If the demands are compatible, the game ends and the players split the difference in addition to their own demand, i.e., player i receives $x_i + (1 - x_1 - x_2)/2 = (1 + x_i - x_j)/2$. When their demands are not compatible, *the yielding stage* begins. At this stage, the players become aware of their own types. However, a player never observes the type of the other player. We assume that player i becomes the chicken type with the probability γ_i, the stubborn type with the probability $\xi_i(1 - \gamma_i)$ and the rational type with the probability $(1 - \xi_i)(1 - \gamma_i)$. (Note that, given that player i is not the chicken type, the conditional probability that she is the stubborn type is equal to ξ_i.) This probability is independent across the two players. We generally suppose that γ_i is positive but small, and that, whenever ξ_i is positive, it is close to zero. This is based on our casual empiricism that the players are quite likely to be rational. The chicken type has the reservation utility $1 - \hat{x} (\geq 0)$ and, at the yielding stage, she yields to the demand of the opponent and accepts his offer if and only if it gives her no less than her reservation utility. As specified later in each section, we generally suppose that $\hat{x}$ is high. (When it is either equal to one or close to it, any of the conditions imposed later will be satisfied.) When the demand of player i is higher than $\hat{x}$, we suppose that the chicken type of player j leaves the negotiation without settlement[1]. In contrast, the other types do nothing in the yielding stage. When only one player, say player i, is the chicken type, she yields and obtains $1 - x_j$, and player j obtains x_j. When both players are the chicken type, they obtain what the other offers and then split the remaining: player i obtains $1 - x_j + (x_i + x_j - 1)/2$. If no settlement is made at the yielding stage, the game moves to the bargaining stage. After the yielding stage but before the bargaining stage, we assume that each player incurs the fixed cost d: $0 < d < 1$. We call this *the bargaining cost*. It corresponds to the time and the

[1] An alternative assumption is that, faced with a demand higher than $\hat{x}$, the chicken type does not yield to that demand and goes into the bargaining stage. The analyses in Section 3 and Section 5 are unaffected because no player has an incentive to make a demand higher than $\hat{x}$ and the settlement at the bargaining stage is dictated by the equitable rule. In Section 4, whether the chicken type can concede or not becomes an issue. If so (or not), she behaves as if she were the rational type (or the stubborn type respectively) in the war of attrition. With this modification, the qualitative nature of the results are unchanged. In the current model, the players' incentive to make a high demand originates from the possibility that the chicken type may accept it. Hence, under either assumption, no player demands more than $\hat{x}$. In order to simplify the analysis, we make this particular assumption Another interpretation is that the chicken type is extremely impatient and thus seeks an immediate settlement either by yielding to the opponent's demand or by taking the outside option.

efforts needed to initiate a formal negotiation. Because of this, the players as a group are better off by concluding the negotiation before the bargaining stage is reached. The rest of the specification of the bargaining stage including what either the rational type or the stubborn type can do is given separately in the next three sections.

The rational type of each player wants to maximize her expected payoff. On the other hand, the other two types are behavior types and do not maximize their expected payoffs. For simplicity, we assume that these types have constant payoffs irrespective either of the obtained amounts or of its timing. Thus, we assume that, at the demand stage, each player tries to maximize the expected payoff of her own rational type. Because of this, when we refer to (expected) payoffs in the following, we always refer to the (expected) payoffs of the rational type. Our equilibrium concept is the perfect Bayesian equilibrium. Namely, a player updates her belief by Bayes' rule whenever possible and takes the best response to it. (When the equitable rule is applied at the bargaining stage, the belief does not affect the players' strategies and thus the equilibrium concept there is equivalent to the subgame perfection. This is because the players do not know their own types at the demand stage.)

3. The sequential demand game under the equitable rule

In this section, once the bargaining stage is reached, we suppose that the settlement is imposed by the following simple rule: when $x_j > x_i$, player i obtains x_i and player j obtains $1 - x_i$, and when $x_i = x_j$, both players obtain $1 - x_i (= 1 - x_j)$. Namely, the more modest demand is used to determine the partition at the settlement. (If the demands are same, the players receive what the other offers.) In the sense that the settlement gives a more equal division out of two demands, we call this rule *the equitable rule.* One reason that we use this particular rule is that it approximates the outcome for the symmetric case of the non-cooperative bargaining game that we examine in the next section. By simplifying the bargaining stage, we aim to illustrate the role of the chicken type in bargaining in the sharpest way. Alternatively, this rule can be viewed as the one that a mediator may use in the final offer arbitration because of its equitability. The analysis of this section applies to both situations.

Given this formulation, the stubborn type plays no role and thus in this section, we simply ignore its possibility and assume $\xi_i = 0$ for $i \in \{1, 2\}$. In order to focus on the effect of the chicken type, we suppose that the bargaining cost is small compared to the highest demand that the chicken type accepts and assume that $\hat{x} > (1+d)/2$. Note that this inequality is satisfied when the reservation utility of the chicken type is zero ($\hat{x} = 1$) because $d < 1$. Throughout the analysis of this section, we assume these two, which we call Assumption A hereafter.

Assumption A

(i) $\xi_i = 0$ for $i \in \{1, 2\}$, and

(ii) $\hat{x} > (1+d)/2$.

Because the equitable rule specified above is not continuous in the demands, a player may want to make her demand lower than her opponent's but as close to it as possible. In other words, the best response is not necessarily well-defined. In reality, there is a minimum unit of money and thus the highest offer among those lower than a certain value is easily specified. Let x_i be player i's demand and let $x_i - \epsilon$ be the highest feasible demand in the monetary unit among those lower than x_i. Then, we make the monetary unit smaller and denote the limit of this demand by $[x_i]_-$. By abusing the notation, we sometimes regard it as the best response of player j when he wants to set his demand to be lower than x_i and as close to it as possible. By the definition of the limit, given the demands $(x_i, [x_i]_-)$, the payoff of player i converges to $1 - x_i$ from the above and that of player j converges to x_i from the below. Throughout this section, when we refer to $[x_i]_-$, we use this convention in the evaluation of payoffs.

Before studying the effect of the chicken type, let us characterize the equilibrium when there is only the rational type. Because the bargaining stage costs d for each player, player 2 would accept the offer of player 1 if the alternative does not improve her payoff by more than d. This creates the first mover advantage for player 1. On the other hand, the equitable rule causes the race to the middle. When player 1 sets her demand too high, player 2 *underbids* her ($x_2 < x_1$) and, under the equitable rule, is able to obtain what he demands. As the consequence of these two effects, player 2 prefers underbidding player 1's demand if and only if $x_1 > (1+d)/2$. Because the underbidding by player 2 is detrimental to player 1, it is optimal for her to demand $(1+d)/2$. Then, player 2 responds to it by the compatible demand $(1-d)/2$. The plus (or minus) term is the first mover advantage (or disadvantage respectively). Except for these terms, their demands are located at the middle. They reach the settlement at the demand stage.

Proposition 0

Suppose that Assumption A is satisfied and also that $\gamma_1 = \gamma_2 = 0$. In the unique equilibrium, player 1 demands $(1+d)/2$ and player 2 demands $(1-d)/2$. They reach the settlement at the demand stage.

Next, consider the case that only player 2 has the possibility of the chicken type. Because the chicken type yields to the demand of the other that gives him no lower than his reservation utility, we may think that player 1 substantially increases her demand from one half and takes advantage of it. As shown in the next proposition, this conjecture turns out to be incorrect when only the second mover may be the chicken type and its probability is less than one half.

Proposition 1

Suppose that Assumption A is satisfied and moreover that $\gamma_1 = 0 < \gamma_2 < 1/2$. In the unique equilibrium, player 1 demands $(1+d)/2$ and player 2 demands $(1-d)/2$. They reach the settlement at the demand stage.

This proposition shows that, when the player with the possibility of the chicken type moves second and the one without it moves first, their demands are exactly equal to the ones that they make when there is no possibility of the chicken type. Namely, in this case, the possibility of the chicken type does not affect the bargaining outcome and, in particular, does not cause incompatible demands. To understand this result, let us examine the payoff of player 1 when she tries to exploit the chicken type of player 2. Specifically, suppose that she makes a demand which is higher than $(1+d)/2$ but not higher than $\hat{x}$. The chicken type of player 2 yields to this demand. Then, player 1 obtains x_1. On the other hand, because player 1 is known to be the rational type, the best response of player 2 is to underbid this demand by the demand of $[x_i]_-$. Given this response, player 1 obtains a payoff close to $1-x_1$. Hence, player 1's expected payoff is approximately given by $\gamma_2 x_1 + (1-\gamma_2)(1-x_1-d)$. Under the assumption that $\gamma_2 < 1/2$, this is strictly decreasing in x_1 and is lower than $(1+d)/2$. Namely, for player 1, the decrease of her payoff against the rational type of player 2 outweighs the increase of it against the chicken type. Because of this, player 1 makes the highest demand that does not cause player 2 to underbid it. In other words, fearing the underbidding by the rational type of player 2, player 1 does not try to take advantage of the chicken type.

In contrast to the above case, if the first mover may be the chicken type, the possibility of the chicken type does cause the other player to take advantage of it when the bargaining cost is relatively small. In such a situation, the next proposition shows that the players make incompatible demands. Define $x_1^* \equiv (\gamma_1 \hat{x} + 1 - \gamma_1)/(2 - \gamma_1)$. We show that player 1 demands this amount when the bargaining cost is sufficiently small.

Proposition 2

Suppose that Assumption A is satisfied and moreover that $\gamma_2 = 0 < \gamma_1 < 1$. When $d < \gamma_1(2\hat{x} - 1)/(2 - \gamma_1)$, player 1 demands x_1^* and player 2 demands $\hat{x}$ in the unique equilibrium. It holds that $1/2 < x_1^* < \hat{x}$ and the demands become incompatible in this case. When $d > \gamma_1(2\hat{x} - 1)/(2 - \gamma_1)$, the players' demands become compatible in equilibrium.

When the bargaining cost is sufficiently high, the proposition shows that the players conclude the negotiation at the demand stage in order to avoid it. On the other hand, when it is low, the players make incompatible demands ($x_1^* + \hat{x} > 1$). In such a case, player 1 makes the moderately high demand of x_1^*. Because player 1's demand is not so high, player 2 finds it as profitable to exploit the possibility of the chicken type as to underbid it. It necessarily involves the settlement at the bargaining stage via player 1's demand if she turns out to be the rational type. Knowing this, player 2 chooses his demand in order to maximize his payoff against the chicken type. Namely, he makes the highest demand that the chicken type yields to. The difference from the previous proposition is caused because, in the current setting, the player who wants to exploit the possibility of the chicken type moves second and does not have to fear the increased demand at the opponent's underbidding in response to her demand.

One interesting aspect of the predicted outcome is that, when the bargaining cost is not so large, player 1 demand x_1^*, which is more than one half but is less than $\hat{x}$. This demand is the threshold above which player 2 underbids player 1. Let us suppose that $d < \gamma_1(2\hat{x} - 1)/(2 - \gamma_1)$ and explain how it is derived. In order to focus on the demands that may be taken in equilibrium, let us consider player 1's demands in $(1/2, \hat{x})$. Given x_1 in this range, it can be shown that player 2's best response is either $\hat{x}$ or $[x_i]_-$. If player 2 demands $\hat{x}$, he obtains $\hat{x}$ against the chicken type and obtains the share $1 - x_1$ against the rational type at the bargaining stage. Hence, his expected payoff is given by $\gamma_1 \hat{x} + (1 - \gamma_1)(1 - x_1 - d)$. On the other hand, if player 2 demands $[x_1]_-$, in the limit, he obtains x_1 against the chicken type and obtains the share x_1 against the rational type at the bargaining stage. Hence, his expected payoff is approximately given by $\gamma_1 x_1 + (1-\gamma_1)(x_1 - d)$. The comparison of the payoffs shows that player 2 is better off by demanding $\hat{x}$ when $x_1 \leq x_1^*$, and by demanding $[x_1]_-$ when $x_1 > x_1^*$. Player 1's share at the demand stage becomes lower than one half when player 2 underbids her demand. Hence, player 1's optimal strategy is to make the highest demand that does not induce player 2 to underbid her, which is x_1^*.

For the incompatible demands to be made, the bargaining cost has to be sufficiently small. To understand how we obtain the condition for the bargaining cost, suppose that the bargaining cost is not so large that $d \leq \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$. Note that the left hand side is larger than the threshold for the bargaining cost in the proposition. (When the bargaining cost is larger than this amount, we can show that demanding $\hat{x}$ never constitutes the best response for player 2 and the settlement through the just compatible demands becomes even more attractive to player 1.) Given that the equitable rule is used in the bargaining stage, player 2's best response is one of the following three, the compatible demand $(1-x_1)$, the highest demand to which the chicken type yields to $(\hat{x})$, and the underbidding $([x_1]_-)$. In the above, we have shown that the threshold between the latter two with respect to player 1's demand is given by x_1^*. Now consider the threshold between the first two. When player 2 makes the compatible demand, he obtains $1-x_1$. Comparing this with the payoff from $\hat{x}$, we can show that player 2 prefers $1-x_1$ when $x_1 < \tilde{x} \equiv (\gamma_1(1-\hat{x}) + (1-\gamma_1)d)/\gamma_1$, and he prefers $\hat{x}$ when $x_1 > \tilde{x}$. Namely, player 1's highest demand that induces player 2 to make the compatible demand is given by $\tilde{x}$. With this demand[2], player 1 obtains the same amount as her payoff. On the other hand, when she demands x_1^*, her expected payoff is given by $\gamma_1 x_1^* + (1-\gamma_1)(x_1^* - d)$. Note that the former is increasing in the bargaining cost while the latter is decreasing in it. Observe that the latter becomes bigger than the former when the bargaining cost d is close to zero. Combining these, we can show that the condition for the latter to be bigger (or smaller) than the former is given by the one described in Proposition 2.

We now study the property of the equilibria described in Proposition 2 when the bargaining cost is small $(d < \gamma_1(2\hat{x}-1)/(2-\gamma_1))$ and the players make incompatible demands.

[2] Under the supposed condition for the bargaining cost, we can show that $\tilde{x} < x_1^*$.

First, let us compute the magnitude of inefficiency. The incompatible demands predicted in this proposition causes each player to pay the bargaining cost when player 1 turns out not to be the chicken type. Thus, the expected total cost is given by $2(1-\gamma_1)d$. The supposition in the proposition shows that the players make incompatible demands when $d < \gamma_1(2\hat{x}-1)/(2-\gamma_1)$ but they make compatible demands when the inequality holds in reverse. Hence, the total cost needs to be lower than $2\gamma_1(1-\gamma_1)(2\hat{x}-1)/(2-\gamma_1) = (2\hat{x}-1)\gamma_1\left(1-\frac{\gamma_1}{2-\gamma_1}\right)$. It implies that the total cost is less than $(2\hat{x}-1)\gamma_1$. Moreover, when $\hat{x}$ is close to one and when γ_1 is small, it implies that the upper bound for the expected total cost is close to γ_1. Namely, the players choose incompatible demands even when doing so reduces the joint surplus by the level close to the probability of the chicken type.

Next, we compute the payoffs of the players in equilibrium. Player 2 obtains $\hat{x}$ when the opponent turns out to be the chicken type and otherwise obtains $(1-x_1^*-d)$. Thus, her expected payoff at the beginning is given by $\gamma_1\hat{x}+(1-\gamma_1)(1-x_1^*-d)$. By substituting the formula into x_1^*, we can show that:

$$
\begin{aligned}
&\gamma_1\hat{x}+(1-\gamma_1)(1-x_1^*-d)-\frac{1-d}{2} \\
&=\gamma_1\hat{x}+(1-d)\frac{2(1-\gamma_1)-1}{2}-(1-\gamma_1)\frac{\gamma_1\hat{x}+1-\gamma_1}{2-\gamma_1} \\
&=\gamma_1\hat{x}+\frac{1-2\gamma_1}{2}-\frac{(1-2\gamma_1)d}{2}-(1-\gamma_1)\frac{\gamma_1\hat{x}+1-\gamma_1}{2-\gamma_1} > 0.
\end{aligned}
$$

It implies that player 2 obtains more than $(1-d)/2$. Namely, player 2 benefits from the possibility of the chicken type. On the other hand, player 1 obtains x_1^*-d because player 1's demand is lower than that of player 2. Recall that player 1 obtains $(1+d)/2$ when there is no possibility of the chicken type. The simple computation shows that $x_1^*-d-(1+d)/2 = \big(\gamma_1(2\hat{x}-1)-3(2-\gamma_1)d\big)/\big(2(2-\gamma_1)\big)$. Hence, when the bargaining cost is sufficiently small so that $d < \gamma_1(2\hat{x}-1)/\big(3(2-\gamma_1)\big)$, the rational type of player 1 also benefits from the possibility of her own chicken type. Observe that x_1^* is constructed as the threshold when player 2's payoff from $\hat{x}$ is equal to that from $[x_1]_-$. Hence, his expected payoffs from the two demands are identical:

$$
\gamma_1\hat{x}+(1-\gamma_1)(1-x_1^*-d) = \gamma_1 x_1^* + (1-\gamma_1)(x_1^*-d) = x_1^*-(1-\gamma_1)d.
$$

This shows that player 2's payoff is higher than that of player 1 by $\gamma_1 d$. Namely, player 2 benefits from being the second mover and able to exploit the possibility of the chicken type.

This makes the contrast with the situation in which there is no possibility of the chicken type and there is the first mover advantage.

Third, as explained above, the demand of x_1^* is the upper bound of the demands that cause player 2 to demand $\hat{x}$. The more likely player 1 is to be the chicken type, the less willing player 2 is to abandon the demand of $\hat{x}$. This is also the case when the reservation utility of the chicken type is lower. Hence, when γ_1 is higher and/or when $\hat{x}$ is higher, player 1 can make a higher demand without causing player 2 to underbid her demand. In particular, it holds that that $x_1^* = 1/2$ when $\gamma_1 = 0$ and that $x_1^* = \hat{x}$ when $\gamma_1 = 1$.

The next corollary summarizes these observations.

Corollary 1

Suppose that Assumption A is satisfied and moreover that $\gamma_2 = 0 < \gamma_1 < 1$. In addition, suppose that $d < \gamma_1(2\hat{x} - 1)/(2 - \gamma_1)$.

(1) The upper bound for the expected total cost is given by $2\gamma_1(1 - \gamma_1)(2\hat{x} - 1)/(2 - \gamma_1)$.

(2) Player 2 obtains a higher payoff with the possibility of the chicken type than without it. When $d < \gamma_1(2\hat{x} - 1)/3(2 - \gamma_1)$, the same is true for the rational type of player 1. Player 2's payoff is higher than that of player 1 by $\gamma_1 d$.

(3) As long as the supposed condition for the bargaining cost holds, the demand of player 1, x_1^*, strictly increases from 1/2 to $\hat{x}$ when γ_1 increases from zero to one. Moreover, it is also strictly increasing in $\hat{x}$.

Finally, we study the case that both players may be the chicken type. The next proposition shows that the equilibrium outcome is exactly same as the one described in Proposition 2 for a sufficiently small bargaining cost.

Proposition 3

Suppose that Assumption A is satisfied and moreover that $0 < \gamma_1 < 1$ and $0 < \gamma_2 < 1/2$. When $d < \gamma_1(2\hat{x} - 1)/\big((1 - \gamma_1\gamma_2)(2 - \gamma_1)\big)$, player 1 demands x_1^* and player 2 demands $\hat{x}$ in the unique equilibrium. It holds that $1/2 < x_1^* < \hat{x}$ and the demands become incompatible in this case. When $d > \gamma_1(2\hat{x} - 1)/\big((1 - \gamma_1\gamma_2)(2 - \gamma_1)\big)$, the players' demands become compatible in equilibrium.

When both players can be the chicken type and the bargaining cost is sufficiently small, their demands become incompatible. It is not caused by the desire of the first mover to take advantage of the opponent's possibility of the chicken type. Because $\gamma_2 < 1/2$, the same logic as in Proposition 1 causes player 1 to avoid the increased demand at player 2's underbidding in response to her demand. Her demand is the highest one that causes player 2 to demand $\hat{x}$. Proposition 2 shows that it is given by x_1^*. Namely, player 1's demand is identical to the one that she would demand when where were no chance that the opponent is the chicken type. As in the situation studied in Proposition 2, the cause of the incompatibility is the desire of the second mover to take advantage of the possibility that the first mover is the chicken type.

The only difference between Proposition 2 and Proposition 3 is the condition for the bargaining cost. When $\gamma_2 > 0$, player 1 may obtain x_1^* without the bargaining cost because of the chicken type. Hence, player 1's expected payoff from the demand of x_1^* is given by $x_1^* - (1 - \gamma_2)d$. This is higher than the expected payoff from x_1^* when $\gamma_2 = 0$. Hence, player 1 has a higher incentive to make this demand when $\gamma_2 > 0$. On the other hand, the highest payoff from the demand that causes player 2 to make the compatible demand is unaffected by γ_2 because it is determined by the best response of the rational type for player 2. Consequently, for the incompatible demands to occur, the condition for the size of the bargaining cost is less stringent when $\gamma_2 > 0$.

Summarizing the propositions in this section, we can say that, given a low bargaining cost and a modest chance of the chicken type, the necessary and sufficient condition for incompatible demands in the sequential demand game under the equitable rule is that the first mover has the possibility of the chicken type.

4. The sequential demand game and the war of attrition

This section studies the situation in which the players play the non-cooperative bargaining game at the bargaining stage. For the model at the bargaining stage, we use a formulation similar to the one in Abreu and Gul (2000). In particular, it is supposed that player i ($i = 1, 2$) becomes stubborn with the positive probability $\xi_i(1 - \gamma_i)(> 0)$ and a war of attrition occurs at the bargaining stage. Note that only the rational type and the stubborn type participate in the war of attrition as the chicken type either yields to the demand of the opponent or leaves the negotiation at the yielding stage. As in the previous section, at the demand stage, the players take both the possibility of the chicken type and their payoffs at the bargaining stage into consideration. Thus, two different kinds of behavior types affect the strategic decisions in different ways and this section studies how they affect the bargaining outcome.

In this section only, we assume that the bargaining stage proceeds in continuous time[3]: $t \in \mathcal{T} = [0, \infty)$. During the bargaining stage, the rational type of a player chooses when she accepts the opponent's offer[4]. On the other hand, the stubborn type of a player is assumed to commit herself to her own demand and thus to wait for the acceptance by the opponent. We maintain our assumption that the types are private information also during the bargaining stage; both players update their beliefs about their opponent's type by using Bayes' rule. Note that player j believes that player i is the stubborn type with the conditional probability ξ_i at the beginning of the bargaining stage as by this stage the players know that their opponent is either the rational type or the stubborn type. When player i accepts player j's demand x_j, the negotiation is over and the players divide the dollar according to the accepted demand: player i receives $1 - x_j$ and player j receives x_j. When both players simultaneously accept the opponent's demand, they obtain what the other offers and then split the remaining: player i obtains $(x_i - x_j + 1)/2$. When the rational type of player i receives the share x_i at time t of the bargaining stage, her payoff is given by $x_i e^{-r_i t} - d$, where $r_i (> 0)$ is the discount rate of player i. (When the settlement occurs prior to the bargaining stage, her payoff is equal to her share.) If no player accepts her opponent's offer, the players simply lose the bargaining cost and thus their payoffs are given by $-d$. Each player tries to maximize the expected value of the discounted payoff that her rational type obtains. (Recall that the shares received by a player matters only when she is the rational type by assumption.)

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[3] We regard the continuous time model as an approximation of the discrete time model. As shown in Abreu and Gul (2000), the equilibrium play in the latter converges to that of the former as the interval between periods goes to zero. This convergence holds no matter how the players' moves are specified as long as both players get sufficiently frequent chances of concession.

[4] Assuming that the stubborn type not only refuses to accept the opponent's offer but also avoids changing her demands, Abreu and Gul (2000) consider a model in which the rational type of a player can modify their demands during the bargaining. They show that, whenever a rational type of a player changes her demands and thus reveals her type, she immediately accepts her opponent's offer when she is not sure of the opponent's type. As we suppose that the stubborn type commits herself to her initial demand, without loss of generality, we limit the strategy of the rationally types at the bargaining stage to the choice between waiting and accepting, omitting the possibility that the rational types change their demands.

4.1. The bargaining stage: the war of attrition

This subsection studies the war of attrition that occurs at the bargaining stage. To simplify the exposition, when we refer to the probabilities in this subsection, we mean the ones conditional on the event that no player is the chicken type. Namely, in this subsection, the probability that player i is the stubborn type is given by $\xi_i (> 0)$ at the start of the bargaining stage and then (weakly) increases as the time goes by. As shown in Abreu and Gul (2000), when each player is stubborn with a positive probability, the players initiate the war of attrition at the bargaining stage and the *unique* continuation equilibrium proceeds as follows. At time 0, one and only one player may accept the opponent's offer with a positive probability. We call it *mass acceptance.* As in the standard war of attrition, the mass acceptance may occur only at its beginning. If the mass acceptance does not happen, the rational type of player i starts accepting the opponent's offer with the instantaneous rate $\rho_i = \frac{r_j(1-x_i)}{x_1+x_2-1}$, which makes the rational type of the opponent feel indifferent between waiting and accepting her offer. Because the stubborn type never accepts, the cumulative probability of acceptance by the rational type reaches one within a finite time. Because the war of attrition is sustained by the possibility of the opponent's acceptance, that time is common between the two players. Let T be such a time. Then, because only the stubborn type remains at time T, the probability that player i does not accept by time t is given by $\xi_i e^{\rho_i(T-t)}$. Now, for specificity, we suppose that, if a player does the mass acceptance, player j is the one who does so. Because player i does not do the mass acceptance, this probability is one at time 0. On the other hand, for player j, the corresponding probability may be less than one and the difference is the probability of the mass acceptance. Denote it by P_j^{ma}: $P_j^{ma} = 1 - \xi_j e^{\rho_j T}$. This argument also shows that the condition for player i not to do the mass acceptance is given by $\xi_i e^{\rho_i T} = 1 \geq \xi_j e^{\rho_j T}$. Solving the equality part on the left with respect to T, substituting it into the inequality part and then rearranging the terms, we obtain $\rho_i/\rho_j \geq \log \xi_i / \log \xi_j$ from this condition. Substituting the formulas into both ρ_i and ρ_j, we can rewrite it as follows:

$$\frac{1-x_i}{1-x_j} \geq \frac{r_i \log \xi_i}{r_j \log \xi_j}. \tag{1}$$

By deriving T from the above condition and then substituting it into the probability of the mass acceptance, we obtain:

$$P_j^{ma} = 1 - \exp\left\{ \log \xi_j \left(1 - \frac{(1-x_j) r_i \log \xi_i}{(1-x_i) r_j \log \xi_j} \right) \right\}. \tag{2}$$

Note that, when the in equality (1) holds strictly (or with the equality), this probability becomes positive (or zero respectively).

Given the demands $x = (x_1, x_2)$, we denote the continuation payoff of player i that is evaluated at the start of the bargaining stage by $\pi_i^w(x)$. For its computation, we exclude the bargaining cost. As in the standard war of attrition, in its midst ($t > 0$), the continuation payoff of a player is equal to what the other player offers. The next lemma, which is originally proven by Abreu and Gul (2000), summarizes the above argument and describes the players' continuation payoffs at the start of the bargaining stage.

Lemma 1 (Abreu and Gul, 2000)

Suppose that $\xi_1, \xi_2 > 0$ and that $x_1 + x_2 > 1$ and the inequality (1) holds. Then, in the unique continuation equilibrium, $\pi_j^w(x) = 1 - x_i$ and $\pi_i^w(x) = P_j^{ma} x_i + (1 - P_j^{ma})(1 - x_j)$, where P_j^{ma} is given by the formula (2).

Define the just compatible demands $x^c = (x_1^c, x_2^c)$ such that they satisfy the equality version of the inequality (1). It holds that:

$$1 - x_i^c = \frac{r_i \log \xi_i}{r_i \log \xi_i + r_j \log \xi_j}.$$

Using this newly defined term, we can rewrite the inequality (1). Namely, the condition for player i not to do the mass acceptance can be written as

$$\frac{1 - x_i}{1 - x_j} \geq \frac{1 - x_i^c}{1 - x_j^c}. \tag{1'}$$

This shows that the player with the more generous offer relative to the ones given by x^c causes the other to do the mass acceptance. Because the mass acceptance is the only source of the extra payoff over what the other offers, there is an incentive for player i to lower a demand toward x_i^c. This situation is similar to what we have under the equitable rule. Note that, when player j demands no more than x_j^c, it is not possible to cause him to do the mass acceptance. When $x_j < x_j^c$, any incompatible demands imply that $x_i > x_i^c$ because $x_i^c + x_j^c = 1$. Then, player i becomes the one who does the mass acceptance. As shown in a similar model by Kambe (1999), when there is no possibility of the chicken type and when the probabilities of the stubborn type are small, the players' demands in equilibrium become close to x^c. (For the current model, we prove this claim as a part of Proposition

4.) In this sense, we can interpret these demands (x_1^c, x_2^c) as the just compatible demands that reflect their inborn bargaining strength. It is sometimes convenient when we express the probability of the mass acceptance by using x^c. The formula (2) can be expressed as follows:

$$P_j^{ma} = 1 - \exp\left\{\log \xi_j \left(1 - \frac{1 - x_i^c}{1 - x_j^c}\frac{1 - x_j}{1 - x_i}\right)\right\}. \tag{2'}$$

Given x_j, let us define the demand of player i that causes the inequality (1') to hold with the equality and call it $\overline{X}_i(x_j)$. Namely, we have $\overline{X}_i(x_j) = 1 - \frac{1-x_i^c}{1-x_j^c}(1 - x_j)$. This function gives a positive value and is strictly increasing in x_j when $x_j \geq x_j^c$. The condition for player j to do the mass acceptance is that player i makes a demand which is higher than $1 - x_j$ but is lower than $\overline{X}_i(x_j)$, given that player j demands more than x_j^c.

To consider small ξ_i's, we parameterize ξ_i's by $\xi_i = \xi^{\alpha_i}$, where $\xi(> 0)$ is a small number and $\alpha_i(> 0)$ are kept constant $(i \in \{1, 2\})$. Note that, when we change only ξ, x^c remains same. Suppose that the probabilities of the stubborn type are small, i.e., ξ is small and that the inequality (1') holds strictly. Then, as observed in Kambe (1999), the formula (2') indicates that the probability of the mass acceptance for any given incompatible demands becomes close to one. Thus, if player j's demand is higher than x_j^c, player i expects to obtain what she demands with a probability close to one when she demands some amount between $1 - x_j$ and $\overline{X}_i(x_j)$. From Lemma 1 and the definition of $\overline{X}_i(x_j)$, we know that player i's payoff is $1 - x_j$ when $x_i > \overline{X}_i(x_j)$. Hence, when $x_j > x_j^c$ and the probabilities of the stubborn type are small, the highest payoff for player i is close to $\overline{X}_i(x_j)$ and is attained by demanding slightly less than $\overline{X}_i(x_j)$. The next lemma proves this claim formally.

Lemma 2

Suppose that $x_j > x_j^c + \triangle$ for some $\triangle > 0$. For any $\delta > 0$, there exists $\underline{\xi}$ such that, for any $\xi < \underline{\xi}$ and any x_j,

(i) $\overline{X}_i(x_j) - \delta < \text{argmax}_{x_i}\ \pi_i^w(x) < \overline{X}_i(x_j)$, and

(ii) $\overline{X}_i(x_j) - \delta < \max_{x_i}\ \pi_i^w(x) < \overline{X}_i(x_j)$.

In light of the equation (1'), the lemma implies that, when $\xi \approx 0$, the payoff of a player in the bargaining stage is determined by the offer that is more generous relative to x^c in the sense of the equation (1'). Note that, when the players are symmetric ($\xi_1 = \xi_2$ and

$r_1 = r_2$), we have $\overline{X}_i(x_j) = x_j$. Hence, in the symmetric-player case, the limit result is equivalent to the equitable rule. In this sense, the following analysis can be interpreted as the extension of the equitable rule to a more general version with the non-cooperative game-theoretic foundation.

4.2. The optimal demands

Now that we know the continuation payoffs at the bargaining stage, we study the optimal demands in this subsection.

In the similar way as assumed in Section 3, we assume that $\hat{x}$ is sufficiently high. In particular, we suppose that the demand that the chicken type yields to is more attractive to the just compatible demand that reflects one's inborn bargaining power.

Assumption B

$\hat{x} > \max\{x_1^c, x_2^c\}$.

We first show that, when $\gamma_1 = 0$, it does not matter whether player 2 has a positive probability of the chicken type or not as long as it is small. This is similar to what we find in Proposition 1. As in Proposition 1, the players make the just compatible demands and the first mover has some advantage due to the bargaining cost.

Specifically, the next proposition shows that, when player 1 is never the chicken type, the players' demands are close to (x_1^c, x_2^c), the just compatible demands that reflect their inborn bargaining strength, and the first-mover advantage is a fraction of the bargaining cost and is equal to $x_2^c d$.

Proposition 4

Suppose that Assumption B is satisfied and that $\gamma_1 = 0 \leq \gamma_2 < x_1^c$.

(i) There exists $\underline{\xi}$ such that for any $\xi < \underline{\xi}$, player 2 makes the just compatible demand in equilibrium.

(ii) For any $\delta > 0$, there exists $\underline{\xi}$ such that for any $\xi < \underline{\xi}$, we have $x_1 \in (x_1^c + x_2^c d, x_1^c + x_2^c d + \delta)$.

As shown by Lemma 2, player 2 has an incentive to make his demand more modest in the sense of the inequality (1') when player 1 makes a high demand. This makes player 1's payoff lower when player 2 turns out not to be the chicken type. As in Proposition 1, when the probability that player 2 is the chicken type is small, this prevents player 1 from exploiting the possibility of the chicken type. Note that the condition for the probability of the chicken type is different from the one specified in Proposition 1. Because of the asymmetry, player 2 does not simply underbid player 1 when he wants to cause the mass acceptance by player 1. When x_1^c is higher, player 2 tends to increase his demand by a larger multiplier given incompatible demands. Then, the loss that player 1 expects against the rational type is higher, which discourages her from making a higher demand. This is why we need $\gamma_2 < x_1^c$ for this proposition.

The lower bound of the first mover advantage in the above proposition is determined by the equation $1 - x_1 = \overline{X}_2(x_1) - d$. The left hand side is the payoff that player 2 obtains when he responds to player 1's demand by the just compatible demand. The right hand side is the supremum of player 2's payoff when he responds to it by an incompatible demand and causes player 1 to do the mass acceptance. (The supremum is obtained when the probabilities of the stubborn type go to zero.) Lemma 2 has shown that it is given by $\overline{X}_2(x_1) - d$. Note that the left hand side is decreasing in x_1 and the right hand side is increasing in it. Hence, in the limit, only when x_1 is no higher than the one specified by this equation, player 2 will respond to it by the just compatible demand. When the probabilities of the stubborn type is positive and the probabilities of the mass acceptance is less than one, player 2's payoff from any incompatible demand becomes less than $\overline{X}_2(x_1) - d$ and thus the threshold computed above gives the lower bound for the first mover advantage.

Next, we show that, when the first mover may be the chicken type, the players do make incompatible demands. This is similar to what we find in Proposition 2. The counterpart of x^* is given by the following definition: $x_1^{**} \equiv \frac{x_1^c - (1-x_1^c)\gamma_1(1-\hat{x})}{1-\gamma_1+\gamma_1 x_1^c}$. In case of the symmetric players ($x_1^c = x_2^c = 1/2$), we naturally have that $x_1^* = x_1^{**}$. When the probability of the stubborn type is small, player 1's demand is close to it and player 2 responds to it by demanding $\hat{x}$. As in Proposition 2, player 1 wants to increase his demand as long as player 2 does not respond to it by lowering his demand from $\hat{x}$ and causing player 1 to do the mass acceptance. On the other hand, player 2 makes the highest demand that the chicken type yields to.

Proposition 5

Suppose that Assumption B is satisfied and that that $\gamma_2 = 0 < \gamma_1$. We assume that $d < \gamma_1(\hat{x} + x_1^c - 1)/(1 - \gamma_1 + \gamma_1 x_1^c)$.

(i) There exists $\underline{\xi}$ such that for any $\xi < \underline{\xi}$, we have $x_2 = \hat{x}$ and $x_1 + x_2 > 1$ in equilibrium.

(ii) For any $\delta > 0$, there exists $\underline{\xi}$ such that for any $\xi < \underline{\xi}$, we have $x_1 \in (x_1^{**} - \delta, x_1^{**} + \delta)$.

The supposition that $d < \gamma_1(\hat{x} + x_1^c - 1)/(1 - \gamma_1 + \gamma_1 x_1^c)$ is the requirement that the bargaining cost is sufficiently low so that player 2 is willing to exploit the possibility of the chicken type. This is similar to what we suppose in Section 3 for the demands to be incompatible.

The next proposition is the counterpart of the first part of Proposition 3. Namely, when both players have the possibility of the chicken type and the bargaining cost is sufficiently small, the equilibrium demands are same as the ones made when only the first mover has that possibility.

Proposition 6

Suppose that Assumption B is satisfied and that $0 < \gamma_i$ for $i \in \{1, 2\}$ and moreover $\gamma_2 < x_1^c$. We assume that $x_1^{**} < \hat{x}$ and that $d < \gamma_1(\hat{x} + x_1^c - 1)/(1 - \gamma_1 + \gamma_1 x_1^c)$, and that $\gamma_2 < (x_1^{**} + \hat{x} - 1)/(2\hat{x} - 1)$.

(i) There exists $\underline{\xi}$ such that for any $\xi < \underline{\xi}$, we have $x_2 = \hat{x}$ and $x_1 + x_2 > 1$ in equilibrium.

(ii) For any $\delta > 0$, there exists $\underline{\xi}$ such that for any $\xi < \underline{\xi}$, we have $x_1 \in (x_1^{**} - \delta, x_1^{**} + \delta)$.

The proposition holds under several conditions. The condition that $\gamma_2 < x_1^c$ is same as assumed in Proposition 4. It requires that the possibility that player 2 is the chicken type is not so high that player 1 fears the underbidding by player more than wants to exploit the possibility that player 2 is the chicken type. In the setting of Proposition 5, we show that player 1 demands x_1^{**} such that $x_1^{**} < \overline{X}_2^{-1}(\hat{x})$. In the symmetric case, it implies that $x_1^{**} < \hat{x}$. If this inequality is not satisfied, the chicken type of player 2 walks away from the negotiation given x_1^{**}. In order to avoid the complexity that it causes, we assume that $x_1^{**} < \hat{x}$. The condition that $d < \gamma_1(\hat{x} + x_1^c - 1)/(1 - \gamma_1 + \gamma_1 x_1^c)$ is similar to the one in Proposition 5. As in the relationship between Proposition 2 and Proposition 3, the condition is relaxed due to the possibility that player 2 is the chicken type. Finally, the condition that $\gamma_2 < (x_1^{**} + \hat{x} - 1)/(2\hat{x} - 1)$ is required.

Let us review the relationship between the effect of the chicken type on the bargaining outcome and that of the stubborn type. When the first mover has no possibility to be the chicken type, (i.e., the situation studied in Proposition 4), the chicken type does not matter but the stubborn type does. On the other hand, when the first mover has the possibility to be the chicken type, (i.e., the situation studied either in Proposition 5 or in Proposition 6), the chicken type as well as the stubborn type matters. Specifically, the demand of the second mover is affected only by the parameter specific to the chicken type while that of the first mover is affected by the properties of both types. The latter result is contrary[5] to what is found in Kambe (1999).

5. The simultaneous demand game under the equitable rule

In this section, we study the simultaneous demand game and suppose that the equitable rule is used to determine the term of settlement at the bargaining stage.

Before conducting the analysis, we introduce a set of notations and add some further structure to the basic model described in Section 2 in order to deal with the complexity caused by the simultaneous move.

Because a player may want to underbid the opponent's demand which is made to underbid her demand in the simultaneous demand game, we need to consider the demands that are smaller than a demand by several monetary units. Let ϵ be the monetary unit as before. Let N be the set of natural numbers and let N^+ be the set of non-negative integers: $N \equiv \{1, 2, \ldots\}$ and $N^+ \equiv \{0\} \cup N$. For $n \in N^+$, we consider $x - n\epsilon$ as the demand that is lower than x by n monetary units. We denote $[x]_{n-}$ as the limit when the monetary unit goes to zero. Note that $[x]_{0-} \equiv x$ and we denote $[x]_{1-}$ by $[x]_-$ to maintain the previous notation. By the construction, when $n < m$, $[x]_{n-}$ is bigger than $[x]_{m-}$ and, when both demands $[x]_{n-}$, their demands are regarded as identical. Given this ordering, we apply the equitable rule with the interpretation that the payoff determined by $[x]_{n-}$ is the limit as ϵ goes to zero. We assume[6] that, for any x, each player demands $[x]_{n-}$ with a positive

[5] Suppose that the probability of the chicken type is small. Then, the supposition in the proposition requires that the bargaining cost needs to be small. Moreover, the demand of player 1 is close to what she would demand when there were no possibility of the chicken type. Considering that the probability of the mass acceptance by player 2 is close to one for a small probability of the stubborn type, the payoffs of the players are close to those without the possibility of the chicken type. Namely, despite the discontinuous change in the demand of player 2, the payoffs change continuously with respect to the probability of the chicken type.

[6] We make this assumption in order to capture the notion of underbidding by a small margin. When the players underbid their demands recursively around a particular amount, this assumption is violated. However, I think that the latter situation should be formulated as the randomization of demands over some interval.

probability for at most one $n \in N^+$; when $\text{Prob}(x_i = [x]_{n-}) > 0$ for some $n \in N^+$, it holds that $\text{Prob}(x_i = [x]_{m-}) = 0$ for any $m > n$. Moreover, we assume that the players prefer a higher demand as long as the ordering against the opponent's demand is same; when $\text{Prob}(x_i = [x]_{n-}) > 0$ for $n \geq 1$, it has to hold that $\text{Prob}(x_j = [x]_{(n-1)-}) > 0$. Because of these assumption, on the equilibrium path, at most $[x]_-$ is made in addition to x itself with a positive probability. Even when we consider deviations, we need to include up to $[x]_{2-}$ and in the subsequent analyses we never use $[x]_{n-}$ for $n > 2$. (In the equilibrium derived later, no player makes $[x]_{n-}$ with a positive probability for any $n \in N$.)

Denote the probability that player i demands $x \in [0,1]^\infty$ by $P_i(x)$. (We have the infinite dimension because of the possibility of the complex underbidding introduced above.) Define the set of the demands that are made with a positive probability by player i to be Ω_i: $\Omega_i \equiv \{x \in [0,1]^\infty \mid P_i(x) > 0\}$. Denote the density of player i's demand by $f_i(x)$ for $x \in [0,1]$. It holds that

$$\int_0^1 f_i(x)dx + \sum_{x \in \Omega_i} P_i(x) = 1.$$

As a convention, we say that a player makes a demand (in equilibrium) either when it is made with a positive probability or any interval including this demand has a positive probability.

We make a slightly stronger assumption than Assumption A in this section.

Assumption A'

(i) $\xi_i = 0$ for $i \in \{1,2\}$, and

(ii) $\hat{x} > 1/2 + 2d$.

As a reference point, we first study the case without the possibility of the chicken type. In such a case, the players have an incentive to underbid the other as long as their demands are not compatible. Because of this, in any pure-strategy equilibrium, the players' demands come close to one half and they become compatible.

Proposition 7

Suppose that Assumption A' is satisfied and also that $\gamma_1 = \gamma_2 = 0$.

In any pure-strategy equilibrium, the players make the just compatible demands and the game ends immediately. Player i's demand x_i is supported in a pure-strategy equilibrium if and only if $x_i \in [(1-d)/2, (1+d)/2]$.

The logic of this proposition is similar to that of Proposition 0. Due to the incentive to underbid a higher demand of the other, the equilibrium demands come down to the middle. On the other hand, because of the bargaining cost, when the other does not demand more than one half plus half of the bargaining cost $(1+d)/2$, the underbidding does not improves the payoff of the player who does it. In the sequential demand game, this leads to the first mover advantage. In the simultaneous demand game, it leads to the range of equilibria in which the payoffs are located around $(1/2, 1/2)$ with the width of d.

Next, let us introduce the possibility of the chicken type. Due to the simultaneity, the player who wants to exploit the possibility of the chicken type can do so without fearing the increased demand by the opponent in response to her demand. In this sense, the situation is similar to the sequential demand game in which the player who can be the chicken type move first. This generally causes the player whose opponent may be the chicken type to make a high demand. The divergence from the prediction of the sequential demand game arises because, knowing such possibility, the other player wants to underbid it by a slightly lower demand. This in turn creates an incentive for the player to underbid it by a small margin. This logic shows that the players randomize their demands in the simultaneous demand game when there is the possibility of the chicken type. It also implies that the players make incompatible demands in contrast with the above case without the possibility of the chicken type. The next proposition characterizes the unique equilibrium, which involves the mixed strategies at the demand stage.

Proposition 8

Suppose that Assumption A' is satisfied, that $\gamma_{max} \equiv \max\{\gamma_1, \gamma_2\} > 0$, and that $d < \gamma_{max}^3(2\hat{x}-1)/\big(2(5-3\gamma_{max})\big)$. Then, the unique equilibrium takes the following form.

There exists $m(> 1/2 + 2d)$ such that both players randomize their demands over the interval $(m, \hat{x})$. When $\gamma_i \leq \gamma_{max}$, player i demands $\hat{x}$ with the probability $\frac{\gamma_{max}-\gamma_i}{1-\gamma_i}$. Except for this, there is no demand which is made with a positive probability. The lower end of the interval is given by $m = \frac{1+\gamma_{max}^2(2\hat{x}-1)}{2}$. Player k randomizes his demand according to $F_k(x) = \frac{1}{1-\gamma_k}\left(1 - \frac{\gamma_{max}\sqrt{2\hat{x}-1}}{\sqrt{2x-1}}\right)$.

As shown in the proposition, even the player whose opponent is known not to be the chicken type demands more than one half and the demands always become incompatible as both in Proposition 2 and in Proposition 3. The difference from the results obtained in those proposition is that, in the simultaneous demand game, the players' demands have non-degenerate distributions. This is because the players try to underbid high demands of the other without knowing the exact demand that the other makes. Each player has an incentive to underbid the demand of the other and thus the equilibrium cannot involve pure strategies.

To understand how the randomization occurs, the remainder of this section explains how we derive the equilibrium strategies of the players.

When the bargaining cost d is sufficiently small, the appendix proves that both players randomize their demands in the same interval between $(m, \hat{x})$ where $m > 1/2$ without any gap and that making a demand with a positive probability occurs only at $\hat{x}$ for at most one player. Let us briefly explain how these properties hold. First, let us explain why the distribution should have the same support $(m, \hat{x})$. The upper bound is given by the highest demand that the chicken type yields to because the driving force behind higher demands is the desire to exploit the chicken type. Moreover, the support should be same for both players because the randomization is caused by the desire to underbid the demand of the other and thus any player does not want to make too low a demand. Next, we argue that there should be no gap in distribution. To understand this, suppose that player i does not make demands in (a, b) even though she makes demands both lower than a and higher than b. Her opponent does not make demands in this range because demanding either b or $[b]_-$ improves his payoff against the player i's demands higher than b and does not affect it otherwise. However, given that both players do not make demands in (a, b), the similar logic as above implies that a player wants to switch her demands from $a - \epsilon$ to b (or $[b]_-$) for sufficiently small $\epsilon(> 0)$. It implies that a gap in distribution cannot be sustained in equilibrium. Finally, we explain why there should not be any demand that is lower than $\hat{x}$ and is made with a positive probability. If player i would demand $x < \hat{x}$ with a positive probability, player j would not make demands in $[x, x + \epsilon')$ for sufficiently small $\epsilon > 0$ and instead would demand $[x]_-$ in order to underbid x. This would create a gap in distribution, which should not occur in equilibrium as argued above.

Now that we know the basic properties of the distribution functions, we derive the concrete forms of the players' equilibrium strategies. Let $f_i(x)$ be the density function of player i's demand. Because making a demand with a positive probability may occur only at the upper end of the interval, we incorporate it in the cumulative distribution of demands.

Namely, we define $F_i(x) \equiv \int_m^x f(y)dy$ for any $x < \hat{x}$ and define $F_i(\hat{x}) \equiv 1$. In this definition, the probability of demanding $\hat{x}$ is given by $1 - F([\hat{x}]_-)$. For any $x \in (m, \hat{x})$, the expected payoff of player i is given by:

$$E\Pi_i(x_i) \equiv \gamma_j x_i + (1-\gamma_j)\left\{(x_i - d)\big(1 - F_j(x_i)\big) + \int_m^{x_i} (1-y-d) f_j(y)dy\right\}.$$

Because there is no gap, it implies that the expected payoff is constant in the interval $(m.\hat{x})$. The above expected payoff is differentiable with respect to x_i. Hence, we need to have the derivative with respect to x_i being equal to zero for any $x \in (m, \hat{x})$:

$$\frac{dE\Pi_i(x)}{dx_i} = 1 - (1-\gamma_j)F_j(x) + (1-\gamma_j)(1-2x)f_j(x) = 0.$$

Rearranging the terms of the second equality, we have:

$$\frac{(1-\gamma_j)f_j(x)}{1-(1-\gamma_j)F_j(x)} = \frac{1}{2x-1}.$$

This can be viewed as a differential equation and the next equation provides its solution with $A_j > 0$ as the parameter:

$$F_j(x) = \frac{1}{1-\gamma_j}\left(1 - \frac{A_j}{\sqrt{2x-1}}\right).$$

The value of the parameter is determines by who demands $\hat{x}$ with a positive probability. Because of the possibility of underbidding, at most one player does so. Suppose that player j does not do so. Then, the above formula implies that:

$$1 = F_j(\hat{x}) = F_j([\hat{x}]_-) = \frac{1}{1-\gamma_j}\left(1 - \frac{A_j}{\sqrt{2\hat{x}-1}}\right).$$

Solving this with respect to A_j, we have:

$$A_j = \gamma_j\sqrt{2\hat{x}-1}.$$

Note that $\gamma_j > 0$ has to hold. Otherwise, it would hold that $A_j = 0$ and the path would not be well defined. (By the same argument, we can conclude that, if $\gamma_j = 0$, it has to hold that $F_j([\hat{x}]_-) < 1$, i.e., the player without the possibility of the chicken type has to demand $\hat{x}$ with a positive probability.) If player j does not demand $\hat{x}$ with a positive probability, in order for player i to have an incentive to make high demands close to $\hat{x}$ which can be underbidden with high probabilities, it is necessary that player j has a positive probability to be the chicken type. Namely, player i's high demands are rewarded by the possibility of yielding. Supposing that $\gamma_j > 0$ and $A_j = \gamma_j\sqrt{2\hat{x}-1}$, we now look at the condition for the lower end of the distribution. Because $F_j(m) = 0$, it holds that:

$$0 = F_j(m) = \frac{1}{1-\gamma_j}\left(1 - \frac{\gamma_j\sqrt{2\hat{x}-1}}{\sqrt{2m-1}}\right).$$

Solving this, we can show that the lower end of the distribution is given by:

$$m = \frac{1+\gamma_j^2(2\hat{x}-1)}{2}.$$

Now look at the distribution of player i's demands. Because $F_i(m) = 0$, we have:

$$0 = F_i(m) = \frac{1}{1-\gamma_i}\left(1 - \frac{A_i}{\sqrt{2m-1}}\right).$$

By solving this equation and substituting the formula derived above into this, we have:

$$A_i = \sqrt{2m-1} = \gamma_j\sqrt{2\hat{x}-1}.$$

Observe that:

$$F_i([\hat{x}]_-) = \frac{1}{1-\gamma_i}\left(1 - \frac{A_i}{\sqrt{2\hat{x}-1}}\right) = \frac{1-\gamma_j}{1-\gamma_i}.$$

Because $F_i([\hat{x}]_-) \leq 1$, this is well defined only when $\gamma_j \geq \gamma_i$. Namely, if player j does not demand $\hat{x}$ with a positive probability, player j's probability to be the chicken type has to be no lower than player i's: $\gamma_i \leq \gamma_j = \gamma_{max}$. Player i demands $\hat{x}$ with the probability $1 - F_i([\hat{x}]_-) = \frac{\gamma_j - \gamma_i}{1-\gamma_i}$.

This derivation shows that the incentive to demand $\hat{x}$ is caused by the desire to exploit the chicken type. Moreover, for any demand that is made in equilibrium and is lower than $\hat{x}$, the incentive to underbid is exactly offset by the incentive to maintain a higher demand to obtain a higher share against the chicken type as well as the rational type with even higher demands. The gain of the share by the underbidding decreases as the demand becomes lower. In order to keep the balance between these competing incentives, the probability of successful underbidding needs to increase. Hence, the probability density becomes higher as the demands become lower.

Using the characterization above, let us investigate the property of the equilibrium that occurs in the simultaneous demand game under the equitable rule. Denote the expected payoff that the rational type of player i obtains by $E\Pi_i$. Because the expected payoffs from the demands that she makes in equilibrium should be identical, they are identical to the one from the demand of m:

$$E\Pi_i = E\Pi_i(m) = \gamma_j m + (1-\gamma_j)(m-d) = m - (1-\gamma_j)d,$$

where $m = \frac{1+\gamma_{max}^2(2\hat{x}-1)}{2}$. Because $m > 1/2 + 2d$, it implies that player i obtains more than one half as her payoff. Note that both players benefit from the possibility that one player is the chicken type, which is similar to the result that we obtain in Proposition 2 (and its corollary).

The above implies that the expected payoff of a player increases in the maximum probability of the chicken type. To understand the effect of the maximum probability of the chicken type, we look both at the case that it is very low and at the case that it is very high. First, consider the case that it is close to zero. Then, both γ_i and γ_j are close to zero. The bargaining cost also needs to be close to zero in order to satisfy the required condition. Given those, the distribution of demands is concentrated near one half and the payoff of any player becomes close to 1/2. Namely, when the probabilities of the chicken type is small and also when the bargaining cost is small, the outcome are on average similar to the one without such possibility. As the other extreme case, consider the case that the maximum probability of the chicken type is close to one. The lower end of the distribution, m, becomes close to $\hat{x}$. (The distribution of demands is concentrated near $\hat{x}$.) When player j has the higher probability to be the chicken type, his expected payoff is close to $\hat{x} - (1-\gamma_i)d$. On the other hand, the expected payoff of player i is close to $\hat{x}$ as γ_j is close to one. In the sense that the higher demands tend to be made with a higher chance of the chicken type, this is similar to what we have in Proposition 2 (and also in Proposition 3).

The next corollary summarizes these observations.

Corollary 2

Suppose that Assumption A' is satisfied, that $\gamma_{max} > 0$, and that $d < \gamma_{max}^3(2\hat{x} - 1)/\big(2(5-3\gamma_{max})\big)$.

(1) It holds that $E\Pi_i = \frac{1+\gamma_{max}^2(2\hat{x}-1)}{2} - (1-\gamma_j)d > 1/2$ for $i \in \{1,2\}$.

(2) For $i \in \{1,2\}$, $E\Pi_i$ increases in γ_{max} and it holds that $E\Pi_i \approx 1/2$ when $\gamma_{max} \approx 0$. When $\gamma_i \leq \gamma_j = \gamma_{max} \approx 1$, it holds that $E\Pi_i \approx \hat{x}$ $E\Pi_j \approx \hat{x} - (1-\gamma_i)d$.

6. Concluding remarks

We have shown that the possibility of the chicken type may cause incompatible demands and may affect the players' payoffs in bargaining. In particular, we have shown that its effect is substantially affected by the ordering at the demand stage. On the other hand, our result shows that the effect of the chicken type is limited by its probability. To understand this point, we can note that the rational type of player i can secure herself the payoff of $1/2-d$ by demanding $1/2$ under the equitable rule and a payoff close to x_i^c-d by demanding a little lower than x_i^c in the game with the war of attrition. Hence, the sum of the payoffs that the rational type of the two players secure themselves is close to one. Thus, in the sense that the gain from exploiting the chicken type is the driving force for the strategic behaviors, the effect of the chicken type is proportional to the magnitude of its possibility. This makes a contrast with the case of the stubborn type. In the war of attrition, its effect does not decrease even when the probabilities of the stubborn type becomes small for both players (as long as their ratio between the players does not change as shown in Kambe, 1999). The difference is caused because the rational type wants to mimic the stubborn type but not the chicken type.

One of the non-trivial assumptions is the choice of timing for the learning of private information. We have assumed that the players learn their types only after they make their demands. Because of this assumption, the players' demands do not have the signalling effect. In order to examine the importance of this assumption, for the moment, let us suppose that the players know their types at the beginning of the game. First, either when the first mover has no possibility of the chicken type in the sequential demand game or when the players make their demands simultaneously, the chicken type yields right after she makes her demand. Hence, it is reasonable to assume that she makes a non-consequential demand such as $\hat{x}$ at the demand stage. With this assumption, our results hold as they are. Next, consider the sequential demand game in which the first mover has the possibility of the chicken type. The question is whether the first mover wants to signal her types or not. The chicken type has no incentive to signal her type as it never improves her payoff. On the other hand, the analysis in Section 3 indicates that even the rational type of the first mover benefits from the possibility of the chicken type when the bargaining cost is sufficiently small. In that case, she has no incentive to signal her type. This informal discussion indicates that both the chicken type and the rational type are likely to make the same demand in equilibrium even with some refinement when the bargaining cost is sufficiently small.

We do not analyze the simultaneous demand game in which the bargaining is done through the war of attrition. It is obvious that a pure strategy equilibrium does not exist when the bargaining cost is sufficiently small. Beyond that, the analysis becomes too complex. Though we conjecture that we can prove a counterpart of Proposition 8 in such a situation, its formal analysis is left for the future research.

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Appendix: the proofs

We collect all the proofs in this appendix. Throughout the appendix, we denote the expected payoff of player i at the beginning of the game by $\Pi_i(x_1, x_2)$. When the argument of this function is obvious, we sometimes omit it.

A.1. The proof of Proposition 0

First, we derive the best response of player 2 at the demand stage given player 1's demand x_1. For player 2, making a demand such that $x_2 \geq x_1$ and $x_1 + x_2 > 1$ is dominated by making the just compatible offer ($x_2 = 1 - x_1$). By doing so, player 2 receives the same share at the demand stage and avoids the bargaining cost. It is easy to see that a demand lower than $1 - x_1$ is not optimal for player 2 as it is dominated by the just compatible demand $1 - x_1$. As the consequence of these, if $x_1 \leq 1/2$, it is best for player 2 to make the just compatible demand. Consider the case that $x_1 > 1/2$. If player 2 makes an incompatible offer, it is best for him to make the demand $[x_1]_-$. By such a demand, player 2 obtains $x_1 - d$. On the other hand, if he makes the just compatible offer, he obtains $1 - x_1$. By comparison, we can conclude that player 2 makes the just compatible offer if $x_1 \leq (1+d)/2$ and he makes the demand $[x_1]_-$ if $x_1 > (1+d)/2$. This conclusion encompasses the case that $x \leq 1/2$. Note that, when $x_1 = (1+d)/2$, the payoff from the just compatible offer becomes equal to that from $[x_1]_-$ in the limit. However, as the latter represent the value in the limit and player 2's payoff is lower than that value except in the limit, player 2 prefers the just compatible offer when this equality holds.

Next, consider player 1's strategy at the demand stage. When she makes the demand higher than $(1+d)/2$, player 2 will underbid player 1 and thus player 1 will obtain $1 - x_1 - d$. Because $(1+d)/2 > 1/2$, this is strictly lower than one half. On the other hand, when her demand is no bigger than $(1+d)/2$, player 2 will make the just compatible demand. Then, her payoff is x_1. This is maximized when $x_1 = (1+d)/2$. Because it is larger than one half, this is the optimal demand for player 1.

Given $x_1 = (1+d)/2$, player 2's best response is to make the just compatible offer as shown above. Because the demands are compatible, they reach the settlement at the demand stage.

A.2. The proof of Proposition 1

Because player 1 is the rational type and because player 2 only cares about the payoff of his rational type by assumption, player 2's best response at the demand stage is identical to the one derived in the proof of Proposition 0.

Now, consider player 1's strategy at the demand stage. When she makes the demand higher than $\hat{x}$, the chicken type of player 2 leaves the negotiation without an agreement. Moreover, for player 1's demand in this range, the best response of player 2 makes player 1's payoff decreasing in her demand. Hence, demanding more than $\hat{x}$ is dominated by demanding $\hat{x}$. When she makes the demand higher than $(1+d)/2$ but no more than $\hat{x}$, player 2 will underbid player 1 by demanding $[x_1]_-$. At the yielding stage, the chicken type of player 2 yields to player 1's demand. Thus, player 1 obtains $\gamma_2 x_1 + (1-\gamma_2)(1-x_1-d)$. Because $\gamma_2 < 1/2$, this is strictly decreasing in x_1. Hence, it holds that $\sup_{(1+d)/2<x_1\leq\hat{x}} \gamma_2 x_1 + (1-\gamma_2)(1-x_1-d) \leq \gamma_2 \frac{1+d}{2} + (1-\gamma_2)\left(1-\frac{1+d}{2}-d\right) < \frac{1+d}{2}$. On the other hand, when her demand is no bigger than $(1+d)/2$, player 2 will make the just compatible demand. Then, player 1's payoff is x_1. This is maximized when $x_1 = (1+d)/2$. By comparing these demands, we can conclude that demanding $(1+d)/2$ is the optimal strategy for player 1.

Given $x_1 = (1+d)/2$, player 2's best response is to make the just compatible offer as shown above. Because the demands are compatible, they reach the settlement at the demand stage.

A.3. The proof of Proposition 2

First, we derive the best response of player 2 at the demand stage given player 1's demand x_1. It is easy to see that a demand lower than $1-x_1$ is not optimal for player 2 as it is dominated by the just compatible demand $1-x_1$. In the following, we restrict our attention to player 2's demands no lower than $1-x_1$. We study several cases depending on the value of x_1 in turn.

(i) $x_1 \leq 1-\hat{x}$

When $x_2 > 1-x_1$, it holds that $x_2 > \hat{x}$. Hence, any demand higher than $1-x_1$ gives zero payoff to player 2 when the other player is the chicken type. Because $\hat{x} > 1/2$ by assumption and thus $x_2 > x_1$, the payoffs at the bargaining stage are determined by player 1's demand. Hence, the expected payoff of player 2 is $(1-\gamma_1)(1-x_1-d)$. It is lower than

$1 - x_1$, which player 2 can obtain by making the just compatible demand. Therefore, for player 1's demand in this range, making the just compatible demand is the best response for player 2.

(ii) $1 - \hat{x} < x_1 \leq \hat{x}$

If player 2 makes an incompatible demand and moreover demands no less than player 1 does ($x_1 + x_2 > 1$ and $x_2 \geq x_1$), it is best for him to demand $\hat{x}$. It is because it attains the highest payoff against the chicken type and keeps the same payoff against the rational type.

If player 2 makes an incompatible demand and moreover demands less than player 1 does ($x_1 + x_2 > 1$ and $x_2 < x_1 \leq \hat{x}$), it is best for him to demand $[x_1]_-$. It is because it attains the highest payoff against both types among those demands satisfying the stated condition. Note that this situation arises only when $x_1 > 1/2$.

These arguments show that the candidates for player 2's best response are making the just compatible demand, demanding $\hat{x}$ and demanding $[x_1]_-$. We now compare player 2's payoffs when he makes his demand among these.

When $x_2 = \hat{x}$, the demands are incompatible and $x_1 \leq x_2$. By the equitable rule, player 2 obtains $1 - x_1$ at the bargaining stage. On the other hand, if player 1 is the chicken type, he obtains $\hat{x}$ at the yielding stage. Hence, his expected payoff at the beginning is $\Pi_2(x_1, \hat{x}) = \gamma_1 \hat{x} + (1 - \gamma_1)(1 - x_1 - d)$. Using this, we compute the difference between player 2's payoff from the just compatible demand and that from $\hat{x}$:

$$\begin{aligned}\Pi_2(x_1, 1 - x_1) - \Pi_2(x_1, \hat{x}) &= 1 - x_1 - \gamma_1 \hat{x} - (1 - \gamma_1)(1 - x_1 - d) \\ &= \gamma_1(1 - \hat{x}) + (1 - \gamma_1)d - \gamma_1 x_1.\end{aligned}$$

When $x_1 = 1 - \hat{x}$, the difference is equal to $(1 - \gamma_1)d$, which is positive. The difference is decreasing in x_1. Hence, when $x_1 < (>)(\gamma_1(1 - \hat{x}) + (1 - \gamma_1)d)/\gamma_1$, making the just compatible demand is better (worse respectively) for player 2 than making the demand of $\hat{x}$. (He is indifferent between these when the equality holds.) Let us denote the threshold by $\tilde{x}_1$:

$$\tilde{x}_1 \equiv \frac{\gamma_1(1 - \hat{x}) + (1 - \gamma_1)d}{\gamma_1}.$$

When $x_1 > 1/2$ and $x_2 = [x_1]_-$, the demands are incompatible. By the equitable rule, player 2 obtains x_1 at the bargaining stage. (Strictly speaking, this payoff is an approximation because the payoff from $[x_1]_-$ is evaluated in the limit. Following the convention stated in the main text, we often omit the explicitly reference to this type of qualification in the following proofs.) Hence, his expected payoff at the beginning is $\Pi_2(x_1, [x_1]_-) = \gamma_1 x_1 + (1-\gamma_1)(x_1 - d)$. Using this, we compute the difference between player 2's payoff from $\hat{x}$ and that from $[x_1]_-$:

$$\begin{aligned}\Pi_2(x_1, \hat{x}) - \Pi_2(x_1, [x_1]_-) &= \gamma_1 \hat{x} + (1-\gamma_1)(1 - x_1 - d) - \gamma_1 x_1 - (1-\gamma_1)(x_1 - d) \\ &= \gamma_1 \hat{x} + 1 - \gamma_1 - (2-\gamma_1)x_1.\end{aligned}$$

When $x_1 = 1/2$, the difference is equal to $\gamma_1 \hat{x} - \gamma_1/2$, which is positive. When $x_1 = \hat{x}$, the difference is equal to $2(\gamma_1 - 1)\hat{x} + 1 - \gamma_1 = (1-\gamma_1)(1-2\hat{x})$, which is negative. Hence, the difference becomes equal to zero between $1/2$ and $\hat{x}$. Specifically, it is equal to zero at $x_1 = x_1^* \equiv (\gamma_1 \hat{x} + 1 - \gamma_1)/(2-\gamma_1)$. By construction, it holds that $1/2 < x^* < \hat{x}$. When $x_1 < x_1^*$, the difference is positive and thus the demand of $\hat{x}$ is better for player 2. On the other hand, when $x_1 > x_1^*$, the difference is negative and thus the demand of $[x_1]_-$ is better for player 2. When the difference is equal to zero, it means that the payoffs from these two strategies are identical in the limit. Hence, except in the limit, the demand of $\hat{x}$ is better for player 2.

As the remaining combination, suppose that $x_1 > 1/2$ and compare player 2's payoff from the just compatible demand with that from $[x_1]_-$. The above argument shows that the difference between these is given by the following:

$$\Pi_2(x_1, 1-x_1) - \Pi_2(x_1, [x_1]_-) = 1 - x_1 - \gamma_1 x_1 - (1-\gamma_1)(x_1 - d) = 1 - 2x_1 + (1-\gamma_1)d.$$

When $x_1 = 1/2$, the difference is equal to $(1-\gamma_1)d$, which is positive. When $x_1 = \hat{x}$, the difference is equal to $1 - 2\hat{x} + (1-\gamma_1)d < 2((1+d)/2 - \hat{x})$, which is negative by Assumption A. Hence, the difference becomes equal to zero between $1/2$ and $\hat{x}$. Specifically, it is equal to zero at $x_1 = (1 + (1-\gamma_1)d)/2$. When $x_1 < (1 + (1-\gamma_1)d)/2$, the difference is positive and thus making the just compatible demand is better for player 2. On the other hand, when $x_1 > (1 + (1-\gamma_1)d)/2$, the difference is negative and thus demanding $[x_1]_-$ is better for player 2. When the difference is equal to zero, it means that the payoffs from these two strategies are identical in the limit. Hence, except in the limit, making the just compatible demand is better for player 2.

We compare the three thresholds derived above for player 2's best response: $\tilde{x}_1$, x_1^*, and $(1+(1-\gamma_1)d)/2$. First, compare the first two:

$$\begin{aligned} x_1^* - \tilde{x}_1 &= \frac{\gamma_1 \hat{x} + 1 - \gamma_1}{2-\gamma_1} - \frac{\gamma_1(1-\hat{x}) + (1-\gamma_1)d}{\gamma_1} \\ &= \frac{\gamma_1(2\hat{x}-1) - (2-\gamma_1)(1-\gamma_1)d}{(2-\gamma_1)\gamma_1}. \end{aligned}$$

If $d \leq \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$, it is non-negative. Then, it holds that $\tilde{x}_1 \leq x_1^*$. In that case, the above argument implies that $x_2 = 1 - x_1$ is optimal when $x_1 \leq \tilde{x}_1$, $x_2 = \hat{x}$ is optimal when $\tilde{x}_1 \leq x_1 \leq x_1^*$, $x_2 = [x_1]_-$ is optimal when $x_1^* < x_1 \leq \hat{x}$. (Note that, when $d = \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$, we have that $\tilde{x}_1 = x_1^*$. Then, the interval $[\tilde{x}_1, x_1^*]$ becomes degenerate. There, both x_1 and $\hat{x}$ are optimal.) If $d > \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$, it holds that $\tilde{x}_1 > x_1^*$. When $x_1 < \tilde{x}_1$, making the just compatible demand is better for player 2 than making the demand of $\hat{x}$. When $x_1 > \tilde{x}_1$, demanding $[x_1]_-$ is better for player 2 than demanding $\hat{x}$. They imply that demanding $\hat{x}$ is a dominated strategy. Hence, if $d > \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$, we can conclude that, if $x_1 > (1+(1-\gamma_1)d)/2$, player 2's best response is to demand $[x_1]_-$ and otherwise it is to make the just compatible demand.

(iii) $\hat{x} < x_1$

When player 2 demands $\hat{x}$, it holds that $x_2 = \hat{x} < x_1$. Hence, at the bargaining stage, he obtains $\hat{x}$ according to the equitable rule. Compare his payoff from making $\hat{x}$ with that from the just compatible demand. Because $\hat{x} < x_1$, we have:

$$\begin{aligned} \Pi_2(x_1, \hat{x}) - \Pi_2(x_1, 1-x_1) &= \gamma_1 \hat{x} + (1-\gamma_1)(\hat{x} - d) - (1-x_1) \\ &> \hat{x} - (1-\gamma_1)d - 1 + \hat{x} \\ &= 2\hat{x} - 1 - (1-\gamma_1)d > 0. \end{aligned}$$

(The last inequality comes from the assumption that $\hat{x} > (1+d)/2$.) It shows that, for player 2, making the just compatible demand is dominated by making the demand of $\hat{x}$. If player 2 makes an incompatible demand and sets it higher than $\hat{x}$, the demand of $[x_1]_-$ is best because it attains the highest payoff against the rational type according to the equitable rule and the chicken type always leaves the negotiation given player 1's demand in this range. If player 2 makes an incompatible demand and sets it no higher than $\hat{x}$, the demand of $\hat{x}$ is best because it attains the highest payoff against both types. Hence, the candidates for the optimal strategy is narrowed to the pair of demands, $\hat{x}$ and $[x_1]_-$. By subtracting the payoff from the latter from that from the former, we have:

$$\begin{aligned}\Pi_2(x_1,\hat{x}) - \Pi_2(x_1,[x_1]_-) &= \gamma_1\hat{x} + (1-\gamma_1)(\hat{x}-d) - (1-\gamma_1)(x_1-d)\\ &= \hat{x} - (1-\gamma_1)x_1.\end{aligned}$$

When $x_1 < \hat{x}/(1-\gamma_1)$, the difference is positive and thus the demand of $\hat{x}$ is optimal for player 2. On the other hand, when $x_1 > \hat{x}/(1-\gamma_1)$, the difference is negative and thus the demand of $[x_1]_-$ is optimal for player 2. Note that, because $\hat{x}/(1-\gamma_1) > \hat{x}$, the former inequality can be satisfied for some x_1 but the latter may not be because $\hat{x}/(1-\gamma_1)$ can be larger than one. When the difference is equal to zero, it means that the payoffs from these two strategies are identical in the limit. Hence, except in the limit, the demand of $\hat{x}$ is optimal for player 2.

Next, given the best responses of player 2 derived above, we derive the optimal demand for player 1. For each case of player 2's best response, we compute the highest attainable payoff of player 1. Note that player 1 only cares about the payoff of her rational type by assumption. Depending on the size of the bargaining cost, we study two cases.

(A) $d \leq \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$

First, let us study the case in which the bargaining cost is relatively small so that the above inequality holds.

(a) $x_1 \leq \tilde{x}_1$

Player 2 will respond to x_1 by the just compatible demand. Hence, player 1's payoff is given by x_1. The maximum payoff is attained by $x_1 = \tilde{x}_1$ at the same amount, $\Pi_1(\tilde{x}_1, 1-\tilde{x}_1) = \tilde{x}_1$.

(b) $\tilde{x}_1 \leq x_1 \leq x_1^*$

Player 2 will respond to x_1 by a higher price $\hat{x}$ and thus player 1 expects to obtain $x_1 - d$. The maximum payoff is attained by $x_1 = x_1^*$ at the payoff $\Pi_1(x_1^*, \hat{x}) = x_1^* - d$. Because $x_1^* > 1/2$, it is more than $1/2 - d$.

(c) $x_1^* < x_1 \leq \hat{x}$

Player 2 will underbid player 1 by demanding $[x_1]_-$. Thus, player 1 expects to obtain $1 - x_1 - d$. Because $x_1 > x_1^* > 1/2$, it is lower than $1/2 - d$.

(d) $\hat{x} < x_1 \leq \hat{x}/(1-\gamma_1)$

Player 2 will underbid player 1 by demanding $\hat{x}$. Thus, player 1 expects to obtain $1-\hat{x}-d$. Because $\hat{x} > 1/2$, it is lower than $1/2-d$.

(e) $x_1 > \hat{x}/(1-\gamma_1)$

Player 2 will underbid player 1 by demanding $[x_1]_-$. Thus, player 1 expects to obtain $1-x_1-d$. Because $x_1 > \hat{x}/(1-\gamma_1) > \hat{x} > 1/2$, it is lower than $1/2-d$.

Because player 1 obtains more than $1/2-d$ by demanding x_1^*, the demands in the cases (c), (d) and (e) are dominated. We now compare player 1's payoffs from the best demands in the first two cases. The difference between these is given by:

$$\begin{aligned}\Pi_1(\tilde{x}_1, 1-\tilde{x}_1) - \Pi_1(x_1^*, \hat{x}) &= \tilde{x}_1 - (x_1^* - d) \\ &= \frac{\gamma_1(1-\hat{x}) + (1-\gamma_1)d}{\gamma_1} - \frac{\gamma_1\hat{x} + 1 - \gamma_1}{2-\gamma_1} + d \\ &= \frac{\gamma_1(1-2\hat{x}) + (2-\gamma_1)d}{\gamma_1(2-\gamma_1)}.\end{aligned}$$

When $d < \gamma_1(2\hat{x}-1)/(2-\gamma_1)$, the above implies that demanding x_1^* is optimal for player 1. When the inequality holds in reverse, player 1's optimal demand induces player 2 to make the just compatible demand.

(B) $d > \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$

Next, let us study the case in which the bargaining cost is relatively large so that the above inequality holds.

(a') $x_1 \leq (1+(1-\gamma_1)d)/2$

Player 2 will respond to x_1 by the just compatible demand. Hence, player 1's payoff is given by x_1. The maximum payoff is attained by $x_1 = (1+(1-\gamma_1)d)/2(> 1/2)$ at the same amount.

(c') $(1+(1-\gamma_1)d)/2 < x_1 \leq \hat{x}$

Player 2 will underbid player 1 by demanding $[x_1]_-$. Thus, player 1 expects to obtain $1-x_1-d$. Because $x_1 > (1+(1-\gamma_1)d)/2 > 1/2$, it is lower than $1/2$. Hence, any demand in this range is not optimal.

The above analysis has shown that, when $\hat{x} < x_1$, player 1's payoff is lower than $1/2-d$. Hence, demanding $(1+(1-\gamma_1)d)/2$ is optimal.

The combination of the above parts (A) and (B) establishes the statement in the proposition.

A.4. The proof of Proposition 3

Because player 2 only cares about the payoff of his rational type by assumption, his best response at the demand stage is identical to the one derived in the proof of Proposition 2. In the similar way, we derive the optimal demand for player 1. In the following, all the cases refer to the same ones in the proof of Proposition 2. As in the proof of Proposition 2, we study two cases depending on the size of the bargaining cost.

(A) $d \leq \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$

First, let us study the case in which the bargaining cost is relatively small so that the above inequality holds.

When $x_1 \leq \tilde{x}_1$ (case a), player 2 will respond to x_1 by the just compatible demand. Hence, her payoff is given by x_1. The maximum payoff is attained by $x_1 = \tilde{x}_1$ at the same amount.

When $\tilde{x}_1 \leq x_1 \leq x_1^*$ (case b), player 2 will respond to x_1 by a higher price $\hat{x}$ and thus player 1 expects to obtain $\gamma_2 x_1 + (1-\gamma_2)(x_1 - d) = x_1 - (1-\gamma_2)d$. The maximum payoff is attained by $x_1 = (\gamma_1\hat{x} + 1 - \gamma_1)/(2-\gamma_1) = x_1^*$ at the payoff $x_1^* - (1-\gamma_2)d$. Note that this is more than $1/2 - (1-\gamma_2)d$.

When $x_1^* < x_1 \leq \hat{x}$ (case c), player 2 will underbid player 1 by demanding $[x_1]_-$. Thus player 1 expects to obtain $\gamma_2 x_1 + (1-\gamma_2)(1-x_1-d)$. Because $\gamma_2 < 1/2$ by the supposition of the proposition, it is strictly decreasing in x_1. Hence, the supremum of the payoff in this case is attained when $x_1 = x_1^*$. Compared to the highest payoff in the above case (b), it is lower because player 1 is underbid here and $x_1^* > 1/2$. Hence, the demand in this case is never optimal.

When $\hat{x} < x_1 \leq \hat{x}/(1-\gamma_1)$ (case d), player 2 will underbid player 1 by demanding $\hat{x}$. Because the chicken type of player 2 leaves the negotiation, player 1 expects to obtain $(1-\gamma_2)(1-\hat{x}-d)$. It is lower than $1/2 - (1-\gamma_2)d$ and is never optimal.

When $x_1 > \hat{x}/(1-\gamma_1)$ (case e), player 2 will underbid player 1 by demanding $[x_1]_-$. Because the chicken type of player 2 leaves the negotiation, player 1 expects to obtain $(1-\gamma_2)(1-x_1-d)$. Because $x_1 > \hat{x}/(1-\gamma_1)$, it is lower than $1/2 - (1-\gamma_2)d$ and is never optimal.

This shows that the candidates for player 1's best demand are narrowed to the first two cases. We compare the payoffs from these two cases. The difference between these is given by:

$$\begin{aligned}\Pi_1(\tilde{x}_1, 1-\tilde{x}_1) - \Pi_1(x_1^*, \hat{x}) &= \tilde{x}_1 - (x_1^* - (1-\gamma_2)d) \\ &= \frac{\gamma_1(1-\hat{x}) + (1-\gamma_1)d}{\gamma_1} - \frac{\gamma_1\hat{x} + 1 - \gamma_1}{2-\gamma_1} + (1-\gamma_2)d \\ &= \frac{\gamma_1(1-2\hat{x}) + (1-\gamma_1\gamma_2)(2-\gamma_1)d}{\gamma_1(2-\gamma_1)}.\end{aligned}$$

It implies that demanding x_1^* is optimal when $d > \gamma_1(2\hat{x}-1)/\big((1-\gamma_1\gamma_2)(2-\gamma_1)\big)$, and that making the demand that causes player 2 to make the compatible demand is optimal when the inequality holds in reverse.

(B) $d > \gamma_1(2\hat{x}-1)/(2-\gamma_1)(1-\gamma_1)$

Next, let us study the case in which the bargaining cost is relatively large so that the above inequality holds.

(a') $x_1 \leq (1+(1-\gamma_1)d)/2$

Player 2 will respond to x_1 by the just compatible demand. Hence, player 1's payoff is given by x_1. The maximum payoff is attained by $x_1 = (1+(1-\gamma_1)d)/2 (> 1/2)$ at the same amount.

(c') $(1+(1-\gamma_1)d)/2 < x_1 \leq \hat{x}$

Player 2 will underbid player 1 by demanding $[x_1]_-$. Thus, player 1 expects to obtain $\gamma_2 x_1 + (1-\gamma_2)(1-x_1-d) = (2\gamma_2-1)x_1 + (1-\gamma_2)(1-d)$. Because $2\gamma_2 - 1 < 0$, it is lower than $\gamma_2((1+(1-\gamma_1)d)/2) + (1-\gamma_2)(1-((1+(1-\gamma_1)d)/2)-d)$. Because $1-((1+(1-\gamma_1)d)/2)-d < (1+(1-\gamma_1)d)/2$, it is lower than the payoff that player 1 obtains by demanding $(1+(1-\gamma_1)d)/2$. Hence, making a demand in this range is never optimal.

The above analysis has shown that, when $x_1 > \hat{x}$, player 1's payoff is lower than 1/2. Hence, demanding $(1+(1-\gamma_1)d)/2$ is optimal.

The combination of the above parts (A) and (B) establishes the statement in the proposition.

A.5. The proof of Lemma 2

Lemma 1 has shown that player i attains a payoff higher than $1-x_j$ when and only when her demand causes player j to do the mass acceptance, i.e., when it is between $1-x_j$ and $\overline{X}_i(x_j)$. Observe that, because $x_j > x_j^c + \triangle$,

$$\overline{X}_i(x_j) - (1-x_j) = 1 - \frac{1}{1-x_j^c}(1-x_j) > 1 - \frac{1}{1-x_j^c}(1-x_j^c-\triangle) > 0.$$

Hence, this interval is well-defined and thus player i's maximum payoff is in fact bigger than $1-x_j$. In the following, we focus on player i's demand in $[1-x_j, \overline{X}_i(x_j)]$.

To prove the claim, we compute the first-order derivative of player i's continuation payoff at the bargaining stage with respect to her own demand as well as the second order one:

$$\frac{d\pi_i^w(x)}{dx_i} = 1 - e^{\alpha_j \log \xi \left(1 - \frac{1-x_i^c}{1-x_j^c}\frac{1-x_j}{1-x_i}\right)} \left(1 - \alpha_j \log \xi \frac{1-x_i^c}{1-x_j^c}\frac{1-x_j}{(1-x_i)^2}\right), \text{ and}$$

$$\frac{d^2\pi_i^w(x)}{dx_i^2} = 2\frac{P_j^{ma}}{dx_i} + (x_i + x_j - 1)\frac{d^2 P_j^{ma}}{dx_i^2} < 0.$$

Because the second order derivative is negative, $\pi_i^w(x)$ is concave in x_i. Note that

$$\left.\frac{d\pi_i^w(x)}{dx_i}\right|_{x_i=\overline{X}_i(x_j)} = \alpha_j \log \xi \frac{1-x_i^c}{1-x_j^c}\frac{1-x_j}{(1-x_i)^2} < 0.$$

It implies that the maximizer is located to the left of $\overline{X}_i(x_j)$. We now evaluate the first-order derivative at the demand a little lower than $\overline{X}_i(x_j)$, $\left.\frac{d\pi_i^w(x)}{dx_i}\right|_{x_i=\overline{X}_i(x_j)-\delta}$. (When $\overline{X}_i(x_j) - \delta \leq 1 - x_j$, the above argument implies that the statement in the proposition holds obviously. Hence, in the following, we consider the case that δ is sufficiently small so that $\overline{X}_i(x_j) - \delta > 1 - x_j$.) Observe that:

$$\begin{aligned}\left.\frac{d\pi_i^w(x)}{dx_i}\right|_{x_i=\overline{X}_i(x_j)-\delta} \\ = 1 - e^{\alpha_j \log \xi \left(1 - \frac{1-x_i^c}{1-x_j^c}\frac{1-x_j}{1-\overline{X}_i(x_j)+\delta}\right)} \left(1 - \alpha_j \log \xi \frac{1-x_i^c}{1-x_j^c}\frac{1-x_j}{(1-\overline{X}_i(x_j)+\delta)^2}\right).\end{aligned}$$

For any $x_j > x_j^c + \triangle$, if ξ is sufficiently small, this is positive uniformly with respect to x_j because $\frac{1-x_i^c}{1-x_j^c}\frac{1-x_j}{1-\overline{X}_i(x_j)+\delta} < \left(1 + \frac{1-x_j^c}{1-x_i^c}\frac{1}{1-x_j^c-\triangle}\delta\right)^{-1} < 1$. It implies that the maximizer is between $\overline{X}_i(x_j) - \delta$ and $\overline{X}_i(x_j)$, namely $\overline{X}_i(x_j) - \delta < \operatorname{argmax}_{x_i} \pi_i^w(x) < \overline{X}_i(x_j)$. This proves the first part of the statement.

Note that, when $x_i = \overline{X}_i(x_j) - \delta$,

$$P_j^{ma} = 1 - \exp\left\{\alpha_j \log \xi \left(1 - \frac{1 - x_i^c}{1 - x_j^c}\frac{1 - x_j}{1 - x_i}\right)\right\} \to 1$$

uniformly with respect to x_j as $\xi \to 0$. Because $\pi_i^w(x) = P_j^{ma} x_i + (1 - P_j^{ma})(1 - x_j)$, player i's payoff is no smaller than $\overline{X}_i(x_j) - \delta$ in the limit. By choosing δ appropriately, we obtain the second statement.

A.6. The proof of Proposition 4

(i) We prove the first claim by looking at three cases of player 1's demand.

First, consider the case that $x_1 \le x_1^c + x_2^c d$. If player 2 demands no lower than $\overline{X}_2(x_1)$, player 1 does not do the mass acceptance and player 2 obtains $1 - x_1 - d$, which is lower than the payoff that he obtains by making the just compatible demand. When $x_2 < \overline{X}_2(x_1)$ and the demands become incompatible, player 2's payoff is lower than $\overline{X}_2(x_1) - d$. Because $x_1 \le x_1^c + x_2^c d$, we can show that $1 - x_1 \ge \overline{X}_2(x_1) - d$. Namely, the just compatible demand gives a higher payoff to player 2 and thus he has no incentive to make this demand. Obviously, making demand less than $1 - x_1$ is not optimal for player 2. This implies that, when player 1 demands no more than $x_1^c + x_2^c d$, player 2 surely makes the corresponding just compatible demand.

Next, consider the case that $x_1^c + x_2^c d < x_1 \le \hat{x}$. Suppose, to the contrary, that player 2 does not make the just compatible demand. Because $x_1^c + x_2^c d < x_1$, we can apply[7] Lemma 2. It shows that, in such a situation, the player 2's best response satisfies $\overline{X}_2(x_1) - \delta < x_2 < \overline{X}_2(x_1)$. Given this response, consider the expected payoff of player 1. As before, we denote the expected payoff of player i at the beginning of the game by $\Pi_i(x_1, x_2)$. When δ is sufficiently small so that $\delta < d$, we have:

$$\begin{aligned}
\Pi_1(x_1, x_2) &= \gamma_2 x_1 + (1 - \gamma_2)(1 - x_2 - d) \\
&\le \gamma_2 x_1 + (1 - \gamma_2)(1 - \overline{X}_2(x_1) + \delta - d) \\
&= \gamma_2 x_1 + (1 - \gamma_2)\frac{1 - x_2^c}{1 - x_1^c}(1 - x_1) - (1 - \gamma_2)(d - \delta) \\
&< \gamma_2 x_1 + (1 - \gamma_2)\frac{1 - x_2^c}{1 - x_1^c}(1 - x_1).
\end{aligned}$$

[7] Note that the following argument depends on the uniformity of the statement of Lemma 2 with respected to x_1.

Because $\gamma_2 < x_1^c$, $\gamma_2-(1-\gamma_2)(1-x_2^c)/(1-x_1^c) = \gamma_2\{1+((1-x_2^c)/(1-x_1^c))\}-(1-x_2^c)/(1-x_1^c) < x_1^c/(1-x_1^c)-(1-x_2^c)/(1-x_1^c) = 0$, the last expression is maximized when x_1 is the smallest. Namely, it is lower than $\gamma_2(x_1^c + x_2^c d) + (1-\gamma_2)\frac{1-x_2^c}{1-x_1^c}(1-x_1^c - x_2^c d)$. This is no higher than $x_1^c + x_2^c d$, which player 1 can obtain for sure as shown above. This is a contradiction and therefore the supposed plays cannot occur in equilibrium.

Finally, consider the case that $\hat{x} < x_1$. Suppose, to the contrary, that player 2 does not make the just compatible demand. The response of the rational type of player 2 is same as the above and, due to the departure of the chicken type, the payoff of player 1 is no higher than the one derived there. This is a contradiction and thus the supposed play cannot occur in equilibrium.

The combination of the above implies that any equilibrium has to entail the just compatible demands.

(ii) From the above argument, we know that any demand no higher than $x_1^c + x_2^c d$ is surely accepted by player 2. It indicates that any demand lower than this amount cannot be optimal for player 1.

Now consider the case that $x_1 > x_1^c + x_2^c d$. When player 2 makes the just compatible demand, he obtains $1 - x_1$. On the other hand, when he makes an incompatible demand, Lemma 2 shows that the highest payoff for player 2 is attained when he makes the demand x_2 such that $\overline{X}_2(x_1) - \delta < x_2 < \overline{X}_2(x_1)$ and then he obtains more than $\overline{X}_2(x_1) - \delta - d$. When $1 - x_1 \leq \overline{X}_2(x_1) - \delta - d$ or $x_1 \geq x_1^c + x_2^c(\delta + d)$, player 2 chooses an incompatible demand. The above argument in (i) shows that, when player 2 makes an incompatible demand, it gives less than $x_1^c + x_2^c d$ to player 1. Hence, player 1's demand cannot be larger than $x_1^c + x_2^c(\delta + d)$.

By appropriately choosing δ, we obtain the desired statement.

A.7. The proof of Proposition 5

The proof proceeds in the similar way as in the proof of Proposition 2. In the following proof, we take ξ_i $(i = 1, 2)$ sufficiently small so that the statement of Lemma 2 holds.

First, we derive the best response of player 2 at the demand stage given player 1's demand x_1. It is easy to see that a demand lower than $1 - x_1$ is not optimal for player 2 as it is dominated by the just compatible demand $1 - x_1$. In the following, we restrict our attention to the demand no lower than $1 - x_1$.

(i) $x_1 \leq 1 - \hat{x}$

Note that $x_1 \leq 1 - \hat{x} < 1 - x_2^c = x_1^c$. Hence, when $x_2 > 1 - x_1$ and the demands are incompatible, the continuation payoff of player 2 in the war of attrition is given by $1 - x_1$. By the same logic as in the part (i) at the proof of Proposition 2, we can show that making the just compatible demand is the best response in this case.

(ii) $1 - \hat{x} < x_1 \leq \overline{X}_2^{-1}(\hat{x})$

Note that, by the condition of this case, $\overline{X}_2(x_1) \leq \hat{x}$. It implies that any demand causing player 1 to do the mass acceptance is lower than $\hat{x}$ and the chicken type of player 1 yields to it at the yielding stage. Moreover, when player 2 makes the demand of $\hat{x}$, Lemma 1 shows that, if some player does the mass acceptance, it is player 2 who does so. Hence, we can apply the same logic as in the part (ii) at the proof of Proposition 2, and can show that the candidates for player 2's best response are making the corresponding just compatible demand, demanding the highest one that the chicken type yields to and making an incompatible demand which causes player 1 to do the mass acceptance: $1 - x_1$, $\hat{x}$, and a demand lower than $\overline{X}_2(x_1)$.

First, compare the just compatible demand with the demand of $\hat{x}$ as player 2's strategy. Given the latter demand, Lemma 1 shows that player 2 obtains $\Pi_2(x_1, \hat{x}) = \gamma_1 \hat{x} + (1-\gamma_1)(1 - x_1 - d)$. Thus, the difference between the payoff from the just compatible demand and that from $\hat{x}$ is given by the same formula derived in the proof of Proposition 2:

$$\Pi_2(x_1, 1-x_1) - \Pi_2(x_1, \hat{x}) = 1 - x_1 - \gamma_1 \hat{x} - (1-\gamma_1)(1 - x_1 - d) = \gamma_1(1-\hat{x}) + (1-\gamma_1)d - \gamma_1 x_1.$$

When $x_1 = 1 - \hat{x}$, the difference is equal to $(1-\gamma_1)d$, which is positive. The difference is decreasing in x_1. Hence, when $x_1 < (>) \tilde{x}_1$, making the just compatible demand is better (worse respectively) than making the demand of $\hat{x}$. (Player 2 is indifferent between these when the equality holds.)

Next, compare the demand of $\hat{x}$ with an incompatible demand that causes player 1 to do the mass acceptance as player 2's strategy. For player 1 to do the mass acceptance, Lemma 1 shows that $x_1 > x_1^c$. For the moment, we focus on player 1's demand such that $x_1 > x_1^c + \triangle$ for $\triangle > 0$. (We later justify this restriction.) Lemma 2 shows that the highest payoff from the latter strategy is slightly lower than $\overline{X}_2(x_1)$ and its maximizer is also slightly lower than $\overline{X}_2(x_1)$. Specifically, let the former be given by $\overline{X}_2(x_1) - \epsilon$ and the latter by $\overline{X}_2(x_1) - \epsilon'$, where $0 < \epsilon, \epsilon' < \delta$. Then, his expected payoff at the beginning is $\Pi_2(x_1, \overline{X}_2(x_1) - \epsilon') = \gamma_1(\overline{X}_2(x_1) - \epsilon') + (1-\gamma_1)(\overline{X}_2(x_1) - \epsilon - d)$. Using this, we compute the difference between the payoff from $\hat{x}$ and that from $\overline{X}_2(x_1) - \epsilon'$:

$$\begin{aligned}
&\Pi_2(x_1, \hat{x}) - \Pi_2(x_1, \overline{X}_2(x_1) - \epsilon') \\
&\quad = \gamma_1 \hat{x} + (1-\gamma_1)(1 - x_1 - d) - \gamma_1(\overline{X}_2(x_1) - \epsilon') - (1-\gamma_1)(\overline{X}_2(x_1) - \epsilon - d) \\
&\quad = \gamma_1 \hat{x} + (1-\gamma_1)(1 - x_1) - \overline{X}_2(x_1) + (\gamma_1 \epsilon' + (1-\gamma_1)\epsilon) \\
&\quad = \gamma_1 \hat{x} + (1-\gamma_1)(1 - x_1) - 1 + \frac{1 - x_2^c}{1 - x_1^c}(1 - x_1) + (\gamma_1 \epsilon' + (1-\gamma_1)\epsilon) \\
&\quad = -\frac{1 - \gamma_1 + \gamma_1 x_1^c}{1 - x_1^c} x_1 + \gamma_1 \hat{x} - \gamma_1 + \frac{x_1^c}{1 - x_1^c} + (\gamma_1 \epsilon' + (1-\gamma_1)\epsilon)
\end{aligned}$$

Observe that, when $\epsilon' = \epsilon = 0$, the payoff from the demand $\overline{X}_2(x_1)$ gives the upper bound for the payoff that player 2 can obtain when his demand causes player 1 to do the mass acceptance. Thus, the solution to the following equation gives the lower bound for the threshold which determines whether demanding $\hat{x}$ is better or not:

$$-\frac{1 - \gamma_1 + \gamma_1 x_1^c}{1 - x_1^c} x_1 + \gamma_1 \hat{x} - \gamma_1 + \frac{x_1^c}{1 - x_1^c} = 0.$$

Solving this equation and calling its solution x_1^{**}, we have:

$$x_1^{**} \equiv \frac{x_1^c - (1 - x_1^c)\gamma_1(1 - \hat{x})}{1 - \gamma_1 + \gamma_1 x_1^c}.$$

Because $\overline{X}_2(x_1^c) = x_2^c = 1 - x_1^c < \hat{x}$, the left-hand side of the above equation is positive at $x_1 = x_1^c$. On the other hand, because $1 - \overline{X}_2^{-1}(x_1^c) = x_1^c$ and $\overline{X}_2^{-1}(x_1)$ is increasing in x_1, we have $1 - \overline{X}_2^{-1}(\hat{x}) < \hat{x}$. Thus, the left-hand side of the above equation is negative at $x_1 = \overline{X}_2^{-1}(\hat{x})$. Combining these, we can conclude that $x_1^c < x_1^{**} < \overline{X}_2^{-1}(\hat{x})$. (Because $\overline{X}_2^{-1}(x_1)$ is increasing in x_1 and $x_2^c < \hat{x}$, it holds that $x_1^c < \overline{X}_2^{-1}(\hat{x})$.) This argument shows that the threshold for demanding $\hat{x}$ to be better is higher than x_1^c by some positive margin. (Note that, because we deal with the limit here, we can choose an arbitrary $\triangle$ when we apply Lemma 2. Thus, this argument is valid.) We return to the comparison of the payoff from $\hat{x}$ and that from $\overline{X}_2(x_1) - \epsilon'$ this time for x_1 such that $x_1 > x_1^{**}$. This justifies the supposition that $x_1 > x_c + \triangle$. The simple computations shows that the best responses of player 2 change at:

$$x_1 = \frac{(1 - x_1^c)(\gamma_1 \hat{x} - \gamma_1 + \gamma_1 \epsilon' + (1-\gamma_1)\epsilon) + x_1^c}{1 - \gamma_1 + \gamma_1 x_1^c} = x_1^{**} + \frac{(1 - x_1^c)(\gamma_1 \epsilon' + (1-\gamma_1)\epsilon)}{1 - \gamma_1 + \gamma_1 x_1^c}.$$

(Note that both ϵ and ϵ' depend on x_1. What we are deriving here is not the exact threshold but the range in which a strategy is known to be the best response.) Lemma 2 shows that $0 < \epsilon, \epsilon < \delta$ for any x_j. Thus, there is some δ' which is of the same order as δ and satisfies $\frac{(1-x_1^c)(\gamma_1\epsilon'+(1-\gamma_1)\epsilon)}{1-\gamma_1+\gamma_1 x_1^c} < \delta'$. Therefore, if $x_1 \leq x_1^{**}$, demanding $\hat{x}$ is better and, if $x_1 \geq x_1^{**}+\delta'$, demanding $\overline{X}_2(x_1)$ is better. (When $x_1^{**} < x_1 < x_1^{**} + \delta'$, which is better depends on the demand x_1.)

We compare the two thresholds derived above: $\tilde{x}_1$ and x_1^{**}. Note that:

$$x_1^{**} - \tilde{x}_1 = \frac{x_1^c - (1-x_1^c)\gamma_1(1-\hat{x})}{1-\gamma_1+\gamma_1 x_1^c} - \frac{\gamma_1(1-\hat{x}) + (1-\gamma_1)d}{\gamma_1}$$
$$= \frac{\gamma_1(\hat{x}+x_1^c-1) - (1-\gamma_1)(1-\gamma_1+\gamma_1 x_1^c)d}{\gamma_1(1-\gamma_1+\gamma_1 x_1^c)}.$$

Under the supposition of the proposition, it is positive because $d < \gamma_1(\hat{x}+x_1^c-1)/(1-\gamma_1+\gamma_1 x_1^c) < \gamma_1(\hat{x}+x_1^c-1)/(1-\gamma_1)(1-\gamma_1+\gamma_1 x_1^c)$. Thus, it holds that $\tilde{x}_1 < x_1^{**}$. Under this inequality, the above argument implies that $x_2 = 1-x_1$ is optimal when $x_1 \leq \tilde{x}_1$, $x_2 = \hat{x}$ is optimal when $\tilde{x}_1 \leq x_1 \leq x_1^{**}$, some demand that causes player 1 to do the mass acceptance is optimal when $x_1 \geq x_1^{**} + \delta'$. When $x_1^{**} < x_1 < x_1^{**} + \delta'$, either of the last two strategies above is optimal.

(iii) $x_1 > \overline{X}_2^{-1}(\hat{x})$

We compare player 2's payoff from making the demand of $\hat{x}$ with that from making the just compatible demand. The above analysis has shown that, at $x_1 = \overline{X}_2^{-1}(\hat{x})$, the former is higher than the latter. Namely, it holds that $\gamma_1\hat{x} + (1-\gamma_1)(1-\overline{X}_2^{-1}(\hat{x}) - d) > 1 - \overline{X}_2^{-1}(\hat{x})$. When $x_1 > \overline{X}_2^{-1}(\hat{x})$ and player 2 demands $\hat{x}$, player 1 does the mass acceptance. Then, the difference between the payoff of player 2 from demanding $\hat{x}$ and that from making the just compatible demand is given by:

$$\Pi_2(x_1, \hat{x}) - \Pi_2(x_1, 1-x_1)$$
$$= \gamma_1\hat{x} + (1-\gamma_1)(1 - x_1 + P_2^{ma}(x_1+\hat{x}-1) - d) - (1-x_1)$$
$$= \{\gamma_1\hat{x} + (1-\gamma_1)(1-x_1-d) - (1-x_1)\} + (1-\gamma_1)P_2^{ma}(x_1+\hat{x}-1).$$

The term in the first bracket is increasing in x_1. Moreover, it holds that

$$\frac{dP_j^{ma}}{dx_j} = -e^{\log\xi_j\left(1-\frac{1-x_i^c}{1-x_j^c}\frac{1-x_j}{1-x_i}\right)}\left(\log\xi_j \frac{1-x_i^c}{1-x_j^c}\frac{1}{1-x_i}\right) > 0.$$

It implies that the second term is also increasing in x_1. Hence, the above inequality implies that the difference is positive. This shows that, for player 2, making the just compatible demand is dominated by demanding $\hat{x}$ and thus is never the optimal strategy for him when $x_1 > \overline{X}_2^{-1}(\hat{x})$.

The similar argument made in the corresponding part in the proof of Proposition 2 can show that, for player 2, either the demand of $\hat{x}$ or some demand that causes player 1 to do the mass acceptance is optimal. Note that, because $x_1 > \overline{X}_2^{-1}(\hat{x})$, player 1 does the mass acceptance when $x_2 = \hat{x}$. Therefore, no matter what is the optimal strategy, player 1 does the mass acceptance when player 2 plays the best response in this case.

Given player 2's best responses derived above, we derive the optimal demand for player 1.

Consider the demand of x_1^{**}. The above analysis shows that player 2 responds to it by demanding $\hat{x}$. Because $x_1^{**} < \overline{X}_2^{-1}(\hat{x})$ from the above derivation, P_2^{ma} converges to one given $(x_1^{**}, \hat{x})$ when ξ goes to zero. For sufficiently small ξ, player 1 obtains a payoff no lower than $x_1^{**} - d - \delta$. Note that, because $x_1^{**} > x_1^c$ as shown above, for sufficiently small δ, player 1's payoff from this strategy is higher than $x_1^c - d$.

First, we argue that this achieves a higher payoff for player 1 than any demand that leads to the just compatible demand by player 2. By the above analysis, the just compatible demand is not made when $x_1 > \tilde{x}_1$ and player 1's maximum payoff from this possibility is given by $\tilde{x}_1$. Taking the difference between player 1's payoff from x_1^{**} and this payoff, we obtain:

$$\begin{aligned}
&\Pi_1(x_1^{**}, \hat{x}) - \Pi_1(\tilde{x}_1, 1 - \tilde{x}_1) \\
&\geq x_1^{**} - d - \delta - \frac{\gamma_1(1-\hat{x}) + (1-\gamma_1)d}{\gamma_1} \\
&= \frac{x_1^c - (1-x_1^c)\gamma_1(1-\hat{x})}{1-\gamma_1+\gamma_1 x_1^c} - \frac{\gamma_1 d + \gamma_1(1-\hat{x}) + (1-\gamma_1)d}{\gamma_1} - \delta \\
&= \frac{\gamma_1(\hat{x} + x_1^c - 1) - \gamma_1(1-\gamma_1+\gamma_1 x_1^c)\delta - (1-\gamma_1+\gamma_1 x_1^c)d}{\gamma_1(1-\gamma_1+\gamma_1 x_1^c)}.
\end{aligned}$$

By the supposition in proposition, we have that $d < \gamma_1(\hat{x} + x_1^c - 1)/(1 - \gamma_1 + \gamma_1 x_1^c)$ or that $\gamma_1(\hat{x} + x_1^c - 1) - (1 - \gamma_1 + \gamma_1 x_1^c)d > 0$. Hence, when we choose sufficiently small δ, the above expression is positive. It implies that demanding x_1^{**} gives a higher payoff to player 1 than any demand that induces player 2 to make the just compatible demand.

Second, we argue that the demand of x^{**} achieves a higher payoff for player 1 than any demand that leads to player 1's mass acceptance given the best response of player 2. The above analysis has shown that player 1 does the mass acceptance only when $x_1 > x_{**}$. In such a situation, Lemma 1 implies that player 1's payoff is given by $1 - x_2 - d$. By the above analysis, we know that the best response of player 2 is either a demand which is higher than $\overline{X}_2(x_1) - \delta$ or $\hat{x}$. The above analysis also shows that $x_1^{**} > x_1^c$. Because $\overline{X}_2(x_1)$ is strictly increasing in x_1, we have $\overline{X}_2(x_1) > \overline{X}_2(x_1^{**}) > \overline{X}_2(x_1^c) = x_2^c$. By choosing a $\delta(> 0)$ such that $\delta < \overline{X}_2(x_1^{**}) - \overline{X}_2(x_1^c)$, we then have $\overline{X}_2(x_1) - \delta > \overline{X}_2(x_1^{**}) - \delta > x_2^c$. Hence, when player 2 responds by $\overline{X}_2(x_1) - \delta$, player 1's payoff is lower than $1 - x_2^c - d = x_1^c - d$. Because $\hat{x} > x_2^c$, we have $x_1^c = 1 - x_2^c > 1 - \hat{x}$. Hence, when player 2 responds by $\hat{x}$, player 1's payoff is lower than $x_1^c - d$. This implies that either response gives a payoff lower than $x_1^c - d$ to player 1.

The above argument shows that the optimal demand for player 1 has to induce player 2 to demand $\hat{x}$. As shown in the above analysis of the case (ii), player 2 will choose to demand $\hat{x}$ when $x_1 \leq x_1^{**}$ but will not do so when $x_1 \geq x_1^{**} + \delta$. Because the probability of player 2's mass acceptance becomes close to one for a small ξ, player 1's demand should not be lower than x_1^{**} by any large margin. The combination of these proves both statements of the proposition.

A.8. The proof of Proposition 6

The best response of player 2 at the demand stage is identical to the one derived in the proof of Proposition 5. We examine various strategies of player 1 at the demand stage in turn.

The argument in proof of Proposition 5 shows that, under the supposition of the proposition, demanding x_1^{**} achieves the payoff which is higher for player 1 than any demand that induces player 2 to make the just compatible demand even when only the former entails the bargaining cost. In the current setting, because the demand of $x^{**}(< \hat{x})$ is accepted by the chicken type, the former achieves a higher payoff than in the setting of Proposition 5. Thus, demanding x_1^{**} dominates any demand which induces player 2 to make the just compatible demand also in the current setting.

We now show that demanding more than $\hat{x}$ is not optimal for player 1, either. We derive a contradiction by supposing that she does so. When player 2 turns out not to be the chicken type, the analysis in the proof of Proposition 5 shows that making some demand in $(x_1^{**} - \delta, x_1^{**} + \delta)$ is optimal for sufficiently small ξ. When player 2 turns out to be the chicken type, player 1 is better off by making some demand in $(x_1^{**} - \delta, x_1^{**} + \delta)$ than by demanding $\hat{x}$ because the chicken type accepts the former but walks away from the latter demand. (Because $x_1^{**} < \hat{x}$ by the supposition, for sufficiently small δ, it holds that $x_1^{**} + \delta \leq \hat{x}$.) Hence, no matter whether player 2 is the chicken type or not, some demand in $(x_1^{**} - \delta, x_1^{**} + \delta)$ is better for player 1 than demanding more than $\hat{x}$.

We next argue that any demand that is no higher than $\hat{x}$ and causes player 2 to make a demand which is lower than $\overline{X}_2(x_1)$ is not optimal for player 1. Suppose not. When player 2 makes a demand less than $\overline{X}_2(x_1)$, the analysis of player 2's best response in the proof of Proposition 5 shows that either $x_2 > \overline{X}_2(x_1) - \delta$ for sufficiently small ξ or $x_2 = \hat{x}$. Because player 1 does the mass acceptance, Lemma 1 shows that player 1's payoff is given by $\gamma_2 x_1 + (1-\gamma_2)(1-x_2-d)$. We first examine the case that $x_2 > \overline{X}_2(x_1) - \delta$. Given such a demand, player 1's payoff is at most:

$$\begin{aligned}
&\gamma_2 x_1 + (1-\gamma_2)(1-\overline{X}_2(x_1)+\delta-d) \\
&\quad = \gamma_2 x_1 + (1-\gamma_2)\left(\frac{1-x_2^c}{1-x_1^c}(1-x_1)+\delta-d\right) \\
&\quad = \left(\gamma_2 - (1-\gamma_2)\frac{1-x_2^c}{1-x_1^c}\right)x_1 + (1-\gamma_2)\left(\frac{1-x_2^c}{1-x_1^c}+\delta-d\right).
\end{aligned}$$

Because $\gamma_2 < x_1^c$, $\gamma_2 - (1-\gamma_2)\frac{1-x_2^c}{1-x_1^c} < x_1^c - (1-x_1^c)\frac{1-x_2^c}{1-x_1^c} = x_1^c - (1-x_2^c) = 0$. Hence, the coefficient of x_1 is negative. Thus, the payoff from this case is no higher than the one evaluated at $x_1 = x_1^{**}$. Compare this payoff with the payoff from demanding x_1^{**}. When player 2 turns out to be the chicken type, the payoffs from these two coincide. On the other hand, the last part of the proof of Proposition 5 shows that demanding x_1^{**} is better for player 1 than demanding a larger sum and causing player 2 to demand less than $\overline{X}_2(x_1)$. Hence, in terms of the expectation, making the demand that causes player 2 to demand less than $\overline{X}_2(x_1)$ is not optimal. We then examine the case that $x_2 = \hat{x} < \overline{X}_2(x_1)$. In this case, player 1's payoff is given by $\gamma_2 x_1 + (1-\gamma_2)(1-\hat{x}-d) < \gamma_2\hat{x} + (1-\gamma_2)(1-\hat{x}-d)$. On the other hand, when player 1 makes the demand of x_1^{**}, her payoff becomes close to $x_1^{**} - (1-\gamma_2)d$ when ξ is sufficiently small. By the supposition, $\gamma_2 < (x_1^{**}+\hat{x}-1)/(2\hat{x}-1)$. It implies that $\gamma_2\hat{x} + (1-\gamma_2)(1-\hat{x}) < x_1^{**}$. Namely, the former is smaller than the latter in the limit and thus demanding x_1 that causes player 2 to make the demand $\hat{x}$ is not optimal for player 1.

The above argument implies that the optimal strategy for player 1 has to be no higher than $\hat{x}$ and it induces player 2 to demand $\hat{x}$. The proof in Proposition 5 shows that, for any δ, such is the case only when $\tilde{x}_1 \leq x_1 < x_1^{**} + \delta$ for sufficiently small ξ. Observe that player 1's payoff is given by $\gamma_2 x_1 + (1-\gamma_2)(1-\hat{x}-d+P_2^{ma}(x_1+\hat{x}-1))$. Because $x_1^{**} < \overline{X}_2^{-1}(\hat{x})$ by the supposition, P_2^{ma} converges to one given $(x_1, \hat{x})$ when $x_1 < x_1^{**} + \delta$ and δ is sufficiently small. This implies that the optimal demand converges to x_1^{**} as ξ goes to zero. This concludes the proof.

A.9. The proof of Proposition 7

Denote the equilibrium demands by (x_1, x_2).

If they are incompatible and $x_i \geq x_j$, player i is better off by demanding $1-x_j$ and saving the bargaining cost. Hence, any pure strategy equilibrium has to involve the just compatible demands.

If they are compatible and $x_i > (1+d)/2$, player j would deviate to the demand $[x_i]_-$ and then obtain the payoff $x_i - d > (1-d)/2$ in the limit. Because $1-x_i < (1-d)/2$, it would increase his payoff. Hence, any equilibrium demand has to satisfy $x_i \leq (1+d)/2$.

Finally, we show that, when $x_i \in [(1-d)/2, (1+d)/2]$, it is supported in the equilibrium where player j demands $1-x_i$. Given such a demand, player i would obtain at most $(1+d)/2 - d$ by deviating to a higher demand. This is never strictly profitable as she expects to obtain at least that much in equilibrium. Deviating to a lower demand reduces her payoff. Hence, she has no incentive to deviate. By symmetry, player j has no incentive to deviate, either.

A.10. The proof of Proposition 8

We prove Proposition 8 by a series of lemmas.

Denote the supremum (or the infimum) of player i's demands by M_i (or m_i respectively):

$$M_i = \max\big\{\sup\{x \in \Omega_i\}, \sup\{x \in [0,1] \,|\, f_i(x) > 0\}\big\}, \text{ and}$$

$$m_i = \min\big\{\inf\{x \in \Omega_i\}, \inf\{x \in [0,1] \,|\, f_i(x) > 0\}\big\}.$$

The first lemma claims that, if a player demands $x(> 1/2)$ with a positive probability, the other player does not demand $[x]_{n-}$ for any $n \in N^{+}$. Due to the possibility of the underbidding, any demand including the ones that are equivalent in the limit is made with a positive probability by at most one player.

Lemma A.1

If $P_i(x) > 0$ for some $x > 1/2$, it holds that $P_j([x]_{n-}) = 0$ for any $n \in N^{+}$. Moreover, it holds that, for any $k \in \{1, 2\}$, $P_k([x]_{n-}) = 0$ for any $n \geq 1$.
(proof)

Suppose that $P_j([x]_{n-}) > 0$ for some $n \in N^{+}$. By the assumption, $P_i([x]_{n-}) = 0$ for any $n \geq 1$. Because $x > 1/2$, underbidding improves player j's payoff and thus what he should demand is $[x]_{-}$. Given this, it is better for player i to switch her demands from x to $[x]_{2-}$. This contradicts the supposition that $P_i(x) > 0$.

The second statement naturally follows from the first statement. *Q.E.D.*

We show that the players' demands cannot be too small as demanding one half always gives them at least one half minus the bargaining cost.

Lemma A.2

For $i \in \{1, 2\}$, it holds that $m_i \geq 1/2 - 2d$.
(proof)

Suppose not. Then, there exists $x_i < 1/2 - 2d$ such that player i demands it in equilibrium.

We consider her deviation to demanding one half and evaluate player i's expected payoff. Depending on the demand of player j, we study three cases in turn.

(i) $x_j > 1 - x_i$

Because $1 - x_i > 1/2 + 2d > 1/2$, the demands are incompatible either before the deviation or after it. Moreover, in either case, player i's demand is lower than that of player j. Hence, the payoff of player i is increased by the deviation.

(ii) $x_j \leq 1/2$

In this case, the demands are compatible either before the deviation or after it. Hence, under the equitable rule, player i improves her payoff by the deviation.

(iii) $1/2 < x_j \leq 1 - x_i$

When player i demands x_i, the demands are compatible. Accordingly to the equitable rule, she obtains $\Pi_i(x_i, x_j) = \frac{1+x_i-x_j}{2}$. When player i switches to the demand of $1/2$, the demands become incompatible. With the probability γ_j, she is faced with the chicken type and obtains 1/2. With the probability $1-\gamma_j$, she is faced with the rational type and, because her demand is lower, she obtains $1/2-d$. Thus, from the demand of 1/2, she obtains $\Pi_i(1/2, x_j) = \gamma_j(1/2) + (1-\gamma_j)(1/2-d) = 1/2-(1-\gamma_j)d$. We compare the two expected payoffs by subtracting the former from the latter:

$$\begin{aligned}\frac{1}{2}-(1-\gamma_j)d-\frac{1+x_i-x_j}{2} &= \frac{x_j-x_i}{2}-(1-\gamma_j)d \\ &\geq \frac{x_j-x_i}{2}-d \\ &> \frac{x_j-1/2+2d}{2}-d \\ &= \frac{x_j-1/2}{2} > 0.\end{aligned}$$

The second inequality comes from the supposition that $x_i < 1/2-2d$. The third inequality comes from the condition for this case. This shows that the deviation increases player i's payoff.

Hence, for any x_j, player i obtains a higher payoff from the deviation. This is a contradiction and thus it has to hold that $m_i \geq 1/2-2d$. *Q.E.D.*

When the bargaining cost is sufficiently small, the player whose opponent may be the chicken type wants to make a high demand to exploit its possibility. Hence, her maximum demand becomes higher than one half by some margin. Given the possibility that the player makes a demand higher than one half, the other player also makes a demand higher than one half by some margin to slightly underbid such demand. Define the maximum of the probabilities of the stubborn type between the two players by γ_{max}: $\gamma_{max} \equiv \max\{\gamma_1, \gamma_2\}$. From the next lemma on, we suppose that at least one player has the possibility of the chicken type: $\gamma_{max} > 0$.

Lemma A.3

Suppose that Assumption A' is satisfied and also that $\gamma_{max} > 0$.

When $d < (\gamma_{max}(\hat{x}-1/2))/(5-3\gamma_{max})$, it holds that $M_i > 1/2+2d$ for $i \in \{1,2\}$.

(proof)

We prove this lemma in two steps. Without loss of generality, suppose that $\gamma_i = \gamma_{max}$.

(i) $\gamma_i > 0$

We want to show that $M_i > 1/2+2d$ when $\gamma_i > 0$ and $d < (\gamma_i(\hat{x}-1/2))/(5-3\gamma_i)$. Suppose not. Then, we have that $M_i \leq 1/2+2d$. Namely, player i's demand is no more than $1/2+2d$. Let x_i be a demand that is made by player i.

Given this demand of player i, we show that the best response of player j is to demand $\hat{x}$ by evaluating his payoffs for various demands.

When $x_j \le 1 - x_i$, the demands become compatible. By Lemma A.2, we know that $m_i \ge 1/2 - 2d$. Hence, $x_j \le 1/2 + 2d$ and moreover $\Pi_j(x_i, x_j) = \frac{1+x_j-x_i}{2} \le 1/2 + 2d$.

When $x_j \ge x_i$ and also $x_j > 1 - x_i$, player j obtains $1 - x_i - d$ against the rational type and thus should demand $\hat{x}$ as it maximizes his payoff against the chicken type. (Because $\hat{x} > 1/2 + 2d$, this is feasible.) Then, his payoff is given by $\Pi_j(x_i, \hat{x}) = \gamma_j \hat{x} + (1-\gamma_j)(1-x_i-d)$. Because $x_i \le M_i \le 1/2 + 2d$, we have $\Pi_j(x_i, \hat{x}) \ge \gamma_i \hat{x} + (1-\gamma_i)(1/2 - 3d)$.

When $1 - x_i < x_j < x_i$, player j's expected payoff is given by $\Pi_j(x_i, x_j) = x_j - (1 - \gamma_j)d < 1/2 + 2d$. The last inequality comes from the inequality $x_j < x_i \le M_i \le 1/2 + 2d$.

We compare $\gamma_i \hat{x} + (1-\gamma_i)(1/2 - 3d)$ with $1/2 + 2d$:

$$\gamma_i \hat{x} + (1-\gamma_i)(1/2 - 3d) - (1/2 + 2d) = \gamma_i(\hat{x} - 1/2) - (5 - 3\gamma_i)d > 0.$$

The inequality comes from the supposition in the statement. It implies that, for any x_i, demanding $\hat{x}$ is the unique best response of player j. However, given this response, player i's best response is $[\hat{x}]_-$, which is greater than $1/2 + 2d$. This is a contradiction.

Now that we know that $M_i > 1/2 + 2d$, the following proves that $M_j > 1/2 + 2d$ in two steps.

(ii) $\gamma_j > 0$

To prove it, we suppose the contrary: $M_j \le 1/2 + 2d$. Because $M_i > 1/2 + 2d$, player i makes a demand higher than $1/2 + 2d$. Such demand is incompatible because $m_j \ge 1/2 - 2d$ as shown in Lemma A.2 and is always underbidden because $M_j \le 1/2 + 2d$. Hence, player i should demand $\hat{x}$ to take the best advantage of the chicken type when she demands more than $1/2 + 2d$. Now consider player j's strategy. When $x_j \le 1/2 + 2d$ as supposed, his highest expected payoff is no greater than $1/2 + 2d$ because $m_i \ge 1/2 - 2d$. On the other hand, when he demands $[\hat{x}]_-$, he expects to obtain at least $\gamma_i \hat{x} + (1-\gamma_i)(1/2 - 3d)$. This is because he obtains at least $1/2 - 2d$ minus the bargaining cost when player i is the rational type and demands no more than $1/2 + 2d$. (When player i demands $\hat{x}$ and turns out to be the rational type, his payoff is even higher.) From the first part, we know that the latter gives the higher payoff. This is a contradiction. Therefore, if $\gamma_j > 0$, the statement in the lemma has to hold.

(iii) $\gamma_j = 0$

To prove it, we suppose the contrary: $M_j \le 1/2 + 2d$. In the following, we look at player i's demand χ such that $\chi > 1/2 + 2d$. Because $M_i > 1/2 + 2d$, player i makes such a demand in equilibrium. We compare the payoff from that demand with the one from some other demand. Because $m_j \ge 1/2 - 2d$, the demands are incompatible given χ. Moreover, because $M_j \leq 1/2 + 2d$, player j makes a lower demand than χ. Hence, player i obtains the payoff of $1 - x_j - d$ from the demand of χ because $\gamma_j = 0$.

(iii-a) $1 - m_j > M_j$

When $1 - m_j > M_j$, we consider player i's switch of demands to $\max\{M_j, 1 - m_j - d\}$. Because $m_j \geq 1/2 - 2d$ by Lemma A.2 and $M_j \leq 1/2 + 2d$ by supposition, it holds that $\max\{M_j, 1 - m_j - d\} \leq 1/2 + 2d$. We consider two cases depending on whether M_j is lower than $1 - m_j - d$ or not.

First, suppose that $M_j \geq 1 - m_j - d$ and consider player i's switch to the demand of M_j. For any x_j such that $1 - M_j < x_j \leq M_j$, the demands are incompatible and player j makes either a lower or an identical demand. Hence, the payoff of player i is unchanged. For any x_j such that $m_j \leq x_j \leq 1 - M_j$, the demands become compatible given $x_i = M_j$. Then, player i obtains $(1 + M_j - x_j)/2$. Compare this with the one from the demand χ:

$$\frac{1 + M_j - x_j}{2} - (1 - x_j - d) = \frac{-1 + M_j + x_j + 2d}{2} \geq \frac{-1 + M_j + m_j + 2d}{2} \geq \frac{d}{2} > 0.$$

By definition, $x_j \geq m_j$ and thus we have the first inequality above. The second inequality comes from the current supposition. Because $1 - m_j > M_j$, the latter contingency has a positive probability. Hence, the expected payoff of player i increases by switching from χ to $1 - m_j - d$. This is a contradiction as player i is supposed to demand χ in equilibrium.

Next, suppose that $M_j < 1 - m_j - d$ and consider player i's switch to the demand of $1 - m_j - d$. When $m_j + d < x_j \leq M_j$, the payoff of player i does not change by the switch as the demands are incompatible and player j makes a lower demand. On the other hand, when $m_j \leq x_j \leq m_j + d$, the demands become compatible given player i's demand of $1 - m_j - d$. Then, player i obtains $(1 + 1 - m_j - d - x_j)/2 = (2 - m_j - d - x_j)/2$. Compare this with the one from the demand χ:

$$\frac{2 - m_j - d - x_j}{2} - (1 - x_j - d) = \frac{-m_j + d + x_j}{2} \geq \frac{d}{2} > 0.$$

By definition, $x_j \geq m_j$ and thus we have the first inequality above. By the definition of m_j, the latter contingency has a positive probability. Hence, the expected payoff of player i increases by switching from χ to $1 - m_j - d$. This is a contradiction as player i is supposed to demand χ in equilibrium.

(iii-b) $1 - m_j \leq M_j$

Because $1 - m_j \leq M_j$ and $m_j \leq M_j$, we have $2M_j \geq M_j + m_j \geq 1$ or $M_j \geq 1/2$.

If $M_j = 1/2$, the above inequality implies that $m_j = 1/2$ and thus player j demands $1/2$ with the probability one. Given this demand, the unique best response of player i is to demand $1/2$ because it saves the bargaining cost without affecting the share that he receives. This contradicts the inequality $M_i > 1/2 + 2d$.

Now suppose that $M_j > 1/2$. We consider the switch of demands by player i from χ to $M_j - \delta$ such that $\delta < d$ and $M_j - \delta > 1/2$. We study three cases depending on the size of player j's demand.

First, suppose that $x_j > M_j - \delta$. Then, the payoff of player i from $M_j - \delta$ is given by $M_j - \delta - d$ because $x_j > M_j - \delta > 1/2$. Because $x_j > M_j - \delta > 1/2$, we have $1 - x_j - d < 1/2 - d < M_j - \delta - d$. Namely, player i is better off by demanding $M_j - \delta$ instead of χ. Note that this contingency occurs with a positive probability because of the definition of M_j.

Second, suppose that $1 - (M_j - \delta) < x_j \leq M_j - \delta$. Then, the switch to $M_j - \delta$ does not change player i's payoff because the demands are still incompatible and player j makes either a lower or an identical demand.

Finally, suppose that $x_j \leq 1 - (M_j - \delta)$. Given the demand of $M_j - \delta$, the demands become compatible and player i obtains $(1 + M_j - \delta - x_j)/2$. Compare this with that from χ:

$$\begin{aligned}\frac{1 + M_j - \delta - x_j}{2} - (1 - x_j - d) &= \frac{-1 + M_j - \delta + x_j}{2} + d \\ &\geq \frac{-1 + M_j - \delta + m_j}{2} + d \\ &\geq \frac{-\delta}{2} + d > 0.\end{aligned}$$

By definition, we have that $x_j \geq m_j$, which causes the first inequality. The second inequality comes from the current supposition that $1 - m_j \leq M_j$. The last equality holds because $\delta < d$ by construction. Hence, player i obtains a higher payoff from $M_j - \delta$ than from χ.

Combining these arguments, we can conclude that the expected payoff of player i is higher from the demand $M_j - \delta$ than from the demand χ. This contradicts with the fact that χ needs to be made by player i in equilibrium.

The above shows that, if $M_j \leq 1/2 + 2d$, the inequality $M_i > 1/2 + 2d$ is never satisfied. Therefore, it also has to hold that $M_j > 1/2 + 2d$ when $\gamma_j = 0$. *Q.E.D.*

The essential part of the above proof is to show that, when player i may be the chicken type and her maximum demand is no more than $1/2 + 2d$, the other player, player j, would want to exploit the possibility of the chicken type by demanding $\hat{x}$. By demanding $\hat{x}$, player j obtains at least $\gamma_i \hat{x} + (1 - \gamma_i)(1/2 - 3d)$ because he obtains at least $1 - (1/2 + 2d) - d$ against the rational type. On the other hand, because $m_i \geq 1/2 - 2d$ by lemma A.2, he obtains at most $1/2 + 2d$ by trying to underbid some of player i's demands and demanding less than $1/2 + 2d$. The former is decreasing in d and the latter is increasing in d and moreover the former is larger than the latter at $d = 0$. The condition that the former is bigger than the latter is the one specified in the lemma. In any equilibrium, one player cannot demand $\hat{x}$

with the probability of one as it leads to the underbidding by the other. It implies that the supposed demands of player i is too low and, in equilibrium, player i needs to demand more than $1/2 + 2d$ with a positive probability.

Note that, combined with Lemma A.2, this lemma implies that the players' demands become incompatible with a positive probability.

The underlying cause of high demands is the desire of the player who wants to exploit the chicken type. Hence, the maximum demand cannot be higher than $\hat{x}$.

Lemma A.4

Suppose that Assumption A' is satisfied and also that $\gamma_{max} > 0$. Then, $M_i \leq \hat{x}$ for $i \in \{1, 2\}$.

(proof)

Without loss generality, let us assume that $M_i \geq M_j$. By assuming $M_i > \hat{x}$, we derive a contradiction. We consider the switch of the demands from the one above $\hat{x}$ to the one below it. It improves the payoff of the player against the chicken type because he leaves the negotiation after the former and yields in the latter. In the following, we show that there is such a switch that improves the payoff against the rational type as well.

First, suppose that $M_i > M_j$. Consider player i's switch of demands from M_i or a little lower than it to $1/2 + 2d$. Because $M_j > 1/2 + 2d$, player i creates the chance of underbidding. Namely, when $x_j > 1/2 + 2d$, the switch improves her expected payoff. On the other hand, when player j makes the demand no more than $1/2 + 2d$, player i's payoff is unchanged. This is a contradiction.

Second, suppose that $M_i = M_j$ and one player demands it with a positive probability. Without loss of generality, we suppose that it is player i who does so. (By Lemma A.1, there exists at most one player who does so.) Then, switching from $\hat{x}$ to $1/2 + 2d$ improves player i's expected payoff by the same reason as above. Thus, this case also leads to a contradiction.

Finally, suppose that $M_i = M_j$ and no player demands it with a positive probability. Consider that player i switches her demands from $M_i - \delta$ to $1/2 + 2d$. When player j demands more than $M_i - \delta$, player i decreases her payoff by this switch because she underbids player j in either case. On the other hand, when player j demands no more than $M_i - \delta$, player i improves her payoff because only her altered demand underbids player j's. When we take sufficiently small δ, we can lower the probability of the former because player j does not demand M_j with a positive probability. Hence, there exists some δ given which the above switch improves player i's expected payoff. Then, by the continuity of the distribution, we can conclude that player i improves her payoff by switching to the demand of $1/2 + 2d$

whenever she is supposed to demand no lower than $M_i - \delta$. By definition, the event $x_i \in (M_i - \delta, M_i)$ occurs with a positive probability. It implies that player i wants to deviate with a positive probability.

In any of the three cases above, we have had a contradiction. Therefore, it has to hold that $M_i \leq \hat{x}$. *Q.E.D.*

Lemma A.4 shows that the supremum of player i's demands cannot be higher than $\hat{x}$. The next lemma shows that, when the bargaining cost is sufficiently small, it is actually equal to $\hat{x}$. Without loss of generality, suppose that the highest demand of player i is no lower than that of player j. When the bargaining cost is sufficiently small, Lemma A.3 shows that player i's demand M_i always leads to the incompatibility. Moreover, by supposition it is always underbidden. The reason that she makes such a demand should be to take advantage of the chicken type. Then, it is best for player i to set $M_i = \hat{x}$ to take the maximum advantage. Given this possibility, the desire to underbid causes player j to make demands up to $\hat{x}$.

Lemma A.5

Suppose that Assumption A' is satisfied, that $\gamma_{max} > 0$, and that $d < (\gamma_{max}(\hat{x} - 1/2))/(5 - 3\gamma_{max})$. Then, it holds that $M_1 = M_2 = \hat{x}$.
(proof)

We prove the statement in two steps.

(i) $\gamma_1 > 0$ and $\gamma_2 > 0$.

We show that, if both $\gamma_1 > 0$ and $\gamma_2 > 0$, it holds that $M_1 = M_2 = \hat{x}$. Suppose not. Because Lemma A.4 shows that $M_i \leq \hat{x}$, it holds that $M_k < \hat{x}$ for some $k \in \{1, 2\}$. Without loss of generality, suppose that $M_i \geq M_j$.

First, consider the case that $M_j < M_i < \hat{x}$. Given this inequality, player i is better off by demanding $\hat{x}$ than by making a demand in $(M_j, M_i]$ because both give the same payoff against the rational type and the former improves her payoff against the chicken type. Hence, this case does not happen.

Second, consider the case that $M_j < M_i = \hat{x}$. By the same reason as above, player i is better off by demanding $\hat{x}$ than by making a demand in (M_j, M_i). Hence, player i does not make any demand in (M_j, M_i). We study two cases depending on whether player i demands M_j with a positive probability or not. We start with the case that she does. Lemma A.1 shows that at most one player makes a given demand with a positive probability. Hence, we have that $P_i(M_j) > 0 = P_j([M_j]_{n-})$ for any $n \geq 0$. Because player j always demands less than M_j, player i is better off by demanding $\hat{x}$ than by demanding M_j due to the possibility of the chicken type. Therefore, this case does not occur. Next, suppose that $P_i([M_j]_{n-}) = 0$ for any $n \geq 0$. We compare player j's strategy of demanding $M_j - \epsilon$ with that of demanding $[\hat{x}]_-$. When player i is the rational type and demands no more than $M_j - \epsilon$, these two strategies give the same payoff to player j. When player i is the rational type and demands more than $M_j - \epsilon$ and less than M_j, player j is worse off. In particular, the former gives $M_j - \epsilon$ and the latter gives at least $1 - M_j$. Either when player i is the rational type and demands $\hat{x}$ or when player i is the chicken type, player j is better off as the former gives $M_j - \epsilon$ and the latter gives $\hat{x}$. When we subtract the expected payoff given the demand of $M_j - \epsilon$ from that given the demand of $\hat{x}$, the difference is at least as large as $\gamma_i(\hat{x} - M_j + \epsilon) - (1 - \gamma_i)\text{Prob}(M_j - \epsilon < x_i < M_j)(2M_j - \epsilon - 1)$. Because $P_i([M_j]_{n-}) = 0$ for any $n \geq 0$, it holds that $\text{Prob}(M_j - \epsilon < x_i < M_j) \downarrow 0$ as $\epsilon \downarrow 0$. Hence, there exists some $\epsilon'(> 0)$ such that this becomes positive for any $\epsilon < \epsilon'$. This is a contradiction. Therefore, this case does not occur, either.

Finally, consider the case that $M_i = M_j < \hat{x}$. Either when $P_i(M_i) > 0$ or when $P_i(M_i) = 0$, the similar argument as in the above paragraph shows that some player (player i in the former or player j in the latter) wants to deviate. Namely, the contradictions will occur given this supposition. Hence, this case does not occur.

(ii) $\gamma_j = 0 < \gamma_i$.

We now want to show that, even if one player has no possibility of the chicken type, it holds that $M_1 = M_2 = \hat{x}$. Suppose not. There are four possible cases, which we consider in turn.

First, consider the case that $M_i < M_j < \hat{x}$. Given this inequality, player j is better off by demanding $\hat{x}$ than by making a demand in $(M_i, M_j]$ because both give the same payoff against the rational type and the former improves her payoff against the chicken type. Hence, this case does not happen.

Second, consider the case that $M_i < M_j = \hat{x}$. Player j has no incentive to make a demand in (M_i, M_j) because the demand of $\hat{x}$ is better for him due to the possibility of the chicken type. Player j does not demand M_i with a positive probability. If he would do so, Lemma A.1 shows that player i would not do so and then, by the same reason as above, player j would be better off by demanding $\hat{x}$. Now consider player i's switch of demands from $M_i - \epsilon$ to $[\hat{x}]_-$. When player j demands no more than $M_i - \epsilon$, these two demands give the same payoff to player i. When player j demands more than $M_i - \epsilon$ and less than M_i, player i is worse off. In particular, the former gives $M_i - \epsilon$ and the latter gives at least $1 - M_i$. When player j demands $\hat{x}$, player i is better off as the former gives $M_i - \epsilon$ and the latter gives $\hat{x}$. When we subtract the expected payoff given the demand of $M_i - \epsilon$ from that given the demand of $\hat{x}$, the difference is at least as large as $\text{Prob}(x_j = \hat{x})(\hat{x} - M_i + \epsilon) - \text{Prob}(M_i - \epsilon < x_j < M_i)(2M_i - \epsilon - 1)$. Because $\text{Prob}(M_i - \epsilon < x_j < M_i) \downarrow 0$ as $\epsilon \downarrow 0$, there exists some $\epsilon'(> 0)$ such that this becomes positive for any $\epsilon < \epsilon'$. This is a contradiction. Therefore, this case does not occur, either.

Third, consider the case that $M_i = M_j < \hat{x}$. We want to claim that neither player demands $M_i(= M_j)$ with a positive probability. From Lemma A.1, we know that at most one player demands it with a positive probability. Suppose that $P_i(M_i) = 0$. Because player i always demands less than M_j, player j is better off by demanding $\hat{x}$ than by demanding M_j due to the possibility of the chicken type. Hence, player j does not demand M_j with a positive probability. Suppose that $P_j(M_j) = 0$. Because $M_i = M_j$, player i's demand of M_i is always underbidden. Since $M_j > 1/2 + 2d$ and player j is known to be the rational type, it is better for him to demand $1/2$. Therefore, neither player demands $M_i(= M_j)$ with a positive probability. We compare player j's strategy of demanding $M_j - \epsilon$ with that of demanding $\hat{x}$. By the similar argument as in the above part (i), we can show that the former is better for player j when ϵ is sufficiently small. This is a contradiction. Therefore, this case does not occur.

Finally, consider the case that $M_j < M_i \leq \hat{x}$. Given this inequality, player i is better off by demanding $1/2$ than by making a demand in $(M_j, M_i]$ because player j is known to be the rational type and the latter demand is always underbidden by him. (From Lemma A.3, we know that $M_j > 1/2+2d$ under the supposition.) Hence, this case does not happen.

Because all four cases have led to a contradiction, we can conclude that $M_1 = M_2 = \hat{x}$.

Q.E.D.

We now focus on the players' demands bigger than $1/2 + 2d$ and no bigger than $\hat{x}$: $1/2 + 2d < x_k \leq \hat{x}$ for any $k \in \{1, 2\}$. Given these demands, lemma A.2 implies that the demands become incompatible. The reason to randomize the demands that lead to incompatibility is the balance between the desire to make a higher demand and the fear of being underbidden. When a player does not make a demand in a certain interval, the other player does not have to fear the underbidding in that interval. Hence, instead of making a demand in that interval, he prefers making the demand at the highest end of that interval. It implies that, if a player does not make a demand in some interval above $1/2+2d$, then the other will not, either. However, if both players do not make demands in a certain interval, due to the lack of the underbidding, at least a player would want to make the demand at the highest end of that interval instead of making demands just below the lowest end of that interval. This leads to a contradiction and there should be no gap in the distribution of demands above $1/2 + 2d$.

Lemma A.6

Suppose that Assumption A' is satisfied, that $\gamma_{max} > 0$, and that $d < (\gamma_{max}(\hat{x} - 1/2))/(5 - 3\gamma_{max})$.

(1) If there is m such that $1/2+2d < m < \hat{x}$ and $\int_{1/2+2d}^{m} f_j(x)dx + \sum_{x\in(1/2+2d,m)} P_j(x) = 0$, it holds that $\int_{1/2+2d}^{m} f_i(x)dx + \sum_{x\in(1/2+2d,m)} P_i(x) = 0$.

(2) There exists m^* such that, for some $j \in \{1,2\}$, $1/2+2d < m^* < \hat{x}$ and $\int_{m^*}^{m^*+\delta} f_j(x)dx + \sum_{x\in[m^*,m^*+\delta)} P_j(x) > 0$ for any $\delta > 0$. Given such m^*, it holds that, for any a and b such that $m^* \leq a < b \leq \hat{x}$, $\int_a^b f_k(x)dx + \sum_{x\in(a,b)} P_k(x) > 0$ for $k = 1, 2$.

(proof)

We prove the lemma in several steps.

(i) Take a and b such that $1/2+2d < a < b \leq \hat{x}$. Suppose that $\int_a^b f_j(x)dx + \sum_{x\in(a,b)} P_j(x) = 0$, and that $\int_b^{b+\delta} f_j(x)dx + \sum_{x\in[b,b+\delta)} P_j(x) > 0$ for some $\delta \geq 0$. Then, we claim that $\int_a^b f_i(x)dx + \sum_{x\in(a,b)} P_i(x) = 0$.

Take any $x \in (a, b)$ and compare player i's strategy of demanding it with that of demanding $[b]_{n-}$ for some n. Specifically, as the latter strategy, suppose that player i demands b when player j does not demand b with a positive probability, and that she demands $[b]_-$ when he does. (By Lemma A.1, player j never demands $[b]_{n-}$ for any $n \geq 1$.) By abusing the notation, we call this demanding $[b]_{n-}$.

By Lemma A.2, we know that player j's demand is no smaller than $1/2 - 2d$. Hence, any demand in (a, b) by player i will lead to the incompatibility due to the supposition that $a > 1/2 + 2d$. Demanding $[b]_{n-}$ by player i also leads to the incompatibility. Hence, given either of the strategies, the players' demands become incompatible.

When player j demands no more than a, player i's strategy of making a demand in (a, b) gives her the same payoff as her strategy of demanding $[b]_{n-}$ because player j underbids player i. When player i demands no less than b, both player i's demand in (a, b) and her demand of $[b]_{n-}$ means that player i underbids player j. By supposition, the probability of this underbidding is positive. Hence, under the equitable rule, the latter strategy gives the higher expected payoff to player i. Therefore, player i never makes a demand in (a, b).

(ii) We now prove the first statement of the lemma.

Because $M_j = \hat{x}$, we have either $P_j(\hat{x}) > 0$ or $\int_{\hat{x}-\delta}^{\hat{x}} f_j(x)dx + \sum_{x\in(\hat{x}-\delta,\hat{x})} P_j(x) > 0$ for any $\delta > 0$. Hence, for any m such that $1/2 + 2d < m < \hat{x}$, it holds that $\int_m^{\hat{x}} f_j(x)dx + \sum_{x\in[m,\hat{x}]} P_j(x) > 0$. Then, the direct application of the above claim gives us the first statement of the lemma.

(iii) We want to show that there exists m^* such that, for some $j \in \{1,2\}$, $1/2+2d < m^* < \hat{x}$ and $\int_{m^*}^{m^*+\delta} f_j(x)dx + \sum_{x\in[m^*,m^*+\delta)} P_j(x) > 0$ for any $\delta > 0$.

From Lemma A.5, we know that $M_i = M_j = \hat{x}$. Lemma A.1 shows that at most one player demands $\hat{x}$ with a positive probability. Without loss of generality, suppose that player j does not demand $\hat{x}$ with a positive probability. Then, the equation $M_j = \hat{x}$ implies that $\int_{\hat{x}-\delta}^{\hat{x}} f_j(x)dx + \sum_{x\in(\hat{x}-\delta,\hat{x})} P_i(x) > 0$ for any $\delta > 0$. Let us take δ such that $\hat{x} - \delta > 1/2 + 2d$. If $P_i(x) > 0$ for some $x \in (\hat{x} - \delta, \hat{x})$, then such x will satisfy the condition for m^*. If $P_i(x) = 0$ for any $x \in (\hat{x} - \delta, \hat{x})$, it holds that $\int_{\hat{x}-\delta}^{\hat{x}} f_j(x)dx > 0$. Define $m^* \equiv \inf\{x \,|\, x \geq \hat{x} - \delta \text{ and } f_j(y) > 0 \text{ for } x < \forall y < \hat{x}\}$. The above inequality implies that $m^* < \hat{x}$ and it satisfies the stated condition.

(iv) We now want to show that, when $\int_{m^*}^{m^*+\delta} f_k(x)dx + \sum_{x\in[m^*,m^*+\delta)} P_k(x) > 0$ for any $\delta > 0$ holds for $k = j$, it also holds for $k = i$.

Suppose not. Then, there exists $\delta' > 0$ such that $\int_{m^*}^{m^*+\delta'} f_i(x)dx + \sum_{x\in[m^*,m^*+\delta')} P_i(x) = 0$. (We take δ' such that $m^* + \delta' < \hat{x}$.) Because $M_i = \hat{x}$, it has to hold that $\int_{m^*+\delta'}^{\hat{x}} f_i(x)dx + \sum_{x\in[m^*+\delta',\hat{x}]} P_i(x) > 0$. Because $m^* > 1/2+2d$, any demand in $[m^*, m^* + \delta')$ leads to the incompatibility. Then, for player j, it is better to demand $[m^* + \delta']_-$ under the equitable rule by the same logic used in the first step above. This is a contradiction. Hence, the statement has to hold for $k = i$ as well.

(v) Take m^* that satisfies the second statement of the lemma and take a and b such that $m^* < a, b < \hat{x}$. We claim that there is no gap in distribution either at (m^*, b) or at $(a, \hat{x})$. Namely, We claim that, for any $k \in \{1, 2\}$, $\int_{m^*}^{b} f_k(x)dx + \sum_{x\in(m^*,b)} P_k(x) > 0$ and $\int_{a}^{\hat{x}} f_k(x)dx + \sum_{x\in(a,\hat{x})} P_k(x) > 0$ for any a and b such that $m^* < a, b < \hat{x}$.

First, let us prove the first inequality. Suppose not. Without loss of generality, suppose that $\int_{m^*}^{b} f_j(x)dx + \sum_{x\in(m^*,b)} P_j(x) = 0$ for some $b > m^*$. Because $M_j = \hat{x}$, it has to hold that $b < \hat{x}$ and $\int_{b}^{\hat{x}} f_j(x)dx + \sum_{x\in(b,\hat{x})} P_j(x) > 0$. The claim at the first step shows that $\int_{m^*}^{b} f_i(x)dx + \sum_{x\in(m^*,b)} P_i(x) = 0$. On the other hand, the forth step shows that $\int_{m^*}^{m^*+\delta} f_k(x)dx + \sum_{x\in[m^*,m^*+\delta)} P_k(x) > 0$ for any k and any δ. Given this, the above inequality implies that $P_j(m^*) > 0$. Then, from Lemma A.1, it has to hold that $P_i(m^*) = 0$. Because $\int_{m^*}^{m^*+\delta} f_i(x)dx + \sum_{x\in[m^*,m^*+\delta)} P_i(x) > 0$ for any $\delta > 0$, it has to hold that $\int_{m^*}^{b} f_i(x)dx + \sum_{x\in(m^*,b)} P_i(x) > 0$. However, this contradicts the prediction proven above.

Second, let us prove the second inequality. Suppose not. Without loss of generality, suppose that $\int_{a}^{\hat{x}} f_j(x)dx + \sum_{x\in(a,\hat{x})} P_j(x) = 0$ for some $a < \hat{x}$. Because $M_j = \hat{x}$, it has to hold that $P_j(\hat{x}) > 0$. Then, the claim at the first step shows that $\int_{a}^{\hat{x}} f_i(x)dx + \sum_{x\in(a,\hat{x})} P_i(x) = 0$ Because $P_j(\hat{x}) > 0$, Lemma A.1 shows that $P_i(\hat{x}) = 0$. Because $M_i = \hat{x}$, it implies that $\int_{a}^{\hat{x}} f_i(x)dx + \sum_{x\in(a,\hat{x})} P_i(x) > 0$. This is a contradiction.

(vi) We argue that, if there is a gap in distribution for a player, the distribution of the other player's demands has a gap at the same place. Namely, take m^* that satisfies the second statement of the lemma and take a and b such that $m^* < a < b < \hat{x}$. We claim that, if for any $\delta > 0$, (1) $\int_{a-\delta}^{a} f_k(x)dx + \sum_{x\in(a-\delta,a]} P_k(x) > 0$, (2) $\int_a^b f_k(x)dx + \sum_{x\in(a,b)} P_k(x) = 0$, and (3) $\int_b^{b+\delta} f_k(x)dx + \sum_{x\in[b,b+\delta)} P_k(x) > 0$ hold for $k = j$, they also hold for $k = i$.

Suppose not. The claim proven at the first step shows that $\int_a^b f_i(x)dx + \sum_{x\in(a,b)} P_i(x) = 0$. Hence, either (1') $\int_{a-\delta}^{a} f_i(x)dx + \sum_{x\in(a-\delta,a]} P_i(x) = 0$ or (3') $\int_b^{b+\delta} f_i(x)dx + \sum_{x\in[b,b+\delta)} P_i(x) = 0$ holds for for some $\delta > 0$.

First, consider the case that the condition (3') holds. Because $M_i = \hat{x}$, we have $\int_{b+\delta}^{\hat{x}} f_i(x)dx + \sum_{x\in[b+\delta,\hat{x}]} P_i(x) > 0$. Then, applying the claim at the first step of this proof, we can conclude that $\int_a^{b+\delta} f_j(x)dx + \sum_{x\in(a,b+\delta)} P_j(x) = 0$. This contradicts the above condition (3).

Next, consider the case that the condition (1') holds. Then, it holds that $\int_{a-\delta}^{b} f_i(x)dx + \sum_{x\in(a-\delta,b)} P_i(x) = 0$. From the preceding paragraph, we know that the condition (3) holds also for player i: $\int_b^{b+\delta} f_i(x)dx + \sum_{x\in[b,b+\delta)} P_i(x) > 0$ for any δ. Then, applying the claim at the first step of this proof, we can conclude that $\int_{a-\delta}^{a} f_j(x)dx + \sum_{x\in(a-\delta,b)} P_j(x) = 0$. This contradicts the above condition (1).

Therefore, the statement of this step has to hold.

(vii) There should be no gap in distribution.

Take m^* that satisfies the second statement of the lemma and take a and b such that $m^* \le a < b \le \hat{x}$. We claim that, for any such a and b, $\int_a^b f_k(x)dx + \sum_{x\in(a,b)} P_k(x) > 0$ for any k.

Suppose not. From the claim at the fifth step, we know that, if there is a gap, it should occur at the interval (a, b) such that $m^* < a < b < \hat{x}$. Without loss of generality, suppose that there is a gap in distribution for player j. Because $M_j = \hat{x}$, it holds that $\int_b^{\hat{x}} f_j(x)dx + \sum_{x\in[b,]\hat{x}} P_j(x) > 0$. By the construction of m^*, $\int_{m^*}^{m^*+\delta} f_j(x)dx + \sum_{x\in[m^*,m^*+\delta)} P_j(x) > 0$ for any $\delta > 0$. Hence, if there is a gap in distribution, we can choose a and b so that the following conditions are satisfied: (1) $\int_{a-\delta}^{a} f_j(x)dx + \sum_{x\in(a-\delta,a]} P_j(x) > 0$, (2) $\int_a^b f_j(x)dx + \sum_{x\in(a,b)} P_j(x) = 0$, and (3) $\int_b^{b+\delta} f_j(x)dx + \sum_{x\in[b,b+\delta)} P_j(x) > 0$. Then, the sixth step above shows that the same conditions hold for player i.

We consider two cases.

(vii-1) $P_k(a) > 0$ for some $k \in \{1, 2\}$.

Lemma A.1 shows that, for any demand, at most one player makes it with a positive probability. When player k switches his demand from a to $[b]_{n-}$, the same logic as in the first step above shows that he can improve his payoff. (We use the same convention for $[b]_{n-}$ as in the proof of the first step.) Hence, this case does not occur.

(vii-2) $P_k(a) = 0$ for any $k \in \{1, 2\}$.

By supposition, we have that $\int_{a-\delta}^{a} f_j(x)dx + \sum_{x\in[a-\delta,a)} P_j(x) > 0$ for any $\delta > 0$. Take $\epsilon > 0$ and evaluate the change in the payoffs of player i from demanding $a - \epsilon$ to that from demanding $[b]_{n-}$. When player j demands no more than $a-\epsilon$, player i's payoff is same given either demand. When player j demands between $a-\epsilon$ and a, the switch of demands leads to the decrease of player i's payoff by at most $a-\epsilon-(1-a)<2a-1$. Because $P_j(a)=0$, it occurs with the probability that $\int_{a-\epsilon}^{a} f_j(x)dx+\sum_{x\in(a-\epsilon,a)} P_j(x)$. Hence, the expected loss of player i from this switch is at most $(2a-1)\left[\int_{a-\epsilon}^{a} f_j(x)dx+\sum_{x\in(a-\epsilon,a)} P_j(x)\right]$. On the other hand, when player j demands no less than b, the switch of demands leads to the increase of the payoff by at least $(b-a)\left[\int_{b}^{1} f_i(x)dx+\sum_{x\in[b,1)} P_i(x)\right]$ (>0). Observe that the former converges to zero as $\epsilon\to 0$. Hence, there exists some $\epsilon'(>)$ such that the expected payoff of player i increases by the switch if $\epsilon<\epsilon'$. This means that player i does not make a demand in $(a-\epsilon',b)$. This is a contradiction.

Therefore, the statement in the lemma has to hold. *Q.E.D.*

When the players randomize their demands, they do not make any demand with a positive probability except for $\hat{x}$. To understand this property, suppose not. For example, suppose that player i demands $a(<\hat{x})$ with a positive probability. Then, player j wants to switch any of his demands a little higher than a to $[a]_-$. By doing so, he will lose a little when he underbids player i but will increase the chance of underbidding substantially. This will create a gap above a, which should not occur as shown in the above lemma. Note that this logic does not apply at the upper end of the distribution. The next lemma formally proves this property.

Lemma A.7

Suppose that Assumption A' is satisfied, that $\gamma_{max}>0$, and that $d<(\gamma_{max}(\hat{x}-1/2))/(5-3\gamma_{max})$.

Take m^* such that $1/2+2d<m^*<\hat{x}$ and $\int_{m^*}^{m^*+\delta} f_j(x)dx+\sum_{x\in[m^*,m^*+\delta)} P_j(x)>0$ for any $\delta>0$ for some $j\in\{1,2\}$. Then, it holds that $P_i(a)=0$ for any $a\in[m^*,\hat{x})$ and any $i\in\{1,2\}$.

(proof)

Suppose not. Without loss of generality, suppose that $P_i(a)>0$ for some $a\in[m^*,\hat{x})$. Lemma A.6 implies that $\int_{a}^{a+\delta} f_j(x)dx+\sum_{x\in(a,a+\delta)} P_j(x)>0$ for any $\delta>0$.

We now compare the payoff of player j from demanding $[a]_-$ with that from demanding $a+\epsilon$ $(0<\epsilon<\delta)$. When player i demands less than a, the two demands give the same payoffs to player j. Thus, for the comparison, we can restrict our attention to the case that $x_i\geq a$. In such a case, the demand of $[a]_-$ approximately gives a to player j. On the other hand, by demanding $a+\epsilon$, player j obtains $1-x_i$ if $x_i\in[a,a+\epsilon]$ and obtains $a+\epsilon$ if

$x_i > a + \epsilon$. When $x_i \in [a, a+\epsilon]$, the former is bigger than the latter because $1 - x_i \leq 1 - a < 1/2 - 2d$ and $a > 1/2 + 2d$. The difference is no smaller than $a - (1-a) = 2a - 1$. When $x_i > a + \epsilon$, the former is smaller than the latter by ϵ. Thus, when we subtract the expected payoff of the latter from that of the former, the difference is at least as large as $(2a-1)P_i(a \leq x_i < a+\epsilon) - \epsilon\text{Prob}(x_i > a+\epsilon) > (2a-1)P_i(a) - \epsilon\text{Prob}(x_i > a)$. Because $2a > 1$, there exists some $\epsilon' > 0$ such that, for any $\epsilon < \epsilon'$, the above expression becomes positive. It implies that player j does not make a demand in the interval $(a, a+\epsilon')$. This contradicts the property of no gap as is proven in Lemma A.6. Therefore, it has to hold that $P_i(a) = 0$. *Q.E.D.*

The above lemmas have shown that the players make demands in an interval $(m, \hat{x}]$ where $m > 1/2 + 2d$ (although they have not precluded the possibility that they make demands less than $1/2 + 2d$). Due to the possible gain from the chicken type as well as that from underbidding, making demands in this interval is better than making demands near one half when the bargaining cost is sufficiently small. Hence, the players do not make demands less than $1/2 + 2d$ and they randomize their demands only over the interval $(m, \hat{x}]$ where $m > 1/2 + 2d$. Summarizing this argument, the next lemma shows that the lower end of the distribution is bigger than $1/2 + 2d$ and lower than $\hat{x}$ for either player: $1/2 + 2d < m_k$ for $k \in \{1, 2\}$.

Lemma A.8

Suppose that Assumption A' is satisfied, that $\gamma_{max} > 0$, and that $d < \frac{\gamma_{max}^3(2\hat{x}-1)}{2(5-3\gamma_{max})}$.

It holds that $1/2 + 2d < m_k < \hat{x}$ for any $k \in \{1, 2\}$.

(proof)

We prove that $m_k < \hat{x}$ by supposing the contrary. Without loss of generality, suppose that $m_i = M_i = \hat{x}$. Lemma A.1 shows that at most one player makes a demand with a positive probability and thus this implies that player i demands $\hat{x}$ with the probability one. Under Assumption A', the unique best response of player j is to demand $[\hat{x}]_-$. However, given this response, the best response of player i is to demand $[\hat{x}]_{2-}$, which is a contradiction.

In the following, we prove $1/2 + 2d < m_k$ in three steps.

As the first step, we show that neither player makes a demand in $(1/2 + 2d, m)$ for some $m > 1/2 + 2d$ when the bargaining cost is small in the sense specified in the proposition. Note that $(\gamma_{max}(\hat{x} - 1/2))/(5 - 3\gamma_{max}) > \gamma_{max}^3(2\hat{x} - 1)/(2(5 - 3\gamma_{max}))$. Thus, from the lemmas A.5, A.6, and A.7, we know that the players randomize their demands over the same interval $(m, \hat{x})$ where $m \geq 1/2 + 2d$ and they do not make a demand in the interval with a positive probability. (We take the limit of the preceding arguments when we consider the case that $m = 1/2 + 2d$.) Moreover, Lemma A.2 shows that $m_j \geq 1/2 - 2d$. Hence, the expected payoff of player i from a demand $x_i \in (m, \hat{x})$ is given by the following formula:

$$\begin{aligned} E\Pi_i(x_i) \equiv &\gamma_j x_i + (1-\gamma_j)\Bigg\{(x_i - d)\big(1 - F_j(x_i)\big) + \int_m^{x_i} (1 - y - d) f_j(y) dy \\ &+ \int_{1/2-2d}^{1/2+2d} (1 - y - d) f_j(y) dy + \sum_{y \in [1/2-2d, 1/2+2d]} (1 - y - d) P_j(y)\Bigg\}. \end{aligned}$$

(Note that, as in the main text, $P_j(\hat{x}) = 1 - F_j([\hat{x}]_-)$.) For the randomization to occur, it needs to be constant in $(m, \hat{x})$. By the analysis in the main text, we know that it is so only when the distribution function takes the following form for $x \in (m, \hat{x})$ with some A_j:

$$F_j(x) = \frac{1}{1-\gamma_j}\left(1 - \frac{A_j}{\sqrt{2x-1}}\right).$$

Because $F_j(\hat{x}) \leq 1$, it has to hold that $A_j \geq \gamma_j\sqrt{2\hat{x}-1}$. Because $F_j(x) \geq 0$, it has to hold that $\sqrt{2x-1} \geq A_j$. By taking the square of both sides, rearranging the terms, and then substituting the above inequality, we obtain $x \geq (A_j^2 + 1)/2 \geq (\gamma_j^2(2\hat{x}-1)+1)/2$. Because the randomization occurs over the same interval, we have that $m \geq (\gamma_{max}^2(2\hat{x}-1)+1)/2$. By the supposition of the lemma, the right hand side is higher than $1/2 + 2d$. This proves that neither player makes a demand in $(1/2 + 2d, m)$ for some $m > 1/2 + 2d$.

Without loss of generality, let us suppose that $\gamma_{max} = \gamma_i > 0$. As the second step, we claim that player j does not make a demand in $[1/2 - 2d, 1/2 + 2d]$. Define q_k to be the probability that player k ($k = 1, 2$) makes a demand in $[1/2 - 2d, 1/2 + 2d]$: $q_k = \text{Prob}(1/2 - 2d \leq x_k \leq 1/2 + 2d)$. When he demands m, he obtains m against the chicken type, obtains at least $(1/2 - 2d) - d$ against the rational type who makes the demand in $[1/2 - 2d, 1/2 + 2d]$, and obtains $m - d$ against the rational type who demands no less than m. (The following argument applies to the degenerate case $m = \hat{x}$ by appropriately choosing the demand of player j in place of m. Specifically, when player i demands $\hat{x}$ with a positive probability and thus $q_i < 1$, we suppose that player j demands $[\hat{x}]_-$. Given this modification, the evaluation of the expected payoffs in the following becomes valid even for the degenerate case.) Hence, his expected payoff from m, $E\Pi_j(m)$, satisfies the following inequality:

$$E\Pi_j(m) \geq \gamma_i m + (1-\gamma_i) q_i \left(\frac{1}{2} - 3d\right) + (1-\gamma_i)(1-q_i)(m-d).$$

On the other hand, when he makes a demand in $[1/2 - 2d, 1/2 + 2d]$, he obtains at most $1/2 + 2d$ against the chicken type, obtains at most $1/2 + 2d$ against the rational type who makes the demand in $[1/2 - 2d, 1/2 + 2d]$, and obtains $(1/2 + 2d) - d$ against the rational type who demands no less than m. (Note that $m > 1/2 + 2d$.) Hence, his expected payoff from a demand $x_j \in [1/2 - 2d, 1/2 + 2d]$, $E\Pi_j(x_j)$, satisfies the following inequality:

$$E\Pi_j(x_j) \leq \gamma_i \left(\frac{1}{2} + 2d\right) + (1-\gamma_i)q_i \left(\frac{1}{2} + 2d\right) + (1-\gamma_i)(1-q_i)\left(\frac{1}{2} + d\right).$$

We now compare these two strategies:

$$\begin{aligned}
&E\Pi_j(m) - E\Pi_j(x_j) \\
&\quad \geq \gamma_i m + (1-\gamma_i)q_i \left(\frac{1}{2} - 3d\right) + (1-\gamma_i)(1-q_i)(m-d) \\
&\qquad - \left[\gamma_i \left(\frac{1}{2} + 2d\right) + (1-\gamma_i)q_i \left(\frac{1}{2} + 2d\right) + (1-\gamma_i)(1-q_i)\left(\frac{1}{2} + d\right)\right] \\
&\quad \geq \gamma_i \left(m - \frac{1}{2}\right) - (5 - 3\gamma_i)d.
\end{aligned}$$

Because $m \geq (\gamma_{max}^2(2\hat{x} - 1) + 1)/2$, the last expression is positive by the supposition of the lemma. It implies that player j is better off by demanding m than by making any demand in $[1/2 - 2d, 1/2 + 2d]$.

As the third and final step, we show that player i does not want to make a demand in $[1/2 - 2d, 1/2 + 2d]$, either. Such a demand causes the incompatibility and moreover it underbids player j's demand because the above argument shows that player j demands more than $1/2 + 2d$. For player i, it is, however, dominated by the demand of m because the latter is larger than $1/2 + 2d$ and still underbids any of player j's demand. It implies that player i does not make a demand in $[1/2 - 2d, 1/2 + 2d]$. *Q.E.D.*

The combination of the preceding lemmas and the derivation following Proposition 8 shows that, if there is an equilibrium, it is unique and the equilibrium distributions of demands are given by the formula derived in the main text. To conclude the proof of Proposition 8, we show that the derived distribution functions form an equilibrium.

First, note that the players' payoffs are constant over the interval $[m, \hat{x}]$ by construction. Thus, there is no incentive to deviate to a demand in $[m, \hat{x}]$,

Second, by using the argument in the proof of Lemma A.2, we can show that any demand less than $1/2 - 2d$ is dominated by the demand of $1/2$.

Third, any deviation to a demand less than m but no less than $1/2 + 2d$ leads to the incompatibility because $m > 1/2 + 2d$. Thus, such demand is dominated by the demand of m because either will underbid the demand of the other with the probability of one and, under the equitable rule, the latter attains a higher payoff.

Finally, consider the deviation to a demand higher than $\hat{x}$. Given such deviation, the chicken type of the other player will leave the negotiation. Moreover, it causes the incompatibility and is always underbidden by the other player. Hence, it is dominated by a demand in $(m, \hat{x})$ because the chicken type yields to it and it creates the chance of underbidding.

Therefore, there is no profitable deviation and thus the derived distribution functions give the unique equilibrium strategies.