Favorite-Longshot Bias in Parimutuel Betting: an Evolutionary Explanation

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Gambling Market

- By definition (and construction), it is an environment with **negative net returns**, which is known publicly.
- Is gambling an irrational act? Perhaps not, for most people who do gambling.
  - Pleasure of gambling can explain why they gamble (perhaps).
- Horse race track - some die hard gamblers with visible problems. But the most of bettors (voters , in Japanese) are ordinary *rational* people who try to beat the market.
- Then a strong form of the efficient market hypothesis might hold: the expected returns on horses should be equated.
- But **is it really so?**

Motivation

- Griffith (1949), studying US horse race track data of 1386 races, first reported that the rate of return of favorites is relatively and significantly greater than that of longshots - **the favorite - longshot bias** (FLB).
- Subsequent studies, for example, Weizman (JPE 1965), Ali (JPE, 1977), Jullien - Salanie (JPE, 2000), and Snowberg-Wolfers (JPE 2010), pointed out that FLBs are also observed in different data sets of horse races in different countries.
  - Snowberg-Wolfers (JPE 2010) uses a very large data set.
- A number of empirical studies (surveyed by Thaler and Ziemba (1988), Hausch and Ziemba (1995) and Jullien and Salanie (2008)) have documented that FLB emerges not only in race tracks among different countries, but also in several gambling markets other than horse races.

FLB in US Data

US: pari-mutuel markets

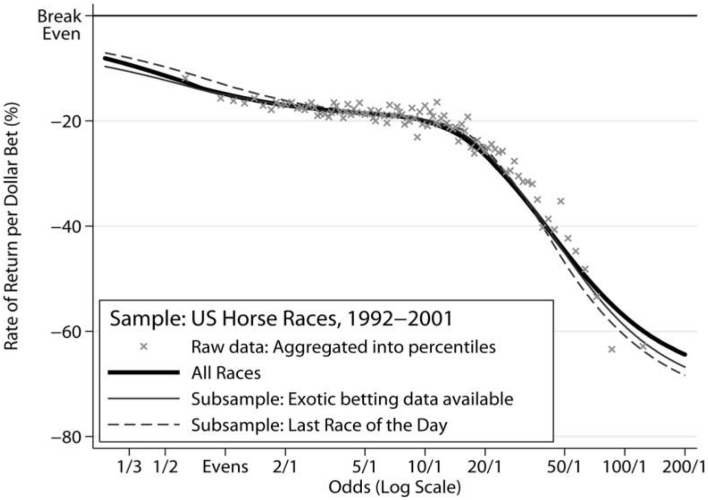


Figure: Figure 1 in Snowberg - Wolfers, JPE 2010

FLB in UK, Australia

UK: bookmakers, Australia: bookmakers competing with a state-run pari-mutuel market

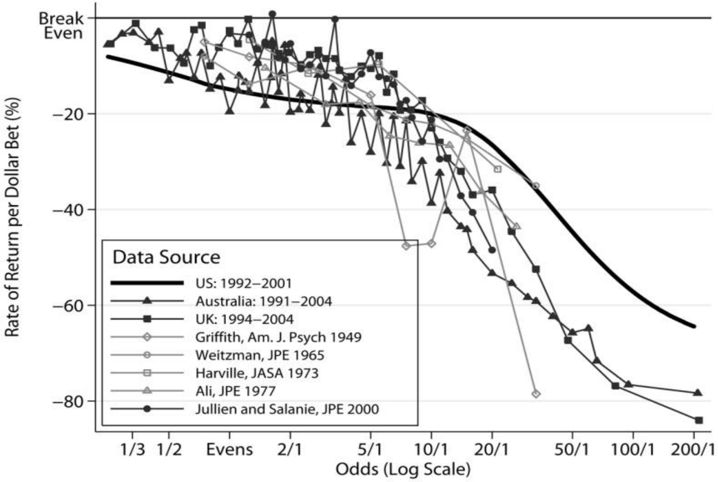


Figure: Figure 2 in Snowberg - Wolfers, JPE 2010

FLB bias: summary of evidence

- Robust in horse races as well as other gambling (e.g. American sports betting)
- Some argues that the examples of such evidence include financial markets.
- But there are some exceptions: Notably for horse races, **Hong Kong and Japan** race tracks show little FLB
  - Both HK and Japan are *pari-mutuel* markets, like US:
  - *pari-mutuel* markets: all the bets are pooled, and after a fraction (*track take*) is subtracted, the remaining is given to the winning bets
- A note on **track take** for win bet: US, 15%-17%, UK, around 13-15% (mostly bookmakers) Australia 12%, Singapore 10% (?), while HK 17.5% or higher, Japan 20% to 25%. France used to be around 30%, now much lower(?)

Plan of Talk

1. Review some theories to explain FLB in pari-mutuel system
    - a simple model of a pari-mutuel betting
    - explain why and in what sense FLB is a puzzle
    - possible theoretical explanations (following Ottaviani and Sorensen (2007)).
    - some drawbacks - <u>silent on the role of track take</u>
2. Our attempt: an evolutionary scenario
    - shows a simple evolutionary model explains FLB.
    - contrast with Friedman hypothesis - why don't "irrational people" get driven out of the markets?
    - <u>the role of track take re-examined</u>
3. Discussion - beyond race track toward economics

A Simple Pari-mutuel Betting Framework

- Two horses, F and L, and players bet on them. F wins with probability $p > \frac{1}{2}$
  - special interest on p somewhat close to one.
- total number of bets equal to 1 by normalization.
  - $y =$ the number of bet on F, thus $1 - y$ is that on L.
- track take $\tau \geq 0$ is subtracted from the total pool so $1 - \tau$ will be paid out to the winning bets
- (gross) payout per bet: $(1 - \tau) / y$ if F wins, $(1 - \tau) / (1 - y)$ if L wins.
  - Odds for F is $((1 - \tau) / y) - 1$ and Odds for L is $((1 - \tau) / (1 - y)) - 1$
- FLB in this setup: $p (1 - \tau) / y > (1 - p) (1 - \tau) / (1 - y)$, which occurs if and only if

$$p > y$$

Bench mark and a basic postulate

- if $\tau > 0$, at least one type of bet has a negative expected return.
- **We do not ask why people gamble!**
- Thus assume that each player bets exactly one unit for the race, irrespective of odds. So in particular, we can just look at the gross returns.
- If the players are risk neutral, then the expected returns must be equated:
$$p\frac{1-\tau}{y} = (1-p)\,\frac{1-\tau}{1-y}$$
- Hence $p = y$ is the only solution - **no FLB bias**

Risk Aversion and perfect competition

- A single representative agent (RA) with vNM function u on ex post wealth choosing a portfolio of bets, market payouts taken as given.
- if RA bets x on F and $1-x$ on L, the payouts are $\frac{1-\tau}{y}x$ if F wins and $\frac{1-\tau}{1-y}(1-x)$ if L wins.
- So RA's problem can be written as

$$\max_{x} pu\left(\frac{1-\tau}{y}x\right)+(1-p)\,u\left(\frac{1-\tau}{1-y}(1-x)\right)$$

- Assume risk aversion - FOC for an interior solution is

$$p\frac{1-\tau}{y}u'\left(\frac{1-\tau}{y}x\right)=(1-p)\,\frac{1-\tau}{1-y}u'\left(\frac{1-\tau}{1-y}(1-x)\right)$$

- Equilibrium: $x=y$, then $y=p$ must follow.- **no FLB bias**

Perfect competition and Mis-perception

- People underestimate a large probability event leading to a small gain.
- So one can introduce probability distortion function ϕ for F's to re-write the FOC:

$$\phi\left(p\right)\frac{1-\tau}{y}u'\left(\frac{1-\tau}{y}x\right)=\left(1-\phi\left(p\right)\right)\frac{1-\tau}{1-y}u'\left(\frac{1-\tau}{1-y}\left(1-x\right)\right)$$

- Equilibrium requires $x=y$, so

$$\frac{\phi\left(p\right)}{y}=\frac{1-\phi\left(p\right)}{1-y}$$

- Hence we have a scenario consistent with FLB, if $\phi\left(p\right)<p$ for $p>\frac{1}{2}$
- Note: then FLB is independent of the size of track take τ

Constrained choice market (1/2)

- Each single infinitesimal player independently chooses F or L, not a portfolio.
- In equilibrium, then the expected utility from payouts must be equated:

$$pu\left(\frac{1-\tau}{y}\right) = (1-p)\,u\left(\frac{1-\tau}{1-y}\right)$$

  which determines an equilibrium y.
- If u is increasing and concave with $u(0) = 0$, from the definition of concavity, for $y > \frac{1}{2}$,

$$yu\left(\frac{\tau}{y}\right) \geq (1-y)\,u\left(\frac{\tau}{1-y}\right)$$

  which is not consistent with $p > y$ and the equilibrium condition.
  - Note: some works assume further that the agents prefers gambling to start with, which immediately imply risk loving.

Constrained choice market (2/2)

- So FLB can be explained, but the common vNM must exhibit **risk loving**.
- Note: one could add misperception story here too, and create FLB with concave u

$$\phi\left(p\right)u\left(\frac{1-\tau}{y}\right)=\phi\left(1-p\right)u\left(\frac{1-\tau}{1-y}\right)$$

- Snowberg and Wolfers argue that with linear u, this model fits the data well, while the risk loving story performs poorly.
- Note that if $u\left(z\right)=z$, **the equilibrium y is independent of the track take.**

Information gap

- Suppose there are two types of agents, **I**nformed experts, and **U**ninformed nonexperts.
- Say U bets evenly on two horses, but I bets as long as the return is positive.
- Then in equilibrium betting on F must be fair gamble, so the expected value of F $p\frac{(1-\tau)}{y}$ must be 1, hence

$$\frac{p}{y} = \frac{1}{1-\tau} > 1$$

  exhibiting FLB.
- Notice that the bias **increases** in track take τ

Summary and Remarks

- It seems difficult to explain FLB with standard risk averse (even neutral) players.
  - our story will also involve risk loving, but the majority will be risk neutral
- The effect of track take is delicate, but generally speaking, **these theories suggest that the higher the track take is, the larger the bias**. unless agents are very risk loving
- But recall that FLB is rather weak in HK and Japan where track take is larger.

Set up

- There is a large pool of potential bettors:
- Two types: risk neutral (F), and longshot (L). Proportion π and $(1-\pi)$.
- Longshot type: bets on L irrespective of the odds.
- Risk neutral type: risk neutral, always bets on the highest expected payout.
  - if FLB exists, all of them bet on F, so they are de facto Favorate type.
- There are many races (i.i.d.) in one period. At the end of each period, some bettors quit.
  - Assume: "ruin" probability depends only on the prevailing odds (constant during the period) and the track take.
- New bettors are selected randomly from the potential bettors so that the total population is kept one.
  - other replacement rules are also possible but we keep it simple.

Population

- $y(t) =$ the fraction of F and so $1 - y(t) =$ fraction of L in period t
- Assume for now that $y(t) < p$, so F bets on horse F, and so FLB exists in period t
- Ruin probability: $\rho_F(y(t), \tau)$ for F, $\rho_L(y(t), \tau)$ for L.
- recall that increasing y makes F less favorable, and increasing τ makes all the bet less favorable, so it is natural to assume

$$\frac{\partial}{\partial y}\rho_F(y(t), \tau) > 0, \; \frac{\partial}{\partial y}\rho_L(y(t), \tau) < 0$$
$$\frac{\partial}{\partial \tau}\rho_\bullet(y(t), \tau) > 0$$

Dynamics

- Total population of exiting bettors $\Delta(t)$ is given by

$$\Delta(t) = \rho_F(y_t, \tau)y_t + \rho_L(y_t, \tau)(1 - y_t). \tag{1}$$

and so

$$y_{t+1} = (1 - \rho_F(y_t, \tau))y_t + \pi\Delta(t). \tag{2}$$

- Substituting (1) to (2), we have

$$y_{t+1} = \{1 - ((1 - \pi)\rho_F(y_t, \tau) + \pi\rho_L(y_t, \tau))\}y_t + \pi\rho_L(y_t, \tau). \tag{3}$$

Long Run Equilibrium

- Let y^* be a steady state of dynamics (3):

$$y^* = \{1 - ((1-\pi)\rho_F^* + \pi\rho_L^*)\}y^* + \pi\rho_L^*$$

where $\rho_F(y^*, \tau) = \rho_F^*$ and $\rho_L(y^*, \tau) = \rho_L^*$. Then

$$y^* = \frac{\pi\rho_L^*}{(1-\pi)\rho_F^* + \pi\rho_L^*}. \tag{4}$$

- Re-writing, we have the basic equilibrium equation

$$\frac{y^*}{1-y^*} = \frac{\pi}{1-\pi}\frac{\rho_L^*}{\rho_F^*}. \tag{5}$$

- It can be shown that it is unique and monotonically stable, thanks to the fact this is only 2 dimensional.

FLB: necessary condition

- So when the initial value $y(0) < p$, if $y^* < p$ then $y(t) \rightarrow y^*$ exhibiting FLB;
  - (2) if $y^* > p$ then $y(t)$ grows and at some point the odds stay at fair value - once $y(t) > p$, then some fraction of rational bettors start betting on L, keeping the market (conditionally) fair.
  - Similarly if $y(0) > p$, the market stays fair.
- Then do we really have $y^* < p$? Clearly, we must have a reasonable size of L type.

FLB in equilibrium: characterization

- Suppose $\pi = p$ - so in principle the market could be "efficient" - F type bets on F and L type bets on L
- The steady state condition $\frac{y^*}{1-y^*} = \frac{p}{1-p}\frac{\rho_L^*}{\rho_F^*}$ then implies:

$$y^* < p \Leftrightarrow \frac{\rho_L^*}{\rho_F^*} < 1$$

  I.e., FLB is equivalent to L type survives more often even when the odds are slightly against them.
- I.e., in the evolutionary story, even the potential population is consistent with the efficient market hypothesis, FLB can occur because of uneven survival probability
- OK, then why the Longshot type is the better fit of the two?

Why could L type survive better?

- In the FLB equilibrium, the *average* return for L type is worse than F type.
- In Friedman's world, in the long run, L type's wealth will be dominated by F type, and so the aggregate property of the market demand function is determined by F type's demand.
  - Hence the efficient market hypothesis will hold. (e.g., Blume - Easely, Sandoroni).
- But this logic will not hold in the environment with negative average returns.
  - In fact, it is often the case that an **optimal** betting strategy in gambling with time limit is go for the **largest variance** bet, not the highest average (Dubins and Savage, "How to Gamble If You Must")
- Moreover, a bettor comes back to the race track not just because s/he makes money, but s/he "enjoyed" the races and had a good time.

Origin for Joy of gaming Idea 1

- Who continues gambling?
- Expected wealth does not necessarily count.
- one returns to the race track if s/he is ahead of certain target during the period
- Gary Loveman (HBS professor, then CEO of Ceasar's Palace Casino) says that: even if one ends up losing \$200 in a \$1 slot machine, s/he might enjoy it if s/he was ahead for some time, while s/he will give up if s/he goes straight down to \$200.
  - So the casino makes sure that a player with bad streaks get some rewards.

Origin for Joy of gaming Idea 2

- Who would quit?
- one might think about quitting if s/he is down in the end
  - of course, not all of them quit, but perhaps a (small) fraction of these people quit
- For both ideas, the chance of survival is more to do with the **variance** for a gamble with negative expected return
  - Notice that betting on L yields a larger variance when the odds are fair.
- Idea 1 will make L type' survival easier than idea 2, but even with idea 2 FLB can appear rather robustly

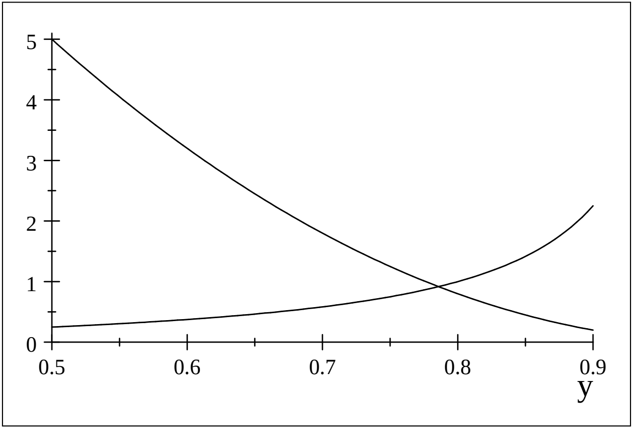
Thought Experiment (I)

- Suppose that the odds are not biased, $\frac{p}{1-p} = \frac{y}{1-y}$, so the average wealth is the same
- Suppose that the final wealth at the end of period is normally distributed with mean μ $(= \mu_L = \mu_F)$ and the standard deviation σ_F and σ_L
  - Thus $\sigma_F << \sigma_L$, and $\mu < 0$ because of track take ($-\mu$ roughly corresponds to the track take, so set $\tau = -\mu$ abusing notation)
- Imagine that players who are ahead surely keep gambling, and fraction κ of those who lose quit:
- Then $\rho_F = \kappa \Pr\{\sigma_F X < \tau\}$ and $\rho_L = \kappa \Pr\{\sigma_L X < \tau\}$ where X is the standard normal, and so $\rho_L < \rho_F$.

Thought Experiment (II)

$$\left(\frac{y}{1-y}\right) / \frac{\pi}{1-\pi} = \frac{\rho_L\left(y;\tau\right)}{\rho_F\left(y;\tau\right)}.$$

- more generally, as a function of y, $\frac{\rho_L}{\rho_F}$ will a downward sloping curve, and $\frac{\rho_L}{\rho_F} < 1$ at $y = p$, if we adopt a joy of gaming story
- On the other hand, $\frac{y}{1-y} / \left(\frac{\pi}{1-\pi}\right)$ is increasing in y and equal to 1 at $y = \pi$



$p = 0.8$

- So if $\pi = p$, we have FLB at the steady state.
  - to put it differently, there is an interval $[0, \bar{\pi}]$ with $\bar{\pi} > p$ such that $\pi \in [0, \bar{\pi}]$ admits a long run equilibrium with FLB.

Role of track take

- When τ increases, which direction the $\frac{\rho_L}{\rho_F}$ curve shifts?
- For the normal case, for a fixed pair of standard deviations, $\sigma_F << \sigma_L$, if τ is large enough, $\frac{\rho_L}{\rho_F}$ is increasing in τ.
  - Intuition: F's distribution is concentrated around μ, so if τ is large, we only look at a thin tail where change in τ does not change (already small) winning chance that much. On the other hand, L's distribution is fatter, so it winning probability decreases faster
- On the other hand, if $\tau = 0$, risk neutral type is resilient, so if τ is very small, either FLB does not exist, or the bias will **increase** as τ increases.
  - At $\tau = 0$, $\frac{\rho_L}{\rho_F} = 1$ and so at the margin the difference in density counts, and that of ρ_L is smaller because of fat tail.

Some Prediction

- Then, this evolutionary model suggests, as track take increases, FLB first increases, and then it diminishes.
- so places with a very high track take such as Japan exhibits a smaller FLB!
- also it predicts a smaller FLB for small track take (but does such a race track exists in the current world?)

What did we learn?

- Despite the compelling Friedman hypothesis, an evolutionary race track model could favor the irrational long shot type over the rational type, explaining the FLB.
- The key is that **a positive track take favors risky behavior**, and **risky behavior is rational** in the environment.
- Moreover, speculatively speaking, the FLB will be large for the medium size of track take, explaining the Japanese case.

Beyond Horse Races

- Since the size of "track take" defers in various form of gambling/gaming, some comparison of FLB' ish bias might be an interesting topic.
- The idea of high risk (doubling up) behavioral type surviving better seems more universal:
  - excessive risk taking by a fund manager, who is already behind the target - e.g. recent collapse of AIJ pension fund in Japan)
  - similar phenomenon even for public office - e.g. Harrisburg kept funding its high tech incinerator.
  - A public project continues despite expected losses - rather than misunderstanding of sunk cost, one should view it as a form of doubling up strategy
  - why do we tend to see sensational book titles? - most titles do not sell well anyway (at least in the Japanese book market), and so risk taking behavior is over- rewarded.
- Perhaps more - economists have been thinking of growing pie environment, but shrinking pie environments are also interesting and real.