Real Options and Signaling in Strategic Investment Games

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Abstract

A game in which an incumbent and an entrant decide the timings of entries into a new market is investigated. The profit flows involve two uncertain factors: (1) the basic level of the demand of the market observed only by the incumbent and (2) the fluctuation of the profit flow described by a geometric Brownian motion that is common to both firms. The optimal timing for the incumbent, who privately knows the high demand, is earlier than that for the low-demand incumbent. This earlier entrance, however, reveals the information of the high demand to the entrant, so that the entrant observing the timing of the incumbent would accelerate the its own timing of the investment that reduces the monopolistic profit of the incumbent. Therefore, the high-demand incumbent may delay the timing of the investment in order to hide the information strategically. The equilibria of this signaling game are characterized, and the conditions for the manipulative revelation are investigated. The values of both firms are compared with the case of complete information.

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1 Introduction

The timings of investments of firms are affected by the uncertainty of a market. In contrast to the traditional net present value (NPV) model, the concept of real options clarifies the nature of the strategic delay of the irreversible investment under uncertainty. Previous studies, for example, Brennan and Schwartz (1985) and McDonald and Siegel (1985), assert that a firm should wait for an investment even if the net present value is positive and the optimal timing of the investment is delayed beyond the traditional Marshallian threshold. This concept has been developed into the real option approach which is analogous to American call options. The real option approach, which has been summarized by Dixit and Pindyck (1994), has been examined in a number of studies.

On the other hand, the timings of investments are also affected by market competition. Thus, the real option approach has recently been extended to investments under competition by combining the real option approach with the game theory. A typical model incorporating the real option approach into game theory is sometimes referred to as an *investment game*, in which two firms decide the timings of option exercises in a duopolistic market. Previous studies, such as Smets (1991), Grenadier (1996), Kulatilaka and Perotti (1998), Huisman and Kort (1999), and Smit and Trigeorgis (2002), investigated competition by symmetric firms. An important implication about the previous studies about the real option under competition is that the threat of preemption by the advantage of the first mover and a negative externality of the investment reduce the value for the options of the firms and accelerate the timing of the investment. Pawlina and Kort (2006) and Kong and Kwok (2007) obtained the results for two asymmetric firms, but the information for the two firms was assumed to be identical.

Asymmetry of information in an investment game also influences the timing of the exercise. Lambrecht and Perraudin (2003) modeled an investment game using incomplete information for the optimal decisions of the investments of two competitive firms, in which the investment cost of each firm is different and is the private information of the firms. In this setting, two firms are assumed to be identical *ex ante* and the prior probabilities of the costs are followed by an identical probability distribution. Hsu and Lambrecht (2007) consider the situation in which one firm has complete information about the investment cost of its rival, whereas the rival firm has incomplete information about the investment cost of the first firm.

These studies examined investment games based on asymmetric information in which the options exercised by one firm do not influence the beliefs of the other firm. However, in the presence of asymmetric information, the behavior of a firm that acts earlier reveals information to the firms that act later. Hence, the firm that acts earlier considers the strategic exercise of the option to hide the information that conflicts with the optimal timing of the exercise. In the present paper, the influence of the strategic transmission of information called signaling on investments is examined under uncertainty and competition. In order to consider the applicability of this concept, a model of an investment game with two asymmetric firms, an incumbent, and an entrant, who have the option to enter a new product market, is specified. The profit flow of each firm has two uncertainty factors. One factor is the potential size of the market, which is referred to as the level of demand that is determined at the beginning of the game. The level of demand is assumed to take one of two possible values, i.e., high or low. The level of demand can be observed only by the incumbent as private information due to the experience of the incumbent, whereas the entrant cannot obtain the information. The other factor is the fluctuation of the profit flow given by a stochastic process that is common to both firms. Hence, there exist two types of incumbent. These incumbents know that the demand is high or low and are referred to hereinafter as high-demand and low-demand incumbents, respectively. In the framework of the present study, the incumbent invests earlier than the entrant for any market level because the market share of the incumbent is assumed to be sufficiently larger than that of the entrant and the investment cost of the incumbent is assumed to be sufficiently smaller than that of the entrant. If both the high- and low-demand incumbents enter the market at the optimal timing truthfully, information of the level of the demand would be revealed to the entrant by observing the timing. Then, the entrant who observes the earlier entry of the incumbent would accelerate the timing of the investment. Since this would reduce the monopolistic profit of the high-demand incumbent, the high-demand incumbent may strategically delay the timing of the investment to hide the information and enter the market at the timing of the low-demand incumbent.

The present study answers three important questions. (1) What conditions cause this manipulative revelation. (2) How are the values of the firms affected in the presence of asymmetric information as compared to complete information. (3) Which factors influence the causes of strategic information revelation.

With regard to question (1), since the low-demand incumbent does not have an incentive to mimic the high-demand incumbent, which may accelerate the timing of the entrant, only the high-demand incumbent has an incentive to mimic the low-demand incumbent strategically by delaying the investment. This derives the conditions for strategic and truthful revelations in an equilibrium. In addition, it is also shown that there exists no pure strategy equilibrium in a certain range, in which there exists an equilibrium in which the high-demand incumbent uses a mixed strategy. Finally, the probability of the mixed strategy for the high-demand incumbent is identified.

With regard to question (2), under the condition in which truthful revelation occurs, neither the entrant nor either the high-demand incumbent nor low-demand incumbent have loss or gain as compared to the case of complete information. In contrast, under the condition for manipulative revelation, the high-demand incumbent increases the values so as to mimic the low-demand incumbent as compared to the case of complete information, whereas the low-demand incumbent decreases the values. The entrant cannot distinguish the level of the demand and enters the market at the expected level of demand. This decreases the value of the entrant for both levels of demand by distorting the optimal timing of the exercise of the option. Under a mixed strategy equilibrium, it is shown that the ex ante value of the high-demand incumbent is identical to that of complete information, whereas the values of the low-demand incumbent and the entrant decrease.

With regard to question (3), the initial condition of the fluctuation of the profit flow is shown not to affect whether the option of the incumbent operates strategically or truthfully. The causes of manipulative revelation depends on the profit flows of both firms. In particular, the smaller duopoly profit of the high-demand incumbent causes the incumbent to act strategically. When the duopoly profit of the high-demand incumbent is sufficiently small, the high-demand incumbent delays market entry in order to hide the information and to enjoy the advantages of the monopoly for a longer period of time. Thus, in this case, the high-demand incumbent enters the market at the optimal timing of the low-demand incumbent. In contrast, when the duopoly profit is sufficiently large, the high-demand incumbent enters the market at the optimal timing truthfully, even if the information of the high demand is revealed.

Similarly, the smaller investment cost of the incumbent is show to result in acting strategically. Note that the profit and cost of the entrant also affect whether the incumbent enters the market strategically or truthfully. The larger investment cost of the entrant is shown to cause strategic operation by the incumbent, due to the strategic interaction between the two firms. The above results are obtained analytically, but the effect of volatility, which is important in a dynamic model under uncertainty, could not be obtained in the present study. However, a numerical example reveals that the larger volatility causes the manipulative revelation.

Whereas the proposed model focuses on an investment game with two competitive firms, the presence of asymmetric information between an owner and a manager or between an investor and a manager also affect investment decisions. Grenadier and Wang (2005) investigated conflicts between managers and owners and presented a model of the investment timing by managers by combining real options with contract theory. Shibata (2009) and Shibata and Nishihara (2010) also examined manager-shareholder conflicts arising from asymmetric information in the context of the real option approach. Note that, recently, signaling and manipulative revelation in this context have been investigated in a few studies. Morellec and Schürhoff (2011) investigated a signaling game between an informed firm and an outside investor, in which the firm decides both the timing of investment and the debt-equity mix. In Morellec and Schürhoff (2011), the presence of asymmetric information and the signaling effect erode the option value of the firm. Grenadier and Malenko (2010) investigated a similar model that considers the conflicts between continuum types of an informed agent and an outsider. Although these models are a signaling game of real options, the present study considers a different situation in that the model of the present study focuses on signaling and timings between competitive firms in a duopoly market.

Information revelation involving several firms was investigated by Grenadier (1999), where each firm has private information about the payoff uncertainty and updates the belief for the payoff by observing the strategies exercised by other firms. Grenadier (1999) focused on informational cascades and projects in which firms are not in competition with each other. Thus, the firms reveal their private information truthfully.

The remainder of the present paper is organized as follows. Section 2 describes the notation used herein and presents a description of the model used herein. Section 3 presents the value of the entrant and non-strategic values of the incumbents, which implies a benchmark of the analysis. In Section 4, a solution of the game achieved through a perfect Bayesian equilibrium and two candidate solutions, Truthful Revelation and Manipulative Revelation, which correspond to a separating equilibrium and a pooling equilibrium, respectively, are presented. Conditions that specify either of the two candidate solutions to the equilibrium are also presented. Although these conditions characterize an equilibrium in pure strategies, in some cases, there is no equilibrium in pure strategies. Section 5 deals with mixed strategies and presents the conditions of the equilibria. Since these equilibria in mixed strategies include the case of equilibria in pure strategies examined in the previous section, the conditions characterize the equilibrium comprehensively. In Section 6, the manner in which the values of firms are affected by the presence of asymmetric information is examines. The gains and losses of the values for both high-demand and low-demand incumbents and the entrant are compared with the case of full information. The conditions of the manipulative revelation for the duopoly profit and the costs of the incumbent and the entrant are also examined. Section 7 presents numerical examples, and Section 8 presents conclusions and discusses future research.

2 The Model

Two asymmetric firms, an incumbent and an entrant, each of which has the option to wait for optimal entry into the market of a new product are considered. The incumbent and the entrant are denoted as firm I and firm E, respectively. The investments for the entry of both firms are assumed to be irreversible, and the sunk cost of the investment of firm i is denoted as K_i for $i = I, E$. The revenue flow of each firm after the entry depends on the market structure (monopoly or duopoly) and two uncertain factors of the profit.

One uncertain factor of the profit represents a stochastic process, denoted by X_s, as a standard real option setting. Here, X_s is interpreted as the unsystematic shocks of the demand over time and is common to both firms.

Suppose that X_s follows a geometric Brownian motion:

$$dX_s = \mu X_s dt + \sigma X_s dz$$

where μ is the drift parameter, σ is the volatility parameter, and dz is the increment of a standard Winner process. Both firms are assumed to be risk neutral, with risk free rate of interest r. Finally, $r > \mu$ is assumed for convergence.

The other uncertain factor of the profit represents a systematic risk and is assumed to be constant over time. This factor is denoted by θ, where $\theta = H$ and $\theta = L$ indicate that the basic level of demand is high and low, respectively. The prior probabilities of drawing $\theta = H$ and $\theta = L$ are denoted as p and $1 - p$, respectively.

When only firm i enters the market, the monopoly profit flow of firm i becomes $\pi_{i1}^{\theta} X_s$. On the other hand, when both firms enter the market, the duopoly profit flow of firm i becomes $\pi_{i2}^{\theta} X_s$. The profit flow of any firm that has not entered the market is assumed to be zero. Here, $\pi_{i1}^{\theta} > \pi_{i2}^{\theta} > 0$ is assumed for the case in which $i = I, E$ and $\theta = H, L$. The monopoly profit is always greater than the duopoly profit for each firm and each level of demand at the same X_s. Moreover, $\pi_{ij}^{H} > \pi_{ij}^{L} > 0$ is assumed for $i = I, E$ and $\theta = H, L$, which indicate that the profit at a high level of demand is always greater than that at a low level of demand.

The incumbent has several advantages over the entrant due to the experience the incumbent gains through past activities. The incumbent has more information, a greater share of the products, and a lower investment cost than the entrant. In detail, the incumbent has the following two advantages. First, while X_s is observable by two firms, the uncertain factor θ can be observed only by the incumbent, i.e., θ is the private information of the incumbent. Second, K_I/π_{I2}^{L} is assumed to be smaller than K_E/π_{E1}^{H}. This assumption holds if the monopoly profit of the incumbent is sufficiently larger than that of the entrant and/or the cost of the investment

K_I is sufficiently smaller than K_E.

3 Value Functions of a Benchmark Case

The proposed model is one of an option exercise game that is investigated under the joint framework of real options and game theory. A number of studies, including Smets (1991), Grenadier (1996), Kijima and Shibata (2002), Kulatilaka and Perotti (1998), Huisman and Kort (1999), Huisman (2001), and Smit and Trigeorgis (2002), have considered symmetric firms in order to examine the preemptive behavior of competition. In these models, if the value of the optimal entry of the leader is greater than the value of the entry for the best reply of the follower, then both firms want to become a leader. In this case, the optimal threshold of the leader is obtained solving a system of equations of equilibria, and the value of the leader is not determined by maximizing the expected profit of either firm. Huisman (2001), Kong and Kwok (2007) and Pawlina and Kort (2006) demonstrated that this preemptive behavior and simultaneous entry would occur under asymmetry of costs and profits. In this case, obtaining the equilibrium values is complicated.

However, if the asymmetry is sufficiently large and the initial value of both firms are sufficiently small to wait for the investment, the lower-cost firm must be the leader, (see Kong and Kwok (2007) and Pawlina and Kort (2006)). Based on the results of Kong and Kwok (2007), the two assumptions, i.e., $K_I/\pi_{I2}^L > K_E/\pi_{E1}^H$ and sufficiently small $X_t = x$, imply that the incumbent must be the leader and that the entrant must be the follower.

Due to this setting, the decisions and the values of both firms are analyzed under the condition in which the incumbent is the leader and the entrant is the follower. In following subsections, the benchmark case is solved by backward induction, i.e., the value of the entrant is solved first and the value of the incumbent as the leader is discussed later.

3.1 Value of the Entrant

In the settings of the present study, the entrant must be the follower, and the entrant is shown later herein to exercise the option at the optimal timing based on the belief of the level of the demand. Thus, it is necessary to consider only the optimal expected payoff of the entrant, which is derived from a standard real option approach. Let $u_E^*(q)$ be the value of the entrant under the condition in which the entrant invests later than the incumbent and believes that high demand will occur with probability q.

The entrant value is given by

$$u_E^*(q) = \max_{t_E} E[\int_{t_E}^{\infty} e^{-r(s-t)}(q\pi_{E2}^H + (1-q)\pi_{E2}^L)X_s ds - e^{-r(t_E-t)}K_E | X_t = x].$$

In this problem, a threshold strategy is sufficient to give the optimal stopping time. Hence the problem is written by deciding the optimal threshold x_E, as follows:

$$u_E^*(q) = \max_{x_E} E[\int_{\tau(x_E)}^{\infty} e^{-r(s-t)}(q\pi_{E2}^H + (1-q)\pi_{E2}^L)X_s ds - e^{-r(\tau(x_E)-t)}K_E | X_t = x].$$

where $\tau(\tilde{x})$ denote the first hitting time at threshold $\tilde{x}$, i.e., $\tau(\tilde{x}) = \inf\{s \geq t | X_s \geq \tilde{x}\}$. Let $x_E^*(q)$ be the optimal threshold for the belief q. The usual calculation of real option analysis (e.g., Dixit and Pindyck (1994)) implies that

$$x_E^*(q) = \frac{\beta}{\beta - 1} \frac{r - \mu}{q\pi_{E2}^H + (1-q)\pi_{E2}^L} K_E \tag{1}$$

where β is defined by

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}}. \tag{2}$$

Let $x_E^H = x_E^*(1)$, $x_E^L = x_E^*(0)$, and let $x_E^M = x_E^*(p)$. Here, x_E^H and x_E^L are the thresholds when the entrant believes that the demands are high and low, respectively. In addition, x_E^M is the threshold when the entrant predicts high demand with prior probability p.

We easily find that

$$x_E^H \le x_E^M \le x_E^L, \tag{3}$$

because $\pi_{E2}^H \ge \pi_{E2}^L$.

3.2 Value of the Incumbent

In contrast to the entrant, the incumbent is the leader and may not enter the market at the optimal timing due to the strategic revelation of the information. Since the incumbent taking into account the strategic exercise chooses the timing of the investment that may not be optimal, the value of the incumbent explicitly expressed by a function of the threshold of the investment by the incumbent. The value of the incumbent also depends on the timing of the entrant and the private information of the level of the demand observed by the incumbent. Let $u_I(x_I, x_E, \theta)$ be the expected profit of the incumbent with the level of the demand θ, when the incumbent invests at the threshold x_I and the entrant invests at x_E under the condition $x_I < x_E$.

Here, $u_I(x_I, x_E, \theta)$ is given by

$$u_I(x_I, x_E, \theta) = E[\int_{\tau(x_I)}^{\tau(x_E)} e^{-r(s-t)}\pi_{I1}^\theta X_s ds - e^{-r(\tau(x_I)-t)}K_I + \int_{\tau(x_E)}^{\infty} e^{-r(s-t)}\pi_{I2}^\theta X_s ds | X_t = x].$$

$u_I(x_I, x_E, \theta)$ can be rewritten as

$$u_I(x_I, x_E, \theta) = v_I(x_I, \theta) - w_I(x_E, \theta),$$

where

$$v_I(x_I, \theta) = E[\int_{\tau(x_I)}^{\infty} e^{-r(s-t)}\pi_{I1}^\theta X_s ds - e^{-r(\tau(x_I)-t)}K_I | X_t = x],$$

and

$$w_I(x_E, \theta) = E[\int_{\tau(x_E)}^{\infty} e^{-r(s-t)}(\pi_{I1}^\theta - \pi_{I2}^\theta) X_s ds | X_t = x].$$

In the following, in order to simplify the analysis, the initial condition x is assumed to be sufficiently small, indicating that the incumbent for any demand has not yet invested at the initial time. Hence, only the case in which $x \leq x_I$ is examined. Since $x_I < x_E$, x is also less than x_E. Under these assumptions, $v_I(x_I, \theta)$ and $w_I(x_E, \theta)$ are expressed as the following proposition, which can be derived by the strong Markov property of the geometric Brownian motion and the calculation for the hitting time.

Proposition 3.1. *$u_I(x_I, x_E, \theta)$ is given by*

$$u_I(x_I, x_E, \theta) = v_I(x_I, \theta) - w_I(x_E, \theta). \tag{4}$$

where

$$v_I(x_I, \theta) = \left(\frac{\pi^\theta_{I1}}{r-\mu} x_I - K_I\right)\left(\frac{x}{x_I}\right)^\beta \quad x \leq x_I \tag{5}$$

and

$$w_I(x_E, \theta) = \frac{\pi^\theta_{I1} - \pi^\theta_{I2}}{r-\mu} x_E \left(\frac{x}{x_E}\right)^\beta, \quad x < x_E. \tag{6}$$

Proof. *See Appendix.*

Note that (3) yields

$$w_I(x^H_E, \theta) \geq w_I(x^M_E, \theta) \geq w_I(x^L_E, \theta) \tag{7}$$

because $w_I(x_E, \theta)$ is the decrease in the threshold x_E.

If x_E is independent of the incumbent decision x_I, then $w_I(x_E, \theta)$ does not depend on x_I. Then, the incumbent can maximize the expected profit only by $v_I(x_I, \theta)$.

The threshold x_E of the entrant in the signaling equilibrium, which is examined in the next section, depends on the threshold of the incumbent x_I. In the remainder of this section, however, the case in which x_E is independent of x_I is examined as a benchmark of the analysis. Let x^θ_I be the optimal threshold of the incumbent with the private information θ under the condition that x_E is independent of x_I. Then, $v_I(x^\theta_I, \theta)$ is given by

$$v_I(x^\theta_I, \theta) = \max_{x_I} v_I(x_I, \theta) = \max_{t_I} E^x\left[\int_{t_I}^\infty e^{-r(s-t)} \pi^\theta_{I1} X_s ds - e^{-r(s-t_I)} K_E\right].$$

Standard calculation of the real option approach[1] implies that

$$x_I^\theta = \frac{\beta}{\beta - 1} \frac{r - \mu}{\pi_{I1}^\theta} K_I, \tag{8}$$

and

$$v_I(x_I^\theta, \theta) = \begin{cases} \frac{K_I}{\beta-1}\left(\frac{x}{x_I^*(\theta)}\right)^\beta & x \leq x_I^\theta, \\ \frac{\pi_{I1}^\theta}{r-\mu}x - K_I & x > x_I^\theta. \end{cases}$$

4 Equilibrium in Pure Strategies

4.1 Definitions of the Solution

For the analysis of the signaling effect, a perfect Bayesian equilibrium is applied as the solution concept. In this model, a solution concept is specified not only by a threshold for each of the players, but also by the entrant belief regarding the level of demand.

An *assessment* consisting of three components $\{(a_I(H), a_I(L)), a_E(\cdot), q(\cdot)\}$ is called, where:

- $a_I(H)$ and $a_I(L)$ are the threshold of the incumbent for private information H and L, respectively,
- $a_E(x_I)$ is the threshold of the entrant for the threshold x_I of the observed incumbent, and
- $q(x_I)$ is the belief of the entrant for the threshold x_I of the observed incumbent.

An assessment $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$ is said to be an equilibrium if the assessment satisfies the following three conditions.

First, $a_I^*(\theta)$ is the optimal threshold of the incumbent for $\theta = H, L$, such that

$$u_I(a_I^*(\theta), a_E^*(a_I^*(\theta)), \theta) = \max_{x_I} u_I(x_I, a_E^*(x_I), \theta). \tag{9}$$

[1] Both $x_I^*(\theta)$ and $v_I(x_I^*(\theta), \theta)$ are calculated based on the smooth pasting condition and the value matching condition of real option approach. These conditions can also be derived from the first-order condition to maximize $v_I(x_I, \theta)$, which is obtained by differentiating (5).

Second, $a_E^*(\cdot)$ is the threshold of the entrant observing the entry of the incumbent at x_I with belief $q^*(\cdot)$, such that

$$a_E^*(x_I) = x_E^*(q^*(x_I)). \tag{10}$$

Finally, $q^*(x_I)$ is the belief of the entrant for the high demand, when the entrant has observed the thresholds of the incumbent x_I, which should be consistent with the equilibrium strategy of the incumbent $(a_I(H), a_I(L))$ in the sense of Bayes rule. The consistent belief $q^*(x_I)$ is calculated as follows. Then $q^*(x_I) = Prob[\theta = H|x_I]$. According to Bayes ’ rule,

$$Prob[\theta = H|x_I] = \frac{Prob[x_I|\theta = H]Prob[\theta = H]}{Prob[\theta = H]Prob[x_I|\theta = H] + Prob[\theta = L]Prob[x_I|\theta = L]}.$$

Substituting $Prob[\theta = H] = p$ and $Prob[\theta = L] = 1 - p$, the consistent belief is expressed by

$$q^*(x_I) = \frac{pProb[x_I|\theta = H]}{pProb[x_I|\theta = H] + (1-p)Prob[x_I|\theta = L]}. \tag{11}$$

In Section 5, the mixed strategies of the incumbent are investigated, so that $Prob[x_I|\theta = H]$ and $Prob[x_I|\theta = L]$ would follow some probability distributions derived from a mixed strategy of the incumbent. However, in this section, since the analysis is restricted to pure strategies, $Prob[x_I|\theta = H]$ and $Prob[x_I|\theta = L]$ can be explicitly written as

$$Prob[x_I|\theta = H] = \begin{cases} 1 & x_I = a_I^*(H) \\ 0 & x_I \neq a_I^*(H), \end{cases} \qquad Prob[x_I|\theta = L] = \begin{cases} 1 & x_I = a_I^*(L) \\ 0 & x_I \neq a_I^*(L). \end{cases} \tag{12}$$

Equations (11) and (12) imply that

$$q^*(x_I) = \begin{cases} p & x_I = a_I^*(H) \quad \text{and} \quad x_I = a_I^*(L), \\ 1 & x_I = a_I^*(H) \quad \text{and} \quad x_I \neq a_I^*(L), \\ 0 & x_I \neq a_I^*(H) \quad \text{and} \quad x_I = a_I^*(L). \end{cases} \tag{13}$$

If $a_I^*(H) \neq x_I$ and $a_I^*(L) \neq x_I$, then any belief $q^*(x_I)$ is consistent.

Thus, in pure strategies, an perfect Bayesian equilibrium is formally defined as follows.

Definition 4.1. *An assessment is said to be a perfect Bayesian equilibrium in pure strategies if the assessment satisfies (9), (10), and (13).*

A perfect Bayesian equilibrium in pure strategies is said to be a *pooling* equilibrium if $a_I^*(H) = a_I^*(L)$. Equation (13) implies that

$$q^*(a_I^*(H)) = q^*(a_I^*(L)) = p.$$

A pooling equilibrium corresponds to the case in which the actions of the incumbent do not convey information about the demand, and the entrant predicts high demand with prior probability p. Therefore, in the pooling equilibrium, the threshold of the entrant in the equilibrium is

$$a_E^*(a_I^*(H)) = a_E^*(a_I^*(L)) = x_I^M$$

because $x_I^M = x_I^*(p)$.

A perfect Bayesian equilibrium in pure strategies is said to be a *separating* equilibrium if $a_I^*(H) \neq a_I^*(L)$. In the separating equilibrium, Eq. (13) implies that

$$q^*(a_I^*(H)) = 1, \quad q^*(a_I^*(L)) = 0.$$

This means that the entrant determines the level of the demand exactly by observing the actions of the incumbent. Hence, the threshold of the entrant in the separating equilibrium is

$$a_E^*(a_I^*(H)) = x_E^H, \quad a_E^*(a_I^*(L)) = x_E^L.$$

4.2 Two Assessments: Truthful Revelation and Manipulative Revelation

In this section, the following two assessments , Truthful Revelation and Manipulative Revelation, are defined. In next section, it is found that either of them can be an equilibrium exclusively.

The assessment is said to be *Truthful Revelation* if it satisfies

$$a_I^*(H) = x_I^H, \quad a_I^*(L) = x_I^L$$
$$a_E^*(x_I) = \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^L & x_I = x_I^L, \end{cases}$$
$$q^*(x_I) = \begin{cases} 1 & x_I \neq x_I^L, \\ 0 & x_I = x_I^L. \end{cases}$$

If Truthful Revelation is an equilibrium, it is a separating equilibrium. In Truthful Revelation, the incumbent for any demand truthfully enters the market at the optimal threshold with respect to the demand. This truthful behavior reveals the information of the demand that the incumbent possesses. The entrant obtains the information about the demand by observing the behavior of the incumbent and enters the market optimally with full information. If the entrant observes that the incumbent enters the market at neither x_I^H nor x_I^L, then any belief of the entrant is consistent. In other words, the belief of the entrant is assigned arbitrarily in the observation of the entrant in this off-equilibrium path. For this unexpected deviation of the equilibrium for the incumbent, the entrant is assumed to believe that the demand is high.

The second assessment is referred to as *Manipulative Revelation*

$$a_I^*(H) = a_I^*(L) = x_I^L$$
$$a_E^*(x_I) = \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^M & x_I = x_I^L, \end{cases}$$
$$q^*(x_I) = \begin{cases} 1 & x_I \neq x_I^L, \\ p & x_I = x_I^L \end{cases}$$

If Manipulative Revelation is an equilibrium, it is a pooling equilibrium. In Manipulative Revelation, the high-demand incumbent does not enters at the optimal threshold of the high demand but rather invests at the threshold of the low demand. This delay of the investment hides the information about the high demand, and the entrant cannot distinguish the demand by observing the behavior of the incumbent. Thus, the entrant predicts the level of the demand according to the prior probability and enters at the threshold for the expectation of the demand. The entrant is assumed to believe that high demand occurs in the off-equilibrium path, as well as Truthful Revelation.

4.3 Conditions for Equilibrium in Strategies

In this subsection, the conditions in which either of the candidates, Truthful Revelation or Manipulative Revelation, is a perfect Bayesian equilibrium in pure strategies is analyzed. Since both candidates are constructed by satisfying the optimality of the entrant and the consistency of the belief of the entrant, it remains to consider the optimality of the incumbent for the strategy $a_E^*(\cdot)$ and belief $q^*(\cdot)$ of a given entrant. Moreover, the low-demand incumbent does not have an incentive to deviate from the optimal timing x_I^L because pretending the high-demand incumbent only accelerates the timing of the investment of the entrant and reduces the value of the incumbent. Hence, only the timing of the high-demand incumbent should be considered.

Consider the necessary conditions for both candidates being equilibria. First, assume that Truthful Revelation is an equilibrium in pure strategies. In Truthful Revelation, the entrant observing the investment of the incumbent at x_I^θ for any $\theta = H, L$ invests at x_E^θ. Since the high-demand incumbent does not have an incentive to hide information in order to delay the investment of the entrant, the following condition holds:

$$u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H). \tag{14}$$

Next, suppose that Manipulative Revelation is an equilibrium in pure strategies. In Manipulative Revelation, the incumbent with information of the high demand strategically delays the investment until the optimal timing for the low demand, and the entrant cannot obtain information about the demand. The entrant observing the investment of the incumbent at x_L then predicts the level of the demand by prior probability p, so that the expectation of the profit flow is π_{E2}^M. The entrant then enters the market at x_E^M, which is optimal for π_{E2}^M. The incumbent with information of the high demand has an incentive to hide information if the expected value for this delayed entrance at x_I^L exceeds that of the optimal entrance at the threshold of the high demand x_I^H. This condition is expressed by

$$u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^M, H). \tag{15}$$

Equations (14) and (15) are not only necessary conditions. The following proposition asserts that Eqs. (14) and (15) are also sufficient conditions of the equilibrium.

Proposition 4.2. **(a)** *Equation (14) holds if and only if Truthful Revelation is a perfect Bayesian equilibrium in pure strategies.*

(b) *Equation (15) holds if and only if Manipulative Revelation is a perfect Bayesian equilibrium in pure strategies.*

Proof. First, it is shown that if assessment $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$ is Truthful Revelation, then it is a perfect Bayesian equilibrium in pure strategies. In order to prove this relationship, it is sufficient to show that the assessment satisfies three conditions, namely, the optimality of the incumbent given by Eq. (9), the optimality of the entrant given by Eq. (10), and the consistency of the belief of the entrant given by Eq. (13). By definition, Truthful Revelation always satisfies the optimality of the entrant given by Eq. (10) and the consistency of the belief given by Eq. (13). However, Truthful Revelation must be formally demonstrated to satisfy the optimality of the incumbent given by Eq. (9) for $\theta = H, L$,

$$u_I(a_I^*(\theta), a_E^*(a_I^*(\theta)), \theta) \geq u_I(x_I, a_E^*(x_I), \theta). \tag{16}$$

and for any $x_I \neq a_I^*(\theta)$.

Let $\theta = L$. Since, in Truthful Revelation, $a_I^*(L) = x_I^L$, $a_E^*(x_I^L) = x_E^L$, and $a_E^*(x_I) = x_E^H$ for any $x_I \neq x_I^L$, we need only show that $u_I(x_I^L, x_E^L, L) \geq u_I(x_I, x_E^H, L)$ for any $x_I \neq x_I^L$. Since x_I^L is the optimal threshold of the incumbent, $v_I(x_I^L, L) \geq v_I(x_I, L)$. Hence,

$$u_I(x_I^L, x_E^L, L) = v_I(x_I^L, L) - w(x_E^L, L) \geq v_I(x_I, L) - w_I(x_E^H, L) = u_I(x_I, x_E^H, L)$$

because Eq. (7) yields $w_I(x_E^H, L) \geq w_I(x_E^L, L)$. Hence, Eq. (16) holds for $\theta = L$.

Second, let $\theta = H$. Then, we have to prove $u_I(x_I^H, x_E^H, H) \geq u_I(x_I, x_E^H, H)$ for any $x_I \neq x_I^L$ and $u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$. Since x_I^H is the optimal threshold of the incumbent, $v_I(x_I^H, H) \geq v_I(x_I, H)$ for any x_I. Hence,

$$u_I(x_I^H, x_E^H, H) = v_I(x_I^H, H) - w(x_E^H, H) \geq v_I(x_I, H) - w_I(x_E^H, H) = u_I(x_I, x_E^H, H).$$

$u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$ holds because of Eq. (14). Then, Truthful Revelation is a perfect Bayesian equilibrium in pure strategies.

Conversely, it is herein proven that if Truthful Revelation is a perfect Bayesian equilibrium in pure strategies, then Eq. (14) holds. Otherwise, assume that $u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^L, H)$. Then, the high-demand incumbent strictly increases the payoff by deviating x_I^L from $a_I^*(H) = x_I^H$ in Truthful Revelation, which means that Truthful Revelation is not an equilibrium. This completes the proof of (a).

The proof of (b) is obtained in a similar manner. □

Since $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^L, x_E^L, H)$, it is found that neither Truthful Revelation nor Manipulative Revelation is a perfect Bayesian equilibrium in pure strategies for $u_I(x_I^L, x_E^M, H) < u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^M, H)$. In this interval, the mixed strategy of the incumbent should be considered in order to ensure the existence of the equilibrium. In Section 5, the equilibria in the mixed strategies are investigated.

5 Equilibria in Mixed Strategies

In order to examine mixed strategies of the incumbent, the notation is extended for actions and a payoff function of the incumbent. Let $x_I(\lambda)$ be a mixed action of the incumbent, where the incumbent chooses x_I^H with probability λ and x_I^L with probability $1-\lambda$ for $0 \leq \lambda \leq 1$. Even if the incumbent uses the mixed action, the entrant observes only a realized action, either x_I^H or x_I^L, in the equilibrium and takes an action, either $a_E(x_I^H)$ or $a_E(x_I^L)$. Here, u_I is extended to the set of mixed actions $x_I(\lambda)$ for $0 \leq \lambda \leq 1$, as defined by

$$u_I(x_I(\lambda), a_E(\cdot), \theta) = \lambda u_I(x_I^H, a_E(x_I^H), \theta) + (1-\lambda) u_I(x_I^L, a_E(x_I^L), \theta)$$

for any x_E and $\theta = H, L$. Here, $u_I(x_I(\lambda), a_E(\cdot), \theta)$ denotes the expected payoff of the incumbent, where the incumbent uses mixed action $x_I(\lambda)$, and the entrant follows $a_E(\cdot)$.

The incumbent strategy $a_I^*(H) = x_I(\lambda)$ and $a_I^*(L) = x_I^L$ is considered because the low-demand incumbent in the equilibrium does not have an incentive to deviate from the optimal timing, which is analogous to the discussion of pure strategies. The consistent belief of the entrant $q^*(\cdot)$ for $a_I^*(H) = x_I(\lambda)$ and $a_I^*(L) = x_I^L$ is derived by Bayes rule, given by Eq. (11). Here, $Prob[x_I|\theta = H]$ and $Prob[x_I|\theta = L]$ are given by

$$Prob[x_I|\theta = H] = \begin{cases} \lambda & x_I = x_I^H \\ 1-\lambda & x_I = x_I^L \\ 0 & x_I \neq x_I^H, x_I^L, \end{cases} \qquad Prob[x_I|\theta = L] = \begin{cases} 1 & x_I = x_I^L \\ 0 & x_I \neq x_I^L, \end{cases} \tag{17}$$

Equations (11) and (17) imply the consistent belief $q^*(\cdot)$, as follows:

$$q^*(x_I^H) = \frac{p\lambda}{p\lambda + (1-p) \times 0} = 1$$

and

$$q^*(x_I^L) = \frac{p(1-\lambda)}{p(1-\lambda) + (1-p) \times 1} = \frac{p(1-\lambda)}{1-p\lambda}.$$

The consistent belief $q^*(\cdot)$ indicates that the entrant observing the investment at x_I^H completely learns the high demand with probability one, because only the incumbent with information of the high demand invests at x_I^H. Hence, the optimal timing of investment of the entrant observing the investment of the incumbent at x_I^H is x_E^H. On the other hand, since both types of the incumbents have positive probabilities of the investment at x_I^L, the entrant observing that the incumbent acted at x_I^L predicts the high demand according to probability $q^*(x_I^L)$. The optimal timing of the investment of the entrant observing the investment of the incumbent at x_I^L is $x_E^*(q^*(x_I^L))$. For simplicity, let $x_E^*(q^*(x_I^L))$ be x_E^λ.

The following assessment $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$, referred to as λ-*Hybrid Revelation*, is a candidate solution, which satisfies the optimality of the entrant and the consistence of the belief.

$$a_I^*(H) = x_I(\lambda), \quad a_I^*(L) = x_I^L$$

$$a_E^*(x_I) = \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^\lambda & x_I = x_I^L, \end{cases}$$

$$q^*(x_I) = \begin{cases} 1 & x_I \neq x_I^L, \\ \frac{p(1-\lambda)}{1-p\lambda} & x_I = x_I^L, \end{cases}$$

Note that λ-Hybrid Revelations for $\lambda = 1$ and $\lambda = 0$ are identical to Truthful Revelation and Manipulative Revelation, respectively. Hence, the condition in which λ-Hybrid Revelation is an equilibrium characterizes any equilibrium comprehensively.

Next, the probability λ in the equilibrium strategies for $u_I(x_I^L, x_E^M, H) < u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^L, H)$ is solved. Let $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$ be λ-Hybrid Revelation. The high-demand incumbent does not have an incentive to deviate from mixed strategy $x_I(\lambda)$ to any strategy for the strategy of the given entrant $a_E^*(x_I^H)$, $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ should be hold. Otherwise, assume that $u_I(x_I^H, x_E^H, H) > u_I(x_I^L, x_E^\lambda, H)$. Then,

$$\begin{aligned} u_I(x_I^H, a_E^*(\cdot), H) &= u_I(x_I^H, x_E^H, H) \\ &> \lambda u_I(x_I^H, x_E^H, H) + (1-\lambda) u_I(x_I^L, x_E^\lambda, H) \\ &= u_I(x_I(\lambda), a_E^*(\cdot), H), \end{aligned}$$

so that the incumbent has an incentive to deviate from mixed strategy $x_I(\lambda)$ to pure strategy x_I^H. Conversely, assume that $u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^\lambda, H)$. Similarly, in this case, the incumbent has an incentive to deviate from mixed strategy $x_I(\lambda)$ to pure strategy x_I^L. Hence, the mixed strategy of the equilibrium of the incumbent $x_I(\lambda)$ satisfies $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$, and solving this equation yields λ in the equilibrium. The results can be summarized as the following proposition.

Proposition 5.1. *The following three cases occur depending on the conditions in which $u_I(x_I^H, x_E^H, H)$ is greater than or less than $u_I(x_I^L, x_E^L, H)$ and $u_I(x_I^L, x_E^M, H)$.*

Case (a) $u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$ *if and only if λ-Hybrid Revelation for $\lambda = 1$, which is identical to Truthful Revelation, is a perfect Bayesian equilibrium,*

Case (b) $u_I(x_I^L, x_E^M, H) < u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^L, H)$ *if and only if λ-Hybrid Revelation for $0 < \lambda < 1$, such that λ that satisfies $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ is a perfect Bayesian equilibrium, and,*

Case (c) $u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^M, H)$ *if and only if λ-Hybrid Revelation for $\lambda = 0$, which is identical to Manipulative Revelation, is a perfect Bayesian equilibrium.*

6 Values of the Incumbents and Comparative Statics

6.1 Values in an Equilibrium

In this subsection, the distortion of the values of firms by the presence of asymmetric information is examined. The gains and losses of the values for both types of incumbent and entrant are compared with the case of full information.

In Case (a), Truthful Revelation is an equilibrium. The value of the incumbent for each demand level $\theta = H, L$ is given by $u_I(x_I^\theta, x_E^\theta, \theta)$. The values of the entrant for demand levels $\theta = H$ and $\theta = L$ are given by $u_E^*(1)$ and $u_E^*(0)$, respectively. The entrant for any demand level is completely informed by signaling in this case, so that none of the firms have gain or loss compared with the case of full information.

In Case (b), λ-Hybrid Revelation is an equilibrium. For the high demand $\theta = H$, the value of the incumbent is $u_I(x_I(\lambda), a_E^*(\cdot), H)$. Since λ is set as $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$, $u_I(x_I(\lambda), a_E^*(\cdot), H)$ is equal to $u_I(x_I^H, x_E^H, H)$ for any λ. This is derived as follows:

$$
\begin{aligned}
u_I(x_I(\lambda), a_E^*(\cdot), H) &= \lambda u_I(x_I^H, x_E^H, H) + (1-\lambda) u_I(x_I^L, x_E^\lambda, H) \\
&= \lambda u_I(x_I^H, x_E^H, H) + (1-\lambda) u_I(x_I^H, x_E^H, H) \\
&= u_I(x_I^H, x_E^H, H).
\end{aligned}
$$

Hence, the *ex ante* expected value of the high-demand incumbent is $u_I(x_I^H, x_E^H, H)$, and no loss exists compared with the case of full information. In contrast, the low-demand incumbent losses are $u_I(x_I^L, x_E^L, L) - u_I(x_I^L, x_E^\lambda, L)$ compared with the case of full information, because the entrant observing that the incumbent enters the market at x_I^L invests at x_E^λ, which is earlier than x_E^L. Since, the entrant also losses the value by distorting the optimal timing of the exercise of the option for both levels of demand. Consequently, the values of all firms in Case (b) are less than or equal to the values in the case of full information.

Note that the high-demand incumbent has no loss for either the ex ante value or the ex post value, as compared to the case of complete information. The mixed strategy realizes trigger x_I^H with probability λ and trigger x_I^L with probability $1-\lambda$. Given the realization of the timing at x_I^H, the entrant enters the market at x_E^H, so that the ex post value of the high-demand incumbent is $u_I(x_I^H, x_E^H, H)$, which is identical to the value in complete information. If the realization of the timing is x_I^L, then the entrant enters the market at x_E^λ, and the ex post value of the high-demand incumbent is $u_I(x_I^L, x_E^\lambda, H)$. Since $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$, the value is also same as that in complete information.

Finally, we consider Case (c), in which Manipulative Revelation is an equilibrium. The values of the high- and low-demand incumbents are given by $u_I(x_I^L, x_E^M, H)$ and $u_I(x_I^L, x_E^M, L)$, respectively. Since $u_I(x_I^L, x_E^M, H) \geq u_I(x_I^H, x_E^H, H)$ holds in Case (c), the high-demand incumbent achieves a positive value for $u_I(x_I^L, x_E^M, H) - u_I(x_I^H, x_E^H, H)$, as compared to full information by mimicking the low-demand incumbent and letting the entrant delay investment. In contrast, the low-demand incumbent losses are $u_I(x_I^L, x_E^L, L) - u_I(x_I^L, x_E^M, L)$, as compared with the case of full information, because the entrant cannot obtain the information of demand and puts the entrance ahead x_E^M from x_E^L. Thus, similarly to Case (b), the entrant losses the value by distorting the optimal timing for both demand levels. Only the high-demand incumbent gains by manipulative revelation, where the low-demand incumbent and the entrant are harmed by the strategic behavior of the high-demand incumbent.

6.2 Comparative Statics

In this subsection, the influences of various factors on strategic information revelation are examined. First, the manipulative revelation is shown to depend on the duopoly profit flow of the high-demand incumbent by solving Eqs. (14) and (15) for duopoly profit flow of the high-demand incumbent π_{I2}^H.

In order to simplify the notation, we define $\xi(\pi_{E2})$ by

$$\xi(\pi_{E2}) = \frac{(\pi_{I1}^H)^\beta - (\pi_{I1}^L)^\beta \phi}{(\pi_{E2}^H)^{\beta-1} - (\pi_{E2})^{\beta-1}}.$$

where

$$\phi = \frac{\beta\pi_{I1}^H - (\beta-1)\pi_{I1}^L}{\pi_{I1}^L}. \tag{18}$$

Proposition 5.1 implies the conditions of manipulative revelation for π_{I2}^H.

Proposition 6.1. Case (a) *Truthful Revelation is an equilibrium if and only if*

$$\pi_{I2}^H \geq \pi_{I1}^H - \frac{\xi(\pi_{E2}^L)}{\beta}\left(\frac{K_E}{K_I}\right)^{\beta-1},$$

Case (b) *λ-Hybrid Revelation for $0 < \lambda < 1$ is an equilibrium if and only if*

$$\pi_{I1}^H - \frac{\xi(\pi_{E2}^L)}{\beta}\left(\frac{K_E}{K_I}\right)^{\beta-1} > \pi_{I2}^H > \pi_{I1}^H - \frac{\xi(\pi_{E2}^M)}{\beta}\left(\frac{K_E}{K_I}\right)^{\beta-1},$$

and

Case (c) *Manipulative Revelation is an equilibrium if and only if*

$$\pi_{I2}^H \leq \pi_{I1}^H - \frac{\xi(\pi_{E2}^M)}{\beta}\left(\frac{K_E}{K_I}\right)^{\beta-1}.$$

Proof. *See Appendix.*

Proposition 6.1 states that larger duopoly profit flow at high demand ensures that the high-demand incumbent truthfully enters the market at the optimal timing because the high-demand incumbent has less incentive to prevent earlier investment of the entrant. In contrast, less duopoly profit flow at the high demand makes the high-demand incumbent to hide information and take advantage of the monopoly for a longer time. Hence, in this case, strategically, the high-demand incumbent enters the market at the optimal timing of the low-demand incumbent in order to hide the information. In the mid-range of the duopoly profit flow, the high-demand incumbent uses a mixed strategy.

Proposition 6.2 is obtained by solving the equations in 6.1 to the ratio between the incumbent and the entrant of costs.

Proposition 6.2. **Case (a)** *Truthful Revelation is an equilibrium if and only if*

$$\frac{K_E}{K_I} \geq \left\{ \frac{\beta}{\xi(\pi_{E2}^L)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}},$$

Case (b) *λ-Hybrid Revelation for $0 < \lambda < 1$ is an equilibrium if and only if*

$$\left\{ \frac{\beta}{\xi(\pi_{E2}^L)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}} > \frac{K_E}{K_I} > \left\{ \frac{\beta}{\xi(\pi_{E2}^M)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}},$$

and

Case (c) *Manipulative Revelation is an equilibrium if and only if*

$$\frac{K_E}{K_I} \geq \left\{ \frac{\beta}{\xi(\pi_{E2}^M)} (\pi_{I1}^H - \pi_{I2}^H) \right\}^{\frac{1}{\beta-1}}.$$

Proposition 6.2 asserts that a sufficiently lower cost of the incumbent or a sufficiently larger cost of the entrant causes the high-demand incumbent enter the market at the optimal timing truthfully. In contrast, under a larger cost of the incumbent or a lower cost of the entrant, the high-demand incumbent has the incentive of the strategic entrance.

7 Numerical Examples for Equilibrium Strategies and Values

In this section, results of comparative statics for equilibrium strategies and the values of the incumbent are presented through numerical examples. Parameters in examples are basically set as $\mu = 0.03$, $r = 0.07$, $p = 0.5$, $\sigma = 0.2$, $x = 0.05$, $\pi_{I1}^H = 12$, $\pi_{I1}^L = 7$, $\pi_{I2}^H = 4$, $\pi_{I2}^L = 4$, $\pi_{E2}^H = 4$, $\pi_{E2}^H = 1$, $K_I = 50$, and $K_E = 100$.

First, the relationship between values $u_I(\cdot,\cdot,H)$ and the duopoly profit incumbent π_{I2}^H of the high-demand incumbent is examined. Figure 1 illustrates the values $u_I(x_I^H, x_E^H, H)$, $u_I(x_I^L, x_E^M, H)$, and $u_I(x_I^L, x_E^L, H)$. For $\pi_{I2}^H \geq 8.0$, $u_I(x_I^H, x_E^H, H)$ is greater than $u_I(x_I^L, x_E^L, H)$. As explained in Proposition 6.1, the high-demand incumbent does not deviate the optimal timing of the investment truthfully, because the duopoly profit of the incumbent is sufficiently large and the incumbent does not have a strong incentive to make the delay the investment of the entrant. Hence, the high-demand incumbent enters the market at the optimal timing, and reveals his information truthfully. In contrast, for $\pi_{I2}^H \leq 2.9$, $u_I(x_I^H, x_E^H, H)$ is less than $u_I(x_I^L, x_E^M, H)$. In this range, the high-demand incumbent invests at the optimal timing of the low demand to hide information for high demand because the duopoly profit of the incumbent is small and the decrement of the profit of the incumbent by the investment of the entrant is critical. For $2.9 \leq \pi_{I2}^H \leq 8.0$, $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^L, H)$, the incumbent uses a mixed strategy as $\lambda-$Hybrid Revelation. In this interval, the value of the incumbent is the same as $u_I(x_I^H, x_E^H, H)$ because the mixed strategy should satisfy the condition $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$. Therefore, the value of the high-demand incumbent in the equilibrium strategy is $u_I(x_I^H, x_E^H, H)$ for $\pi_{I2}^H \leq 2.9$ and is $u_I(x_I^L, x_E^M, H)$ for $\pi_{I2}^H \geq 2.9$.

Figure 2 illustrates the probability λ that the high-demand incumbent invests at the optimal timing for the high demand in the equilibrium strategy, i.e., the incumbent enters to the market truthfully. For $\pi_{I2}^H \leq 2.9$, Manipulative Revelation is a perfect Bayesian equilibrium so that $\lambda = 0$, while for $\pi_{I2}^H \geq 8.0$, Truthful Revelation is a perfect Bayesian equilibrium so that $\lambda = 1$. For $2.9 < \pi_{I2}^H < 8.0$, the incumbent uses a completely mixed strategy, and λ has a positive value, which increases in π_{I2}^H.

Next, the effect of volatility is examined. Figure 3 illustrates the relationship between the values of the high-demand incumbent and the volatility. If the volatility is small, the incumbent invests truthfully, whereas if the volatility is large, the incumbent invests strategically. If the volatility is moderate, the incumbent uses a mixed strategy.

The relationship between the investment cost and the value of the incumbent is then investigated. Figure 4 depicts the relationship between the values of the high-demand incumbent and its cost of the investment. As explained in Proposition 6.2, the values decrease non-linearly with investment cost and increase with profit flow. If the cost is small, Truthful Revelation occurs, whereas if the cost is large, Manipulative Revelation occurs. If the cost is moderate, the incumbent uses a complete mixed strategy, in which $\lambda-$ Hybrid Revelation for some $0 < \lambda < 1$ is an equilibrium.

Finally, the impact of the value of the incumbent on the investment cost of the entrant is investigated. Note that the value of the incumbent is affected not only by the own cost of the incumbent, but also by the cost of the rival because the smaller cost of the entrant, which pushes forward the investment of the entrant, reduces the value of the incumbent. Figure 5 depicts the relationship between the values of the high-demand incumbent and the cost of the entrant. As shown shown in Proposition 6.2, if the cost of the entrant is large, then the timing of the investment of the entrant is late. Since the effect of the investment of the entrant on the value of the incumbent is negligible, the high-demand incumbent invests truthfully. On the other hand, the incumbent invests strategically for the case in which the cost of the entrant is small. For a moderate interval of the cost of the entrant, the incumbent uses a mixed strategy.

8 Conclusion

The present paper examines an investment game for an incumbent and an entrant for optimal entries into a new market in which only the incumbent has the information of whether the demand, is high or low, and the entrant predicts the demand by observing the investment timing of the incumbent. Whether the incumbent reveals the information truthfully is investigated while taking into account the signaling effect by using the concept of a perfect Bayesian equilibrium. A condition in which the incumbent with information of high demand invests strategically in the equilibrium is characterized. A condition in which the incumbent to use a mixed strategy in the equilibrium is also demonstrated.

If the duopoly profit for the high-demand incumbent is small, then the incumbent invests strategically, whereas the incumbent invests truthfully if the duopoly profit is sufficiently large. The incumbent also invests strategically, if the volatility or the cost of the incumbent is large, or if the cost of the entrant is small.

Since this is the first study of signaling model for an investment game under real option approach, extension of the model would be interesting in future research. Preemptive behavior should be considered by eliminating the assumption in which the incumbent is the leader and the entrant is the follower. Other stochastic processes could also be considered in order to extend the model.

Appendix

Proof of Proposition 3.1: This proposition is derived by a standard calculation of the first hitting time (see for example Dixit and Pindyck (1994), pp. 315?316). Let $\hat{t}$ be the first hitting time at which X_s reaches a fixed threshold $\hat{x}$, where $X_0 = x$. Dixit and Pindyck (1994, pp. 315–316) reported that

$$E[e^{-r\hat{t}}] = \left(\frac{x}{\hat{x}}\right)^{\beta} \tag{19}$$

and

$$E[\int_0^T X_s e^{-rs} ds] = \frac{x}{r-\mu} - \frac{\hat{x}}{r-\mu}\left(\frac{x}{\hat{x}}\right)^{\beta} \tag{20}$$

where β is given by Eq. (2). Equation (20) implies that

$$E[\int_T^{+\infty} X_s e^{-rs} ds] = E[\int_0^{+\infty} X_s e^{-rs} ds] - E[\int_T^{+\infty} X_s e^{-rs} ds] = \frac{\hat{x}}{r-\mu} \left(\frac{x}{\hat{x}}\right)^\beta \tag{21}$$

First, we consider $v_I(x_I, \theta)$. If $x > x_I$, then the incumbent immediately enters the market, i.e., $t_I = t$. Hence, by the Markov property of the geometric Brownian motion, we have

$$\begin{aligned} v_I(x_I, \theta) &= E[\int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^\theta X_s ds - e^{-r(t_I - t)} K_I | X_t = x] \\ &= E[\int_t^{\infty} e^{-r(s-t)} \pi_{I1}^\theta X_s ds - K_I | X_t = x] \\ &= E[\int_0^{\infty} e^{-rs} \pi_{I1}^\theta X_s ds | X_0 = x] - K_I \\ &= \frac{\pi_{I1}^\theta}{r-\mu} x - K_I. \end{aligned}$$

If $x \le x_I$, then t_I is the first hitting time of the stochastic process reaches a fixed threshold x_I. Then, according to the Markov property of the geometric Brownian motion,

$$\begin{aligned} v_I(x_I, \theta) &= E[\int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^\theta X_s ds - e^{-r(t_I - t)} K_I | X_t = x] \\ &= E[\int_{t_I}^{\infty} e^{-rs} \pi_{I1}^\theta X_s ds - e^{-rt_I} K_I | X_0 = x] \\ &= E[\int_{t_I}^{\infty} e^{-rs} \pi_{I1}^\theta X_s ds | X_0 = x] - K_I E[-e^{-rt_I} | X_0 = x]. \end{aligned}$$

Equations (19) and (21) imply that $v_I(x_I, \theta) = \left(\frac{\pi_{I1}^\theta}{r-\mu} x_I - K_I\right) \left(\frac{x}{x_I}\right)^\beta$

Similarly, $w_I(x_E, \theta)$ for $x < x_E$ is

$$\begin{aligned} w_I(x_E, \theta) &= E[\int_{t_E}^{\infty} e^{-r(s-t)} (\pi_{I1}^\theta - \pi_{I2}^\theta) X_s ds | X_t = x] \\ &= E[\int_{t_E}^{\infty} e^{-rs} (\pi_{I1}^\theta - \pi_{I2}^\theta) X_s ds | X_0 = x] \\ &= \frac{\pi_{I1}^\theta - \pi_{I2}^\theta}{r-\mu} x_E \left(\frac{x}{x_E}\right)^\beta. \end{aligned}$$

□

Proof of Proposition 3.1:

This proposition is derived by solving Eqs. (14) and (15) for π_{I2}^H. In this proof, the condition of Case (a) is shown to be obtainable by solving Eq. (14). The conditions of Case (b) and Case (c) are obtained in a similar manner.

According to Eq. (4), the inequality (14) is expressed by

$$v_I(x_I^H, H) - w_I(x_E^H, H) \geq v_I(x_I^L, H) - w_I(x_E^L, H),$$

which can be rewritten as

$$v_I(x_I^H, H) - v_I(x_I^L, H) \geq w_I(x_E^H, H) - w_I(x_E^L, H). \tag{22}$$

Then, Eq. (5) implies

$$v_I(x_I^H, H) - v_I(x_I^L, H) = \left(\frac{\pi_{I1}^H}{r-\mu} x_I^H - K_I\right)\left(\frac{x}{x_I^H}\right)^\beta - \left(\frac{\pi_{I1}^H}{r-\mu} x_I^L - K_I\right)\left(\frac{x}{x_I^L}\right)^\beta.$$

Substituting Eq. (8) into the above expression, we obtain

$$v_I(x_I^H, H) - v_I(x_I^L, H) = \beta^{-\beta}(\beta-1)^{\beta-1} x^\beta K_I^{1-\beta}(r-\mu)^{-\beta}\{(\pi_{I1}^H)^\beta - \phi(\pi_{I1}^L)^\beta\}, \tag{23}$$

where ϕ is defined by Eq. (18). Equation (6) also implies that

$$w_I(x_E^H, H) - w_I(x_E^L, H) = \frac{\pi_{I1}^H - \pi_{I2}^H}{r-\mu} x_E^H \left(\frac{x}{x_E^H}\right)^\beta - \frac{\pi_{I1}^H - \pi_{I2}^H}{r-\mu} x_E^L \left(\frac{x}{x_E^L}\right)^\beta \tag{24}$$

According to Eq. (1), x_E^H and x_E^L are given by

$$x_E^H = x_E^*(1) = \tfrac{\beta}{\beta-1}\tfrac{r-\mu}{\pi_{E2}^H} K_E, \quad x_E^L = x_E^*(0) = \tfrac{\beta}{\beta-1}\tfrac{r-\mu}{\pi_{E2}^L} K_E.$$

Hence, by substituting these expressions into Eq. (25), we obtain

$$w_I(x_E^H, H) - w_I(x_E^L, H) = \beta^{1-\beta}(\beta-1)^{\beta-1} x^\beta K_E^{1-\beta}(r-\mu)^{-\beta}(\pi_{I1}^H - \pi_{I2}^H)\{(\pi_{I1}^H)^{\beta-1} - (\pi_{I1}^L)^{\beta-1}\}. \tag{25}$$

According to Eqs. (23) and (25), inequality (22) implies that

$$K_I^{1-\beta}\{(\pi_{I1}^H)^\beta - \phi(\pi_{I1}^L)^\beta\} \geq \beta K_E^{1-\beta}(\pi_{I1}^H - \pi_{I2}^H)\{(\pi_{I1}^H)^{\beta-1} - (\pi_{I1}^L)^{\beta-1}\}.$$

Consequently, this yields

$$\pi_{I2}^{H} \geq \pi_{I1}^{H} - \frac{1}{\beta}\left\{\frac{(\pi_{I1}^{H})^{\beta} - \phi(\pi_{I1}^{L})^{\beta}}{(\pi_{I1}^{H})^{\beta-1} - (\pi_{I1}^{L})^{\beta-1}}\right\}\left(\frac{K_E}{K_I}\right)^{\beta-1},$$

which completes the proof. □

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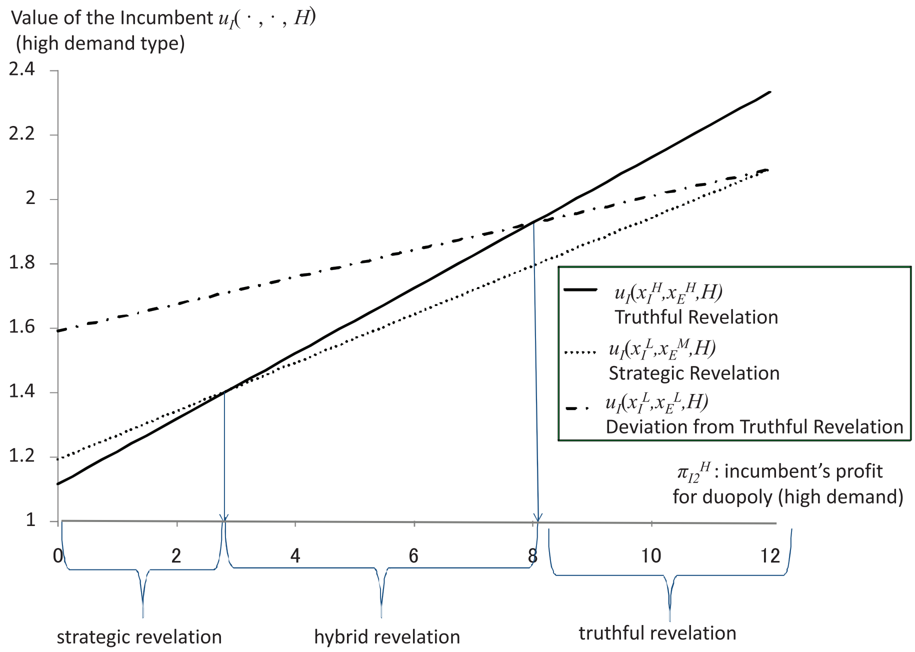

Figure 1: Values of the high-demand incumbent $u_I(\cdot, \cdot, H)$ and the duopoly profit of the high-demand incumbent π_{I2}^H

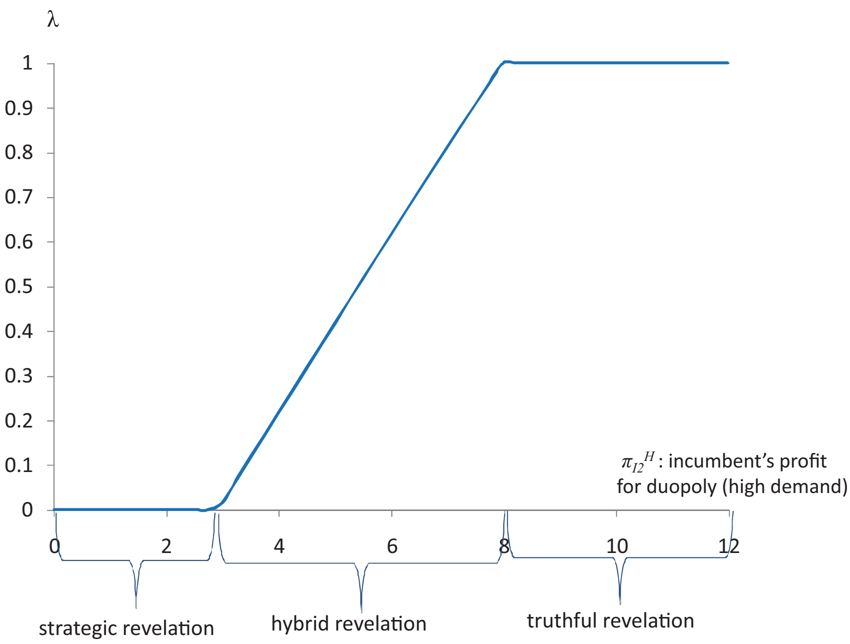

Figure 2: Probability of investment of the high-demand incumbent at the optimal timing for the high demand and the duopoly profit of the high-demand incumbent π_{I2}^{H}

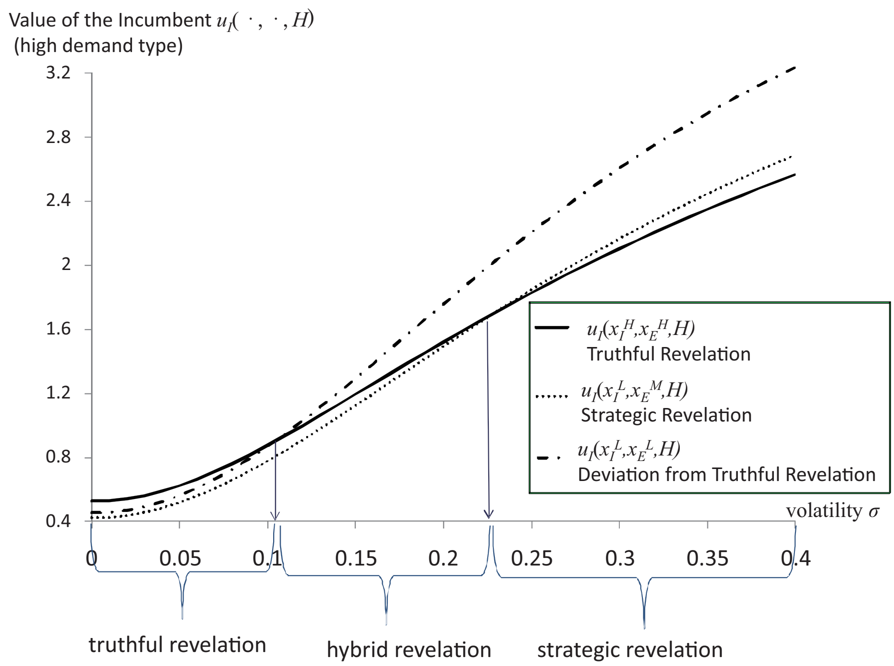

Figure 3: Values of the high-demand $u_I(\cdot, \cdot, H)$, high-volatility σincumbent

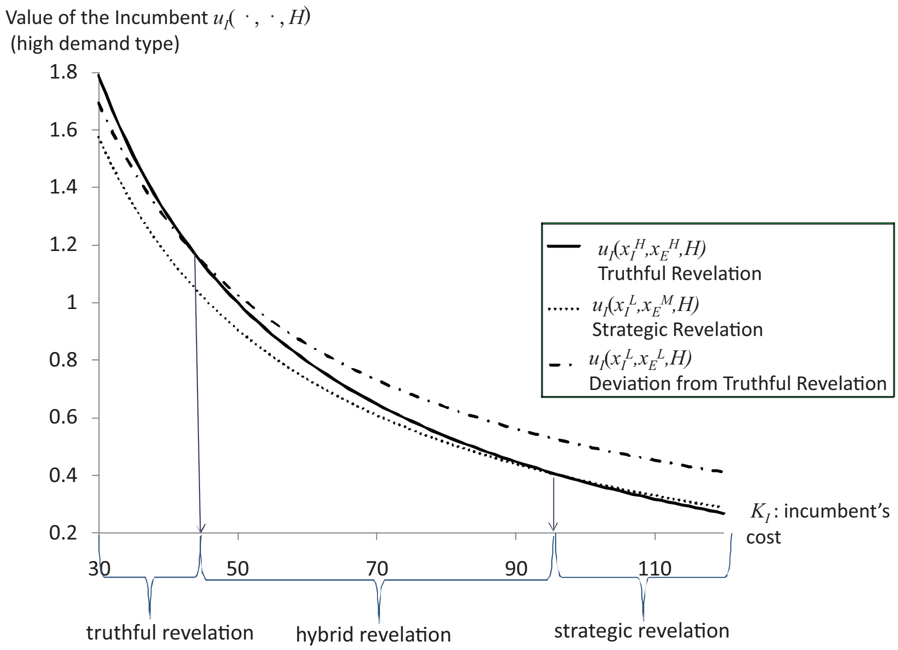

Figure 4: Values of the high-demand $u_I(\cdot,\cdot,H)$, high-cost K_I incumbent

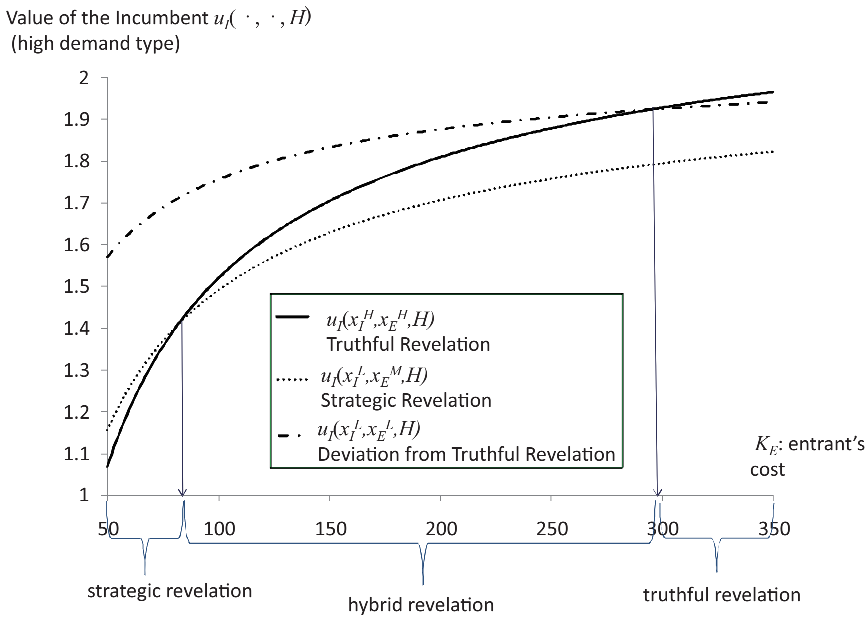

Figure 5: Values of the high-demand $u_I(\cdot,\cdot,H)$, high-cost K_E incumbent