The Diversity of Information Acquisition Strategies in a Noisy REE Model with a Common Signal and Independent Signals

by

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Abstract

In this paper, I study a noisy REE model of asset market with two types of costly private signals: a common signal with an identical error term and independent signals with dispersed error terms. Studying investors' endogenous information acquisition, I show that (i) investors observing the common signal and those observing the independent signals are likely to coexist in the equilibrium, (ii) at most, three equilibrium strategies can coexist, and (iii) when the equilibrium has three strategies, the evolutionary learning dynamics of investors can exhibit detours and cyclical oscillation.

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Andrei Shleifer (2000) argued that behavioral finance has revealed that very little is understood about the behavior of investors in real security markets. This paper attempts to answer an important question Shleifer posed in his study: "Why do different investors have such different models of what are good investments?" (Shleifer 2000, 195) Indeed, diverse investment strategies coexist in security markets. Some strategies are based on fundamental analysis while others are based on technical analysis or chart analysis.

Shleifer's question will be answered in part by Grossman and Stiglitz (1980). Their noisy rational expectations equilibrium (hereafter REE) model focuses on two different strategies, informed and uninformed. Informed investors create fundamental information about a risky asset, incurring information costs. In contrast, uninformed investors extract information about securities from the price system without incurring any costs. When informed investors find that an asset is under-priced, they bid its price higher; hence, the asset price conveys information from informed fundamental analysts to uninformed price watchers. Grossman and Stiglitz showed that if both the private information of informed investors and the price do not perfectly reflect the fundamental information, then the two types of strategies can coexist.[1]

This paper further explores the diversity of information strategies, particularly focusing on the diversity of information sources accessed by informed investors. In the real world, there are quite a large number of information sources, including sources available to the public, that can provide useful information in predicting future security prices. According to recent studies of investor behavior, investors' attention is not dispersed over the entire set of information sources, but rather it is concentrated on a small number of common information sources, such as the *Wall Street Journal* or the *Financial Times*.

Huberman and Regev (2001) provide us with an example that clearly illustrates the concentration of investors' attention. In their study, they investigated the effect of public information on the stock price of a biotechnology company, EntreMed, which was developing new cancer-curing drugs. The first report about EntreMed's breakthrough in cancer research was published in November 1997 as a scientific piece in *Nature* and as hard news in various popular newspapers (including *The New York Times*). Inconsistent with market efficiency, the news did not have a significant impact on EntreMed's stock price. More than five months later, news of the breakthrough was reported again as a special report on the front page of the Sunday edition of *The New York Times*. This time the story had a significant and persistent impact on the stock price. Huberman and Regev also showed that another piece of news on EntreMed that appeared on the front page of the *Wall Street Journal* had an immediate impact on the stock price. Their case study reveals two important aspects of investor behavior. First, the focus of the majority of investors is on a much smaller set of information sources than the whole set of publicly available information. Thus, many investors ignore much information that is publicly available, such as the information that was published in *Nature*. Second, the information sources ignored by the majority of investors can provide potentially profitable information. Finding that publicly available information yields abnormal returns would not be surprising for many readers who observe a large number of empirical studies in the field of behavioral finance. The important effect of investors' attention being concentrated on only a small number of information sources is that this small number of sources has a significant impact on the market prices, as was evident in the case of EntreMed. Therefore, if a front-page article in the *Wall Street Journal* is too optimistic about the future of a company, which then leads to investors' optimistic opinion, the stock price will become higher than it should be. However, this would not happen if investors' attention was dispersed over the entire set of information sources. In that case, as the law of large numbers predicts, optimistic and pessimistic opinions would cancel each other out in the process of market trade, which would lead to reasonable average opinions and reasonable market prices.

The concentration of investors' attention on only a small number of easily accessible sources could be partially explained by information cost. If an investor wants to save searching cost by quickly accessing profitable information from the vast number of information sources, it makes sense to access the information sources that are already publicly recognized as trusted sources.

Of course the existence of abnormal returns does not mean that all investors ignore the profitable information sources. Some successful investors search for profitable information from the vast information sources that are ignored by the majority of investors. The best example of this is James B. Rogers Jr., co-founder of the Quantum Fund. Rogers has consistently claimed that a thorough investigation of the security investors choose to invest is important to being successful in investment. This perspective is clearly seen in the following response Rogers gave to a question about why he follows so many diverse markets throughout the world:

"I don't see how you can invest in American steel without understanding what is going on in Malaysian palm oil." (Schwager 1989, 306)

Julian H. Robertson Jr., founder of the investment firm Tiger Management Corp, also emphasizes the importance of conducting a thorough investigation of investment opportunities. In an interview with the Japanese public broadcasting company NHK, Robertson said that his company employs 130 analysts compared to just 40 traders, and that the company's 130 analysts conduct thorough research of the industry of each firm they are considering. To do that, the analysts travel to all corners of the world and conduct interviews with the top managers of the companies they are interested in. Clearly, these analysts are engaged in independent investigations to search for useful information from a sea of ignored information sources. Compared to the investors that just check a small number of common information sources, these investors' opinions are thought to be more balanced. The information sources used by these investors are not well known or publicly recognized; therefore, it is less likely that their research overlaps by depending on the same information sources. Indeed, how many investors dealing with American steel pay attention to Malaysian Palm oil?

These observations reveal that informed investors face an important strategic decision: depend solely on the common information sources, as the majority of investors do, or explore uncommon information sources, as James Rogers does. In the real world, a variety of informed strategies can coexist. The coexistence is not obvious, however, from an economic point of view. If one particular strategy always provides information with better cost performance, all informed investors will adopt that strategy while other strategies will be abandoned and will therefore not be observed in the equilibrium.

This paper presents a theoretical model to study the diversity of information strategies, and shows that their coexistence is likely to be observed in the equilibrium because of asymmetric informational value of asset prices. The model I have developed in this paper is an extension of Grossman and Stiglitz's (1980) noisy REE model with endogenous information acquisition; the critical difference is that the model presented here has two different types of private signals, a common signal and independent signals. Investors observing the common signal face the identical error term, as assumed in Grossman and Stiglitz. In contrast, the error terms of the independent signals are dispersed and independent, as assumed in Hellwig (1980) and Admati (1985). The two types of private signals correspond to the two types of information sources, common or uncommon. The common signal represents the information that comes from the common information sources. The identical error term represents the information error in the common information source, such as an optimistic article in the *Wall Street Journal*. In contrast, the dispersed error term of the independent signals represents the dispersed attention that is spread over the vast uncommon information sources; so even if an extreme event in the Malaysian palm oil industry gives James Rogers an optimistic opinion, different independent signals can make other investors pessimistic. In addition to these private signals, all investors can use the security price as a public signal.

In the model constructed with two types of private signals and one public signal, investors have four types of alternative strategies:

Strategy B (Common informed): Taking only the common signal.

Strategy G (Independent informed): Taking only the independent signal. First we assume the accuracy level is fixed, and then we allow investors to choose an accuracy level.

Strategy H (Hybrid informed): Taking both the common and independent signals.

Strategy U (Uninformed): No private signals.

Of course strategies that give a lower expected payoff to investors will not be chosen or observed in the equilibrium. In the first part of the paper, I analyze how many of the strategies can survive and coexist with other strategies as equilibrium strategies. The main result is that *Strategies B and G can coexist in the equilibrium if their signals are equally accurate and the cost for strategy G is higher than that of strategy B, though not significantly.* The condition in the above statement implies that strategy B has a better cost performance if the role of the security price as a conveyer of information is ignored. Despite that fact, strategy G can coexist with strategy B in the equilibrium.

I then demonstrate that the survival of strategy G in the equilibrium is caused by an asymmetric informational value of security price. For example, suppose there are no informed investors except strategy B investors who read the *Wall Street Journal*. Then, the security price contains information published in the *Wall Street Journal*. The informational value of the security price is zero for strategy B investors who know the original information, while it is strictly positive for potential strategy G investors. That is, the informational value of the security price is asymmetrical. By observing the security price, strategy G investors can infer information published in the *Wall Street Journal* without purchasing and reading it. Because of this asymmetry in the informational value of the security price, even if each private signal has equal accuracy, strategy G's total information, taking the security price into account, is more accurate than that of strategy B. Therefore, strategy B investors will have an incentive to change their strategy to strategy G as long as the additional information cost is not too high; thus, the state without strategy G cannot be sustained as equilibrium.

When the information costs for informed strategies are not too low, the uninformed strategy (Strategy U) coexists with strategies B and G. I demonstrate that the equilibrium state with the three strategies B, G, and U is robust under the assumption of linear information technology. First, even if investors can choose accuracy levels continuously, the accuracy level chosen by strategy G investors is unique in the equilibrium. In addition, the hybrid strategy (Strategy H) cannot coexist with the uninformed strategy in the equilibrium. Thus, the number of strategies that can coexist in an equilibrium state is three at most.

The second part of the paper is devoted to the study of learning dynamics. The equilibrium analysis in the first half of the paper is conducted under the implicit assumption that investors make rational inferences that are consistent with the strategy distribution in the equilibrium state, and that they can choose the best strategy against the other investors' strategic choices. At this point, it is natural to wonder whether naive investors in the real world behave as the equilibrium theory predicts. Learning studies in economics and game theory are useful for determining the answer; that is, learning studies can determine whether naive, or boundedly rational, economic agents learn to play such a complicated equilibrium strategy if they face the same situation repeatedly. This paper attempts to determine whether investors can learn to play the best information acquisition strategy when there are three equilibrium strategies—B, G, and U. Assuming that investors can make a rational inference on the true asset value based on their own information, I study the dynamics of investors' strategy adjustment by applying evolutionary game theory.

For this application, I adopt and analyze two typical dynamic processes applicable to our model, replicator dynamics and best response dynamics, as alternative descriptions of investors' learning process. As a result of numerous simulation studies on a wide range of initial strategy distributions and parameters of the model, the following conclusions were obtained: *The model has a unique stable equilibrium. Independent from the choice of dynamic processes, the learning process converges to the unique equilibrium.* After thousands of numerical simulations, the author could not find any cases that had multiple stable equilibria or cases in which the dynamic process did not converge under moderate adjustment speed.[2] This is a positive result from the viewpoints of naive learning or mathematical algorithm study.

Although both best response dynamics and replicator dynamics lead investors to the unique equilibrium strategy distribution, the properties of the paths are totally different.

For all the cases studied in numerical simulation, best response dynamics, which assumes that investors know the best strategy, lead investors straight to the equilibrium after a few changes in the direction of adjustment. In contrast, the learning process under replicator dynamics, which represents more naive learning, creates smooth curved lines, including large detours for most of the cases, and then the paths change direction toward the equilibrium. In some cases, the detours end up in cyclical oscillation converging to the equilibrium, resembling water going through a huge funnel. Along the cyclical oscillation path, the most favorable strategies for investors change in the repeated alphabetical order B, G, U, B, G, U... This cycle is thought to be partly caused by the asymmetric informational value of the security price and partly caused by the nature of replicator dynamics. This cycle might be relevant to the changes in investors' behavior and asset price dynamics, including the asset price bubble.

Finally, this study is closely related to two existing studies in finance. First is the work of Grossman and Stiglitz (1980), whose analysis showed the general impossibility of informational efficiency. Like their model, the equilibrium of my model is informationally inefficient. By introducing different types of private signals, this paper makes it possible to discuss the correspondence between private and public signals in the model and the different levels of information efficiency—weak form, semi-strong form, and strong form.[3] Another area of financial research related to this paper is the study of herd behavior. There are two different herds in my model—the price-watching uninformed and the informed that adopts a common signal. My conclusion is that the reason for herding is much simpler than the reasons presented in the current literature on herd behavior. Put simply, investors herd to save information cost. Before concluding the paper, I will further explore the relationship between this paper and these two studies on informational efficiency and herd behavior.

The paper is organized as follows: Section 1 presents the basic model of two different types of costly private signals. Section 2 analyzes the equilibrium of the basic model and provides the main theorem on the diversity of information acquisition strategies. Section 3 demonstrates the robustness of the equilibrium including three strategies.

Section 4 investigates the evolutionary learning behavior of investors. Section 5 discusses related literature and Section 6 concludes.

1. The Basic Model with Three Information Acquisition Strategies

For this paper, I theoretically study the diversity of endogenous information acquisition strategies in a noisy REE model in the manner of Grossman and Stiglitz (1980). I extend their model by giving investors an additional information acquisition strategy, which reveals dispersed private information similar to the models built by Hellwig (1980) and Admati (1985). In this section, we provide the assumptions of the model. We start from a simple situation in which investors have only three information strategies. Later, in section 3, investors are allowed to choose accuracy levels of independent signals and combine two different types of private signals.

A. Assets, Investors, and Information

In my analysis, I consider a three-period model, days 1, 2, and 3. There is one risk-free asset, which has a constant price of 1, and one risky asset. The value of the risky asset in day 3, denoted v, is determined as the sum of two independently normally distributed random variables y and z, with means μ and 0 and variances σ_y^2 and σ_z^2, respectively. The value of y is determined at the beginning of day 2, and investors are able to obtain signals regarding y in day 2. The value of z is determined in day 3, and it simultaneously determines the value of v. The existence of z assures that any investment in the risky asset is literally risky.

Following Admati (1985), we assume that there is a continuum of rational investors. Let $I = [0,1]$ be the set of all rational investors. The investors are indexed $i \in I$. We use Lebesgue measure m to evaluate the size of subsets in I. To save space, we express the Lebesgue measure of subset E in I by m_E, not by $m(E)$. (Note that $m_I = 1$.) Since each investor corresponds to a real number, each investor has measure zero, that is, $m_{\{i\}} = 0$ for all $i \in I$. This means that each investor is too insignificant to affect the market price. The investors have identical CARA utility with common coefficient of risk aversion $a >$

0 and make decisions so as to maximize expected utility.

As stated in the introduction, we assume there are two different types of private signals. The first type is what we call the common signal, denoted s_b, which represents information from common information sources, such as the *Wall Street Journal* or top analyst recommendations. All the traders who chose to adopt the common signal observe

(1) $s_b = y + \varepsilon_b$.

The error term, ε_b, is drawn from a normal distribution with mean 0 and variance σ_b^2. Label b means "bad" because it is an unfavorable signal from a social point of view; since all investors with the common signal are influenced by the common error term, the common error disturbs the price. Each investor has to incur information cost, c_b, to obtain the common signal.

The second type of private signal is what we call independent signals, which represent information from uncommon information sources, such as informal interviews with CEOs of a corresponding company conducted by individual investors. An arbitrary investor, say investor i, who chooses to adopt an independent signal observes

(2) $s_i = y + \varepsilon_i$ for all $i \in G$

where G is the set of investors observing independent signals. Unlike the common signal, investors taking independent signals observe dispersed signals. The error terms of independent signals, $\{\varepsilon_i\}_{i\in G}$, are drawn independently from an identical normal distribution with mean 0 and variance σ_g^2. Following Admati (1985), we assume ε_i is m-measurable function[4] and the strong law of large numbers holds. That is,

(3) $\int_{i\in G} \varepsilon_i dm = 0$ almost surely.

Note that this equation does not depend on m_G. When m_G is positive, G contains infinite investors under the assumption of a continuum of investors; thus, the analogy of strong law of large numbers works. When m_G is zero, we can ignore the sum. In either case, the equation holds. Information cost for an independent signal is c_g. In section 3, we allow investors taking independent signals to choose the level of accuracy, σ_g^2. For the moment, however, we assume the accuracy level is given. Conversely to label *b*, label *g* means "good" since the error terms do not affect the price.

B. Decision Timing

The timing of investors' decision-making is as assumed in Grossman and Stiglitz (1980).

Day 1: Investors choose one information acquisition strategy from three pure strategies.

Strategy B (Common informed, or bad type): Taking only the common signal.

Strategy G (Independent informed, or good type): Taking only the independent signal.

Strategy U (Uninformed): Taking no private signals.

Strategies B and G are costly, while strategy U is costless. Let *B*, *G*, *U* be the sets of investors adopting strategies B, G, U, respectively. They are assumed to be measurable and satisfy $m_B + m_G + m_U = 1$.[5]

Day 2: After the value of y has been determined, investors receive private signals on the realization of y corresponding to their strategy choice in day 1. The price of the risky asset, denoted p, is a public signal. Using observed signals, investors decide on the demand schedule for the risky asset as a function of p. Apart from trade by rational investors, there are noise trades reflecting the demand and supply by liquidity traders or naive arbitragers with biased belief. We denote the per capita net demand of the noise trade as x, which is assumed to be independently and normally distributed with mean 0 and variance σ_x^2.

Equilibrium price p is determined so as to let the total net demand for the risky asset equal zero.

Day 3: Values of z and v are realized, and then investors' final wealth and utility are determined accordingly.

Random variables x, y, z, ε_b, $\{\varepsilon_i\}_{i\in G}$ are assumed to be independent and to have strictly positive variances. The rational investors are supposed to know their distributions.

2. Equilibrium of the Basic Model

In this section, we study the model equilibrium's property. We can assume that rational investors play the strategic game in day 1, expecting rational results of market trade in days 2 and 3. We therefore start from the equilibrium analysis of the asset market in day 2, where strategy distributions are already given. We then solve the Bayesian Nash equilibrium (hereafter BNE) of the strategic game in day 1 in which the expected utility is consistent with the asset market equilibrium in day 2.

2.1 REE of the Asset Market in Day 2

After investors commit to a pure strategy in day 1, they play the rational expectations equilibrium (REE) of the asset market in day 2. That is, (i) all investors are price takers, (ii) investors' inferences are rationally driven from their beliefs, and (iii) their beliefs are consistent with the equilibrium strategies of all investors.[6]

Under the assumption of CARA utility, it is known that the optimal demand does not depend on investors' initial wealth. Thus we omit it and focus on the net return from investment in the risky asset, $(v-p)\,x_i - c_i$, where x_i is the quantity of the risky asset purchased by investor i in day 2 and c_i is the information cost corresponding to investor i's strategy choice in day 1. The maximization problem for investor i with information Ω_i is given as follows:

(4) $\max_{x_i} E\left[u(R_i)\|\Omega_i\right]$ where $u(R) = -\exp[-aR]$ and $R_i \equiv (v-p)x_i - c_i$.

u is the identical CARA utility function for all investors and R_i is the net return of investor i.

Rational investors know that the demands of other rational investors affect the equilibrium price of the risky asset; they therefore make rational inferences about underlying information based on the price. To be able to make inferences based on the price, these traders must conjecture a form for the price function, and in equilibrium, this conjecture must be correct. It is natural to expect that the price function is a function of all the private signals and the noise demand. Note, however, that each private signal is divided into two parts: fundamental value of asset y and the error term. Moreover, under the strong law of large numbers (equation (3)), the error terms of independent signals are almost surely canceled out. So, suppose that the conjectural price function is of the form

(5) $p = \beta_0 + (\beta_b + \beta_g)(y - \mu) + \beta_b \varepsilon_b + \beta_x x$,

where β_0, β_b, β_g, and β_x are coefficients to be determined. Parameters β_b and β_g represent the impacts of strategies B and G investors, respectively, to the equilibrium price.

Under this conjecture, we can apply the projection theorem[7] which assures that the distribution of y conditional on investor's private and public signals has a normal distribution. The theorem also gives the means and the variances.

(6) $E[y \mid s_b, p] = \mu + \dfrac{(-\beta_b\beta_g\sigma_b^2 + \beta_x^2\sigma_x^2)\sigma_y^2(s_b - \mu) + \beta_g\sigma_b^2\sigma_y^2(p - Ep)}{\beta_g^2\sigma_b^2\sigma_y^2 + \beta_x^2(\sigma_b^2 + \sigma_y^2)\sigma_x^2}$,

(7) $\mathrm{var}[y \mid s_b, p] = \dfrac{\beta_x^2\sigma_b^2\sigma_x^2\sigma_y^2}{\beta_g^2\sigma_b^2\sigma_y^2 + \beta_x^2(\sigma_b^2 + \sigma_y^2)\sigma_x^2}$,

for $i \in G$

(8) $E[y \mid s_i, p] = \mu + \dfrac{(\beta_b^2\sigma_b^2 + \beta_x^2\sigma_x^2)\sigma_y^2(s_i - \mu) + (\beta_b + \beta_g)\sigma_g^2\sigma_y^2(p - Ep)}{(\beta_b + \beta_g)^2\sigma_g^2\sigma_y^2 + (\sigma_g^2 + \sigma_y^2)(\beta_b^2\sigma_b^2 + \beta_x^2\sigma_x^2)}$,

(9) $\mathrm{var}[y \mid s_i, p] = \dfrac{(\beta_b^2\sigma_b^2 + \beta_x^2\sigma_x^2)\sigma_g^2\sigma_y^2}{(\beta_b + \beta_g)^2\sigma_g^2\sigma_y^2 + (\sigma_g^2 + \sigma_y^2)(\beta_b^2\sigma_b^2 + \beta_x^2\sigma_x^2)}$,

(10) $E[y \mid p] = \mu + \dfrac{(\beta_b + \beta_g)\sigma_y^2(p - Ep)}{(\beta_b + \beta_g)^2\sigma_y^2 + \beta_b^2\sigma_b^2 + \beta_x^2\sigma_x^2}$, and

(11) $\mathrm{var}[y \mid p] = \dfrac{(\beta_b^2\sigma_b^2 + \beta_x^2\sigma_x^2)\sigma_y^2}{(\beta_g + \beta_b)^2\sigma_y^2 + \beta_b^2\sigma_b^2 + \beta_x^2\sigma_x^2}$.

It is worth noting that the conditional means of y are linear functions of signals and the conditional variances are constants. This is a unique property of multidimensional normal distribution. Given the conditional distribution of y, we can derive investor i's demand function for the risky asset.

(12) $x_i = \dfrac{E[y \mid \Omega_i] - p}{a(\mathrm{var}[y \mid \Omega_i] + \sigma_z^2)} \equiv D(\Omega_i)$.

Note that all investors' information includes public information, p. In equilibrium, the total net demand for the risky asset equals zero.

(13) $\int_{i\in I} D(\Omega_i)dm + x = 0$.

Putting investors' demand functions of each strategy and applying the strong law of large numbers (equation (3)), we have a linear equation of y, ε_b, x, and p. Solving the equation for p leads to another price function of the form conjectured in (5). The REE demands that the conjectural price function is consistent with the derived price function. By analyzing the requirement, we can show the uniqueness of the REE.

PROPOSITION 1: For the given strategy distribution and parameters, there exists a unique equilibrium function such that $\beta_0=\mu$, $\beta_b\geq 0$, $\beta_g\geq 0$, $\beta_x>0$, and β_b (β_g) is strictly positive if and only if $m_B>0$ ($m_G>0$, respectively).

Proof: See the Appendix.

Note that the necessary and sufficient condition for β_b >0 is consistent with the statement that β_b represents the impacts of strategy B investors to the equilibrium price. When z, the *ex post* uncertainty, is always zero, we obtain the closed form solution.

PROPOSITION 2: When $\sigma_z^2 = 0$, $\beta_b = Q_b\beta_x$, $\beta_g = Q_g\beta_x$, and

$$\beta_x = \frac{\sigma_b^2\sigma_g^2\sigma_y^2[(Q_b^2\sigma_b^2+\sigma_x^2)(Q_g m_B + a\sigma_x^2) + (Q_b+Q_g)\sigma_x^2(1-m_B)]}{(Q_b+Q_g)^2\sigma_b^2\sigma_g^2\sigma_x^2\sigma_y^2(1-m_B) + (Q_b^2\sigma_b^2+\sigma_x^2)\{[\sigma_y^2[\sigma_b^2\sigma_x^2 m_G + (Q_g^2\sigma_b^2+\sigma_x^2)\sigma_g^2 m_B] + \sigma_b^2\sigma_g^2\sigma_x^2\}}$$

where $Q_b = \dfrac{a\sigma_g^2\sigma_x^2 m_B}{a^2\sigma_b^2\sigma_g^2\sigma_x^2 + \sigma_b^2 m_G m_B}$, $Q_g = \dfrac{m_G}{a\sigma_g^2}$.

Proof: See the Appendix.

We use this closed form solution for the numerical simulation of learning dynamics in section 4.

2.2 BNE of the Strategic Game in Day 1

The natural equilibrium concept applicable to the model setting provided above is the Bayesian Nash equilibrium (BNE) which requires that (i) investors maximize their expected utility, and (ii) the expectation must be consistent with the equilibrium strategy distribution and the REE of the asset market in day 2.

Given the REE in day 2, investors expect in day 1 that the net return from the investment in the risky asset, $(y - p)\ x_i - c_i$, has non-central chi-squared distribution. The distribution depends on the strategy choice by investor *i*, not on the realization of *y* or signals. Since the expected utility from the return resembles a moment generation function, we can easily calculate the certainty equivalent of the net return as follows:

(14) $CE_B(\mathbf{m}) \equiv \frac{1}{2a}[\ln \mathrm{var}[v - p(\mathbf{m})] + \ln \tau_{v|s_b,p}(\mathbf{m})] - c_b$,

(15) $CE_G(\mathbf{m}) \equiv \frac{1}{2a}[\ln \mathrm{var}[v - p(\mathbf{m})] + \ln \tau_{v|s_g,p}(\mathbf{m})] - c_g$, and

(16) $CE_U(\mathbf{m}) \equiv \frac{1}{2a}[\ln \mathrm{var}[v - p(\mathbf{m})] + \ln \tau_{v|p}(\mathbf{m})]$

where $\mathbf{m} \equiv (m_B, m_G, m_U)$ and $\tau_{v|\Omega}(\mathbf{m}) \equiv 1/\mathrm{var}[v \mid \Omega(\mathbf{m})]$.

CE_S stands for the certainty equivalent of the net return for investor adopting strategy S. That is, the corresponding expected utility is $u(CE_S)$. $\tau_{v|\Omega}$ is the precision of the random variable v conditional on information Ω, which is defined as the reciprocal of the variance of v conditional on information Ω. Since information of all investors consists of public signal p which is a function of strategy distribution **m**, the quality of information depends on strategy distribution **m**. To indicate the private signal of a strategy G investor, we used s_g instead of s_i because the precision is identical for all strategy G investors. Like the conditional variance of y, all the precisions of v do not depend on the signal realization.

In a Grossman-Stiglitz type model, information acquisition by informed investors has two different channels of external effects. The first channel is through the profit margin: an increase in the number of informed investors makes the security price close to the fundamental value, which in turn decreases the expected profit margin per trade, making it difficult for all investors to make money from the arbitrage. The second channel is through the role the security price plays as a conveyer of information, which can be called informational externality.

The CARA-Gaussian setting allows us to separate the two channels of external effects explicitly. The first externality is represented by the first logarithm in the square brackets of the certainty equivalents, which is a function of the expected profit per unit trade, and the second externality is represented by the second logarithm, which is a function of the accuracy of conditional inference. This separation makes the analysis easier to conduct. In strategy choice, only the second externality matters because the first externality does not affect the difference of the certainty equivalent.

To understand the results below, it is worth noting that each informed strategy has different informational externalities to different strategies. For example, information created by strategy G investors is useful for all investors, including other strategy G investors. On the contrary, information created by strategy B investors is useless for strategy B investors. It is, however, useful for uninformed and strategy G investors as long as the price does not contain much profitable information. When the price contains sufficiently good information, an increase in strategy B investors can be unfavorable for all investors because their information dilutes the informational value of the asset price.

Since each investor cannot affect strategy distribution **m** or the certainty equivalents of any strategy, the BNE must satisfy the following necessary condition in terms of certainty equivalent.

EQUILIBRIUM CONDITION: If strategy distribution $\mathbf{m}^* = (m_B^*, m_G^*, m_U^*)$ is a BNE, then all strategy S with $m_S^* > 0$ must satisfy

(17) $CE_S(\mathbf{m}^*) \geq \max[CE_G(\mathbf{m}^*), CE_B(\mathbf{m}^*), CE_U(\mathbf{m}^*)]$.

Using this condition, we obtain the main theorem on the diversity of information acquisition strategies:

THEOREM: Suppose $\sigma_b^2 = \sigma_g^2$ and

(18) $$c_b < \frac{1}{2a} \ln \frac{\tau_{v|s_b}}{\tau_v} = \frac{1}{2a} \ln \frac{(\sigma_b^2 + \sigma_y^2)(\sigma_y^2 + \sigma_z^2)}{\sigma_b^2 \sigma_y^2 + \sigma_z^2 (\sigma_b^2 + \sigma_y^2)},$$

then there exists a positive number $\delta > 0$, and as long as $c_g \in (c_b, c_b + \delta)$, any equilibrium state that exists must include strategies B and G with strictly positive measure.

Proof: See the Appendix.

As shown in the proof, the diversity of informed strategies is caused by the asymmetry in informational value of the asset price. Suppose there are no informed investors except strategy B investors. In this state, security price is useless for the strategy B investors because they already know the information conveyed by the asset price. In contrast, the security price is valuable for potential strategy G investors because they do not know the common signal. That is, price is more valuable information for strategy G investors than for strategy B investors in such a state. Indeed, this fact protects strategy G investors from extinction. The asymmetry therefore depends on the strategy distribution. In equilibrium, the difference in information costs must be compensated by the difference in the information value of the asset price.

The equality of signal accuracy is assumed to make it clear why the two different informed strategies coexist in the equilibrium. However, the assumption is not necessary to have the coexistence. In fact, we can find similar conditions for coexistence in general cases.

We can easily show that if the information costs for types G and B are not too low, the uninformed strategy is also qualified as the equilibrium strategy; thus, the equilibrium has three different types of strategies—G, B, and U—as we see in the real security markets.

In the next section, we examine the robustness of the equilibrium with three different strategies when we allow investors to obtain multiple signals.

3. The Robustness of the Equilibrium with Three Strategies

Up to this point, we have restricted investors' available strategies to three strategies—B, G, and U. In this section, we allow strategy G investors to choose their level of investigation and to take both type G and type B signals. We then show that the equilibrium with three strategies derived in the previous section is robust. In other words, the equilibrium state is still equilibrium even if we allow a variety of information strategies.

In the following subsection, we allow strategy distribution **m** to include type G investors with different accuracy levels and investors with both types of private information. We assume that for any **m**, there is a REE and the equilibrium price function is of the form of equation (5) without proof.[8]

3.1 Uniqueness of Individual Investigation Level

First, we allow strategy G investors to choose their level of investigation. We assume the linear technology for the individual investigation.

Suppose that strategy G investors can obtain additional independent signals by paying constant cost c for each one. When strategy G investor i decides to get K signals $\{s_{ik}\}$ k=1,...,K, she/he has to pay $C(K) = \bar{c} + Kc$ where $\bar{c}$ is the fixed cost for independent investigation. Note that the average of K independent signals $\Sigma s_{ik}/K$ is the sufficient statistics for the observation, and the average has mean y and variance σ_ε^2/K. By analogy, we allow K to be any positive real number. That is, we denote type G signal with investigation level $K \in [0, \infty)$ as

(19) $s_i(K) = y + \varepsilon_i(K)$ where $\varepsilon_i(K) \sim N(0, \sigma_g^2 / K)$.

The corresponding cost is $C(K) = \bar{c} + Kc$, and the precision of y conditional on $s_i(K)$ and p is

(20) $\tau_{y|s_g(K),p}(\mathbf{m}) = \tau_{y|p}(\mathbf{m}) + K\sigma_g^{-2}$.

This is true even if **m** includes type G investors with heterogeneous investigation levels because $p(\mathbf{m})$ and $s_i(K)$ are independent conditional on y. Note that the indicator i is removed because the precision does not depend on the realization of $\varepsilon_i(K)$. The linear technology means linear relationship between the cost and the condition precision of y, as shown in Figure 1.

Now investors have a continuous set of type G strategies. However, it is impossible to have an equilibrium state with different investigation levels as long as the equilibrium state has some uninformed investors. We show this as follows:

First of all, each single investor has measure zero and cannot affect the strategy distribution $\mathbf{m}$ and the equilibrium price function $p(\mathbf{m})$. Therefore, the certainty equivalent of the net return for strategy G investors with investigation level K is

$$(21)\quad CE_{G(K)}(\mathbf{m}) \equiv \frac{1}{2a}\left[\ln \mathrm{var}[v - p(\mathbf{m})] - \ln\left((\tau_{y|p}(\mathbf{m}) + K\sigma_g^{-2})^{-1} + \sigma_z^2\right)\right] - \bar{c} - Kc\ ,$$

and the second derivative w.r.t. K is

$$(22)\quad \frac{d^2}{dK^2} CE_{G(K)}(\mathbf{m}) = -\frac{\tau_{v|s_g(K),p}(\mathbf{m})}{2a\sigma_g^4}[\tau_{y|s_g(K),p}(\mathbf{m}) + 2\sigma_z^2] < 0\,.$$

This means that the function $CE_{G(K)}$ is strictly concave of K for any strategy distribution $\mathbf{m}$. Therefore, it is impossible to have multiple investigation levels coexist in an equilibrium state because, at most, only one optimal accuracy level can exist. In addition, if there is no fixed cost, uninformed strategy and type G informed strategy cannot coexist because uninformed strategy is included in type G strategy as a special case with investigation level 0.[2] The following proposition concludes the results.

PROPOSITION 3: The invetigation level chosen by type G investors must be unique in equilibrium.

The uniqueness of the investigation level depends on the assumption that cost function

[2] Since the uniqueness of the investigation level crucially depends on the concavity of $CE_{G(K)}$ function, it holds for non-linear cost function $C(K)$ as long as $CE_{G(K)}$ function is strictly concave.

$C(K)$ is symmetric among investors. In the case of heterogeneous cost functions we will have the different investigation levels in the equilibrium. Thus the implication of Proposition 3 is that there is no factor generating diversity of investigation levels except heterogeneity of investigation technology. In other words, the market has a force making type G investors to choose the common investigation level. In the sense, the existence of different investigation levels itself does not cause the diversity of information acquisition strategies.

3.2 Combination of Types G and B Signals

Now we allow investors to combine two different types of private signals. An arbitrary investor, say investor j, can take signals $s_j(K)$ and s_b by paying the cost of $C(K)+c_b=\bar{c}+Kc+c_b$. We call this new strategy a hybrid informed strategy. The combination gives a more accurate estimate for v but it is more costly. In fact, the additional information is not worth the additional cost, as the following proposition reveals:

PROPOSITION 4: (i) Suppose a state with the three different strategies G, B, and U is equilibrium when the hybrid strategy is not available for investors. The state then remains equilibrium even if the hybrid strategy is available. (ii) Any state with both uninformed and hybrid strategies cannot be equilibrium.

Proof: See the Appendix.

The proposition does not mean the hybrid strategy is always excluded from the equilibrium state. It is excluded only when the equilibrium state contains uninformed strategy. The existence of uninformed strategy implies that the benefit and the cost of information acquisition is equivalent for any costly equilibrium strategies. This equivalence makes the hybrid strategy less attractive for investors. Additional type of information is helpful, but does not worth the additional cost.

In theory, the hybrid strategy might be chosen in the equilibrium if the information costs are so low that no investors choose uninformed strategy. The condition would be less realistic given the existence of uninformed strategies in the real financial market.

Based on this study we can conclude that the equilibrium with three strategies is robust. In other words, the number of equilibrium strategies observable in an equilibrium state is at most three, even if investors have many more strategies available. This result naturally leads to the following questions: "Do investors like James Rogers really know nothing about what the common investors do? Isn't he a hybrid informed?" Determining answers to these questions is beyond the scope of this paper; empirical and/or theoretical studies into this area are left for future research.

4. Learning Dynamics of Strategy Choice

In this section, we study learning dynamics of strategy choice by investors. The equilibrium analysis in the previous sections is studied under the implicit assumption that investors make rational inferences and choose the best strategy. It is natural to ask if naive investors in the real world behave as the equilibrium theory predicts. Learning studies in economics and game theory are useful for determining the answer; that is, learning studies can determine whether naive, or boundedly rational, economic agents learn to play such a complicated equilibrium strategy if they face the same situation repeatedly. In our model, there are two things that investors need to learn in order to play the equilibrium strategies: (i) how to make an inference of the true asset value v by using private and public signals, and (ii) what strategy to play.[9]

On the first matter, existing literature, for example, Bray (1982), provides positive results. That is, boundedly rational agents using only ordinary least regressions can learn to play the REE. This result is not too surprising. Under the assumption that the state variables v, y, z and the private signals have a multi-dimensional normal distribution, the rational inference provides the conditional expectations as linear functions of signals. Even if the investors do not know the true coefficients of the functions, they can calculate good approximations of the functions by using ordinary least square regression and sufficiently large sample; this conclusion is based on a tenet of statistics theory.

Unlike in the first matter about how to make an inference, the results of existing learning studies in the second matter, about what strategy to play, are ambiguous. Convergence to the equilibrium depends on the structure of the model and the specification of learning dynamics. In other words, it is worth studying this matter of learning for each model under the reasonable specification of learning process.

As its secondary purpose, learning theory is also used as algorithms to find a numerical solution for theoretical models. As is the case with our model, models in economics theory are sometimes too complicated for researchers to find a closed form solution and to study the property of the solution analytically, such as existence, uniqueness, and comparative statics. Actually, the equilibrium with three different strategies found in the first part of this study is complicated enough to justify a numerical approach.

In order to focus on strategy learning, we assume that investors know how to make rational inference and how to maximize their expected payoff under their current strategy choice, but that they do not know whether or not they are choosing the best strategy. We also assume $\sigma_z^2 = 0$ so that then we can apply proposition 2 and obtain the closed form solution for the REE in day 2 for any given strategy distribution. This allows us to calculate the expected utility for the given strategy distribution.

For this application, I adopt and analyze two learning processes applicable to our model—best response dynamics and replicator dynamics—as alternative descriptions of investors' learning process. For the purpose of numerical simulation, we use the discrete version of these dynamics. As common assumption, the learning dynamics start from an arbitrary strategy distribution, only a fraction of investors are allowed to change their strategy, and their strategy adjustment generates evolution of strategy distribution.

Best response dynamics assumes that investors who change their strategy know the best response to the present strategy distribution and they then switch their strategy to the best response strategy. Therefore, only the populations of the best response strategies increase while the other strategies' populations decrease in the strategy distribution.

Compared to best response dynamics, replicator dynamics is a description of a more naive learning process. Its dynamic process is given by the following simple difference equations:

(23) $m_{S,t+1} - m_{S,t} = \gamma \cdot m_{S,t}(u(CE_S(\mathbf{m}_t)) - \bar{u}(\mathbf{m}_t))$

where $S \in \{B,G,U\}$ and $\bar{u}(\mathbf{m}_t) \equiv m_{B,t}u(CE_B(\mathbf{m}_t)) + m_{G,t}u(CE_G(\mathbf{m}_t)) + m_{U,t}u(CE_U(\mathbf{m}_t))$.[3]

That is, the growth rate in the population of each strategy is proportional to the difference between its payoff and average payoff. Replicator dynamics was originally studied as a description of biological evolution. It was used to describe changes in the population of a species in an ecological system. Here we employ an alternative interpretation as a description of the social learning process.[10] For example, when investors change their strategy based on noisy sample observation of other investors' payoff, the evolution of strategy distribution follows the difference equations shown above. Noisy sample observation means that investors might switch to a worse strategy with small probability. In my model this assumption seems likely because investors' payoff is stochastic. Note that the population increase in the second best strategy can be larger than that of the first best strategy under replicator dynamics. This happens when the second best strategy is slightly worse than the first best and has relatively large population.

Figures 2.1a and 2.1b show the evolutions of strategy distribution under best response dynamics for two typical cases—a and b. Each point in the right triangle corresponds to

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[3] Incidentally, the difference equations for best response dynamics are as follows:

$m_{S,t+1} - m_{S,t} = \gamma \cdot (1 - m_{S,t})$ if strategy $S \in \{B,G,U\}$ is strictly the best response at period *t*.

$m_{S,t+1} - m_{S,t} = \gamma \cdot (-m_{S,t})$ if strategy $S \in \{B,G,U\}$ is not the best response at period *t*.

If there are two best response strategies, the sum of the increase in the strategies is equal to $\gamma \cdot (-m_{S,t})$ where strategy *S* is not the best response. If three strategies are indifferent, there is no change in the population.

a strategy distribution **m**. The apices give the states with each single strategy. For both cases, the whole set of strategy distribution is divided into three areas with different best responses to the strategy distribution itself. For example, state "All B" is included in the area whose best response is strategy G for both cases a and b. Under best response dynamics, strategy distribution moves straight to apex "All G", corresponding to the best response. Therefore, the direction of strategy distribution evolution in the area heads southeast toward the apex. The equilibrium of the model is represented by the intersection of the border lines of the three areas. These figures help us visually examine the existence and the uniqueness of the equilibrium for each case. The author has studied similar figures for thousands of cases and could not find any cases with multiple equilibria. Moreover, the learning dynamics always converges to the equilibrium. In fact, the process is quite simple. After one or two changes in direction, best response dynamics leads investors to the equilibrium. Needless to say, this property comes from the underlying assumption of best response dynamics: investors know the correct best response.

In contrast, Figures 2.2a and 2.2b show the evolution of strategy distributions under replicator dynamics for cases a and b. As you can see, the appearance of the naive learning process is totally different from that of best response dynamics. Strategy adjustment under replicator dynamics draws a smooth curved line starting from each initial distribution. Most of the curved lines exhibit large detours, and then the paths change direction toward the equilibrium. The common result for the two different learning dynamics is the convergence to the equilibrium. No cases without convergence are observed.

In addition to the detours of learning process, the author also found cyclical dynamics in the neighborhood of the equilibrium for some cases, as shown in Figure 2.2a. The cycles are not so clear that we can track the paths moving around the equilibrium. This is because the speed of convergence is quite high. If we carefully examine the local behavior of the learning paths in the neighborhood of the equilibrium, however, we can make sure there is a cycle. To be honest, the author cannot say how likely the cases with cycle occur. It is even difficult to say what parameters are crucial for the cycle. Common properties for the cases with cyclical paths are (i) the equilibrium share of uninformed investors is much smaller than that of informed investors, and (ii) the equilibrium share of type G informed is not larger than that of type B informed. These properties seem relevant to the property of replicator dynamics, under which the relative population size of each strategy affects the increase in the relative population.

Along the cyclical oscillation paths, the most favorable strategies for investors change in alphabetical order—B, G, U, B, G, U... This cycle is thought to be partly caused by the asymmetric informational value of the security price and partly caused by the nature of replicator dynamics. Lacking friction, the equilibrium is so absorptive that the cyclical oscillation does not persist for long. To change the conditions, the author introduced some frictions, such as time-lag in recognition and reaction, and found that such friction can make the equilibrium less absorptive and allows the oscillation to persist longer.

Cyclical learning dynamics can be relevant to the instability of security markets. Along the cyclical learning paths, the information value of the security price, asset p, and the trading volume are predicted to oscillate with changes in strategy distributions. These changes would cause dynamic changes in asset price volatility and in the effectiveness of common information sources like the *Wall Street Journal*. Though its relevancy should be studied empirically in future research, we have some observations suggesting a relationship between market instability and investors' strategy choice. George Soros, another co-founder of the Quantum Fund, has suggested that he would have lost money if he had not changed strategy periodically because he recognizes that market environments are dynamic. Furthermore, Malkiel (1999) shows that there are few fund managers who consistently outperform the rest. These observations might be explained by the cyclical learning path presented here.

5. Related Literature

This study is closely related to two existing studies in finance.

Informational Efficiency

This paper follows up on the Grossman and Stiglitz (1980) study that is concerned with the general impossibility of informational efficiency. By introducing different types of private signals, this paper makes it possible to discuss the different levels of informational efficiency: weak form, semi-strong form, and strong form. The public signal corresponds to the information relevant to weak form efficiency. The information corresponding to the common signal is publicly available and known to be relevant. Thus, this information has a close relation to the semi-strong form efficiency. The readers might therefore expect the independent signals to correspond to the information relevant to strong form efficiency. As discussed in the introduction, the information of independent signals will include some publicly unavailable information, such as exclusive interviews with firm managers that might contain insider information. But it will also include publicly available information, as illustrated in the example of EntreMed, that does not attract much investor attention. So it would be fair to say that the information relates to both semi-strong and strong forms of efficiency. The results of the paper suggest the impossibility of any form of efficiency relevant to the signals, at least as long as the information cost is omitted. The learning study suggests that the level of efficiency might oscillate with the strategy distribution when the learning dynamics exhibit cycles.

Herd Behavior

In his seminal paper on herd behavior, Lee (1993) stated that "the many predictions of economic theory crucially depend on the assumption that agents in the economy can make correct inferences about an underlying state by aggregating their own (independent) information and the information available from observing other agents' behavior." (395) He then demonstrated that when the action set of agents is too poor to reveal the information they have obtained, herd behavior, formally an inefficient informational cascade, might occur as a result of rational decision; thus, the aggregation of information does not proceed efficiently.

In my model, two types of herd behavior are shown: strategy B investors are herding on common information sources and uninformed investors are herding on the price. Their behavior disturbs the aggregation of information and allows the common noises, ε_b and x, to affect the asset price.

The literature on herd behavior has identified two alternative sources that make people herd. One is sequential decision-making and poor action set, as suggested by Lee. Another source is reputation or career concern, as suggested by Scharfstein and Stein (1990), which implies that if you want to be regarded as intelligent as others, it can be a good idea to follow or imitate the others' decision. A recent paper by Dasgupta and Prat (2007) suggests that the combination of these two sources generates herding in stock markets.

The source of herding in my paper, however, is different from these two sources. My study has revealed that investors herd to save information cost. When information is costly, investors do not stay literally uninformed. Rather, they choose easy information sources, and as the example of EntreMed suggests, their attention is focused on a small number of information sources. It is therefore likely that many investors pay attention to the behavior of only a few star investors.

Cost saving is thought to coexist with other sources and strengthens people's incentive to follow and mimic others' behavior in various situations. In order to better understand herd behavior, we would need to conduct more empirical or experimental research into why we herd in each situation.

6. Conclusions

In this paper, we studied a noisy REE model with endogenous information acquisition a la Grossman and Stiglitz (1980) including two different types of costly private signals, the common signal with an identical noise term and the independent signals with dispersed independent noise terms. Paying particular attention to investors' strategic choice of information acquisition, it was demonstrated that (i) investors observing the common signal and those observing the independent signals are likely to coexist in the equilibrium, (ii) at most, three equilibrium strategies can coexist, and (iii) when the equilibrium has three strategies, the evolutionary learning dynamics of investors can exhibit detours and cyclical oscillation.

Figure Legends

Figure 1. Linear technology of independent investigation.

Linear technology means that, when investors of type G information are allowed to choose the number of independent signals to get, an additional independent signal, which increases the precision of *y* by σ_g^{-2}, increase the cost by a fixed amount *c*.

Figure 2.1a. Evolution of strategy distribution under best response dynamics

This graph shows how strategy distribution evolves under best response dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
1.4	1.1	2	2.2	0.7	0.32	0.53

Figure 2.1b. Evolution of strategy distribution under best response dynamics

This graph shows how strategy distribution evolves under best response dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
3	1	0.1	1	0.7	0.05	0.3

Figure 2.2a. Evolution of strategy distribution under replicator dynamics

This graph shows how strategy distribution evolves under replicator dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
1.4	1.1	2	2.2	0.7	0.32	0.53

Figure 2.2b. Evolution of strategy distribution under replicator dynamics

This graph shows how strategy distribution evolves under replicator dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
3	1	0.1	1	0.7	0.05	0.3

Appendixes

Proof of PROPOSITION 1: Based on the assumption of $\sigma_x^2 > 0$, no investor has perfect information about y. If $\beta_x = 0$, the price function implies some investors with positive measure can infer the true value of y, which contradicts the fact. When $m_B > 0$, strategy B investors can infer true value of y from the price and their own private information. Otherwise, other investors can infer true value of y from the price. Thus, we have $\beta_x \neq 0$.

Putting the conditional means and variances (6)-(11) into each strategy's demand function, and putting the demand functions into the market clearing condition, we have an equation with private and public signals. Taking the unconditional expectations of the market clearing equation, we have $Ep = \mu$. Then, solving the market clearing equation for p yields another price function.

(A1) $$p = \mu + \frac{-A_b V_g V_u m_B (y + \varepsilon_b - \mu) - A_g V_u V_b m_G (y - \mu) - a V_b V_g V_u \cdot x}{B_g V_u V_b + B_b V_g V_u + B_u V_b V_g}$$

where

$A_b = (-\beta_b \beta_g \sigma_b^2 + \beta_x^2 \sigma_x^2)\sigma_y^2$,

$A_g = (\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2)\sigma_y^2$,

$B_b = [(\beta_g - \beta_g^2)\sigma_b^2 \sigma_y^2 - (\sigma_b^2 + \sigma_y^2)\beta_x^2 \sigma_x^2] m_B$,

$B_g = [(\beta_g + \beta_b)(1 - \beta_g - \beta_b)\sigma_g^2 \sigma_y^2 - (\sigma_g^2 + \sigma_y^2)(\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2)] m_G$,

$B_u = [(\beta_g + \beta_b)(1 - \beta_g - \beta_b)\sigma_y^2 - \beta_b^2 \sigma_b^2 - \beta_x^2 \sigma_x^2)] m_U$,

$V_b = \beta_x^2 \sigma_b^2 \sigma_x^2 \sigma_y^2 + \sigma_z^2 [\beta_g^2 \sigma_b^2 \sigma_y^2 + \beta_x^2 (\sigma_b^2 + \sigma_y^2)\sigma_x^2] > 0$,

$V_g = \sigma_g^2 \sigma_y^2 (\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2) + \sigma_z^2 [\sigma_g^2 \sigma_y^2 (\beta_g + \beta_b)^2 + (\sigma_g^2 + \sigma_y^2)(\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2)] > 0$, and

$V_u = \sigma_y^2 (\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2) + \sigma_z^2 [(\beta_g + \beta_b)^2 \sigma_y^2 + \beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2] > 0$.

The REE demands that the conjectural price function is coincident with this derived price function. Thus, the coefficients of the price function are given as a solution of the following equations.

(A2) $\beta_0 = \mu$

(A3) $\beta_b = -A_b V_g V_u m_B / (B_g V_u V_b + B_b V_g V_u + B_u V_b V_g)$

(A4) $\beta_g = -A_g V_u V_b m_G /(B_g V_u V_b + B_b V_g V_u + B_u V_b V_g)$

(A5) $\beta_x = -a V_b V_g V_u /(B_g V_u V_b + B_b V_g V_u + B_u V_b V_g)$

Remember $\beta_x \neq 0$ and note that the coefficients, except for β_0, have the common denominator, and then we have

$$\frac{\beta_b}{\beta_x} = \frac{A_b \theta_b}{a V_b} = \frac{(-\beta_b \beta_g \sigma_b^2 + \beta_x^2 \sigma_x^2)\sigma_y^2 m_B}{a[\beta_x^2 \sigma_b^2 \sigma_x^2 \sigma_y^2 + \sigma_z^2[\beta_g^2 \sigma_b^2 \sigma_y^2 + \beta_x^2(\sigma_b^2 + \sigma_y^2)\sigma_x^2]]}, \text{ and}$$

$$\frac{\beta_g}{\beta_x} = \frac{A_g m_G}{a V_g} = \frac{(\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2)\sigma_y^2 m_G}{a[\sigma_g^2 \sigma_y^2(\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2) + \sigma_z^2[\sigma_g^2 \sigma_y^2(\beta_g + \beta_b)^2 + (\sigma_g^2 + \sigma_y^2)(\beta_b^2 \sigma_b^2 + \beta_x^2 \sigma_x^2)]]}.$$

By setting $Q_b = \beta_b / \beta_x, Q_g = \beta_g / \beta_x$, we have simultaneous equations of Q_b and Q_g.

(A6) $$Q_b = \frac{(-Q_b Q_g \sigma_b^2 + \sigma_x^2)\sigma_y^2 m_B}{a[\sigma_b^2 \sigma_x^2 \sigma_y^2 + \sigma_z^2[Q_g^2 \sigma_b^2 \sigma_y^2 + (\sigma_b^2 + \sigma_y^2)\sigma_x^2]]}$$

(A7) $$Q_g = \frac{(Q_b^2 \sigma_b^2 + \sigma_x^2)\sigma_y^2 m_G}{a[\sigma_g^2 \sigma_y^2 (Q_b^2 \sigma_b^2 + \sigma_x^2) + \sigma_z^2[\sigma_g^2 \sigma_y^2 (Q_g + Q_b)^2 + (\sigma_g^2 + \sigma_y^2)(Q_b^2 \sigma_b^2 + \sigma_x^2)]]}$$

We show that these equations have a unique pair of non-negative solutions. Note that equation (A7) implies $0 \le Q_g < m_G / a\sigma_g^2$ because the RHS is so bounded. We can solve (A6) for Q_b.

(A8) $$Q_b = \frac{\sigma_x^2 \sigma_y^2 m_B}{a[\sigma_b^2 \sigma_x^2 \sigma_y^2 + \sigma_z^2[Q_g^2 \sigma_b^2 \sigma_y^2 + (\sigma_b^2 + \sigma_y^2)\sigma_x^2]] + Q_g \sigma_b^2 \sigma_y^2 m_B}$$

Equation (A8) and $Q_g \geq 0$ imply $0 \le Q_b < m_B / a\sigma_z^2$ because the RHS is so bounded. Therefore, the numerator of equation (A6) should be non-negative.

(A9) $-Q_g Q_b \sigma_b^2 + \sigma_x^2 \ge 0$

Suppose first that both m_B and m_G are strictly positive. Then (A7) and (A8) mean Q_b and Q_g are also strictly positive. Equation (A8) implies that Q_b is a function of Q_g. Let $F_b(Q_g)$ be the function. Conversely, equation (A7) gives us Q_g as a function of Q_b. Let $F_g(Q_b)$ be the function. The derivative of F_b is

(A10) $$F_b'(Q_g) = -Q_b^2 \frac{\sigma_b^2(2a\sigma_z^2 Q_g + m_B)}{\sigma_x^2 m_B} \le 0.$$

To derive the derivative of F_g, take the reciprocal of (A7) and then differentiate both sides, and we have

(A11) $$F_g'(Q_b) = -\frac{2a\sigma_g^2\sigma_z^2(Q_g+Q_b)Q_g^2[-Q_bQ_g\sigma_b^2+\sigma_x^2]}{(Q_b^2\sigma_b^2+\sigma_x^2)[(Q_b^2\sigma_b^2+\sigma_x^2)m_G+2a\sigma_g^2\sigma_z^2(Q_g+Q_b)Q_g^2]} \le 0 \text{ by (A9).}$$

That is, both functions are decreasing and bounded.

Here we show $F'_bF'_g<1$. Equations (A10) and (A11) yield

(A12) $$F_g'F_b' = \frac{2a\sigma_b^2\sigma_g^2\sigma_z^2Q_b^2Q_g^2(Q_b+Q_g)[-Q_bQ_g\sigma_b^2+\sigma_x^2][2a\sigma_z^2Q_g+m_B]}{m_B\sigma_x^2(Q_b^2\sigma_b^2+\sigma_x^2)[m_G(Q_b^2\sigma_b^2+\sigma_x^2)+2a\sigma_g^2\sigma_z^2Q_g^2(Q_b+Q_g)]}.$$

The numerator is non-negative and the denominator is strictly positive, and the difference, the numerator minus the denominator, is

(A13) $$2a\sigma_g^2\sigma_z^2Q_g^2(Q_b+Q_g)\{\sigma_b^2Q_b^2[-Q_bQ_g\sigma_b^2+\sigma_x^2][2a\sigma_z^2Q_g+m_B]-m_B\sigma_x^2(Q_b^2\sigma_b^2+\sigma_x^2)\}$$
$$-m_Bm_G\sigma_x^2(Q_b^2\sigma_b^2+\sigma_x^2)^2.$$

This should be negative for $F'_bF'_g<1$. The sufficient condition is that the part in bracket {} is negative. Note that the part is a quadratic function of σ_x^2 with strictly negative coefficient of σ_x^4. Thus, the part is always negative if the discriminant of the quadratic function is negative. Let Δ be the discriminant. Then we have

(A14) $$\Delta = \sigma_b^4Q_b^3Q_g(a^2Q_bQ_g\sigma_z^4-2aQ_gm_B\sigma_z^2-m_B^2).$$

Again this is a quadratic function of σ_z^2 with strictly negative coefficient of σ_z^4. Note that when σ_z^2 is zero, Δ is negative. Therefore, Δ is always negative as long as

(A15) $$\sigma_z^2 < \frac{Q_g+\sqrt{Q_g^2+Q_bQ_g}}{aQ_bQ_g}m_B.$$

The boundary is larger than $2m_B/aQ_b$. Remember $Q_b < m_B/a\sigma_z^2$, or equivalently $\sigma_z^2 < m_B/aQ_b$. Therefore, the inequality above always holds, Δ is always negative, and we have $F'_bF'_g<1$. In addition to the fact that F_b and F_g are both strictly positive, non-increasing bounded functions, this implies that the graphs of F_b, F_g have only one intersection, which is the unique solution of equations (A6) and (A7).

When m_B is zero and m_G is strictly positive, Q_b equals zero, and Q_g is determined uniquely by function F_g; $Q_g=F_g(0) > 0$. Similarly, when m_B is strictly positive and m_G is zero, $Q_b=F_b(0) > 0$, $Q_g=0$. And when $m_B = m_G$, $Q_g = Q_b = 0$. This completes the proof of the uniqueness of Q_b and Q_g, and it also proves " $Q_b > 0 \Leftrightarrow m_B > 0$ " and

"$Q_g > 0 \quad \Leftrightarrow \quad m_G > 0$."

Putting $\beta_b = Q_b\beta_x, \beta_g = Q_b\beta_x$ into (A5), we then obtain a linear equation of β_x with strictly non-zero coefficient. This linear equation gives the unique solution of β_x as a function of Q_b and Q_g. Since the pair is unique, β_x is also unique, and this completes the proof of uniqueness.

$\beta_x > 0$ is easily proved by contradiction. Suppose β_x is negative, then B_b, B_g, and B_u are all strictly negative. Then, (A5) implies $\beta_x > 0$. Contradiction. Thus, $\beta_x > 0$ implies β_b and β_g are both non-negative because Q_b and Q_g are non-negative. Moreover, β_b (β_g) is strictly positive if and only if $m_B > 0$ ($m_G > 0$, respectively). This completes the proof of Proposition 1.

QED

Proof of PROPOSITION 2: When σ_z^2 is zero, (A7) and (A8) are rewritten as follows:

(A7') $Q_g = \dfrac{m_G}{a\sigma_g^2}$

(A8') $Q_b = \dfrac{\sigma_x^2 m_B}{a\sigma_b^2\sigma_x^2 + Q_g\sigma_b^2 m_B}$

Putting (A7') into (A8') leads to $Q_b = \dfrac{a\sigma_g^2\sigma_x^2 m_B}{a^2\sigma_b^2\sigma_g^2\sigma_x^2 + \sigma_b^2 m_G m_B}$ and $Q_g = \dfrac{m_G}{a\sigma_g^2}$.

Putting $\beta_b = Q_b\beta_x, \beta_g = Q_b\beta_x$ into (A5), we obtain a linear equation of β_x as stated in the proof of proposition 1. When σ_z^2 is zero, we can solve the equation and obtain

$$\beta_x = \frac{\sigma_b^2\sigma_g^2\sigma_y^2[(Q_b^2\sigma_b^2 + \sigma_x^2)(Q_g m_B + a\sigma_x^2) + (Q_b + Q_g)\sigma_x^2(1 - m_B)]}{(Q_b + Q_g)^2\sigma_b^2\sigma_g^2\sigma_x^2\sigma_y^2(1 - m_B) + (Q_b^2\sigma_b^2 + \sigma_x^2)\{[\sigma_y^2[\sigma_b^2\sigma_x^2 m_G + (Q_g^2\sigma_b^2 + \sigma_x^2)\sigma_g^2 m_B] + \sigma_b^2\sigma_g^2\sigma_x^2\}}.$$

QED

Proof for THEOREM: (i) Suppose $m_U^* = 1$. Proposition 1 implies $\beta_b = \beta_g = 0$ in the equilibrium state. That is, the asset price p does not contain any useful information. So actually we have

(A16) $\tau_{v|s_b,p}(m_U=1)=\tau_{v|s_b}=\dfrac{1}{\tau_{y|s_b}^{-1}+\sigma_z^2}=\dfrac{\sigma_y^2+\sigma_b^2}{\sigma_y^2\sigma_b^2+\sigma_z^2(\sigma_y^2+\sigma_b^2)}$, by $\sigma_b{}^2=\sigma_g{}^2$, and

(A17) $\tau_{v|p}(m_U=1)=\tau_v=\dfrac{1}{\sigma_y^2+\sigma_z^2}$.

Thus, we have

$$CE_B(m_U=1)-CE_U(m_U=1)=\frac{1}{2a}[\ln\tau_{v|s_b,p}(m_U=1)-\ln\tau_{v|p}(m_U=1)]-c_b$$

$$=\frac{1}{2a}\ln\frac{\tau_{v|s_b}}{\tau_v}-c_b=\frac{1}{2a}\ln\frac{(\sigma_y^2+\sigma_b^2)(\sigma_y^2+\sigma_z^2)}{\sigma_y^2\sigma_b^2+\sigma_z^2(\sigma_y^2+\sigma_b^2)}-c_b>0$$

by assumption. This contradicts the equilibrium condition; hence, we have $m_U^*<1$.

(ii) Next, suppose $m_B^*=0$. This and result (i) imply $m_G^*>0$. By proposition 1, we have $\beta_b = 0$, $\beta_g > 0$ in the equilibrium state. This means the informational value of the asset price p for types B and G investors is exactly the same. Actually, we have

(A18) $\tau_{v|s_b,p}(m_B=0,m_G>0)=\tau_{v|s_g,p}(m_B=0,m_G>0)$ by $\sigma_b{}^2=\sigma_g{}^2$.

Therefore, as long as $c_b < c_g$ we have

(A19) $CE_G(m_B=0,m_G>0)-CE_B(m_B=0,m_G>0)=c_b-c_g<0$.

This contradicts the equilibrium condition. Hence, we have $m_B^*>0$ as long as $c_b < c_g$.

(iii) Finally, suppose $m_G^*=0$. This and result (i) imply $m_B^*>0$. In the equilibrium state, we have $\beta_b > 0$, $\beta_g = 0$ by proposition 1. That is, the asset price contains only type B information. While this information is useless for strategy B investors because they already have it, it is useful for strategy G investors. So we have

(A20) $\tau_{v|s_b,p}(m_B>0,m_G=0)=\tau_{v|s_b}=\tau_{v|s_g}<\tau_{v|s_g,p}(m_B>0,m_G=0)$, by $\sigma_b{}^2=\sigma_g{}^2$.

This asymmetry in the informational value of the asset price allows strategy G investors to coexist with strategy B investors even though it is more costly. This is true as long as type G information is not significantly more costly than type B information. To identify the upper bound, denoted δ, we introduce the hypothetical equilibrium state $\hat{\mathbf{m}}$. We define it as an equilibrium strategy distribution under the hypothetical assumption that strategy G is not available. $\hat{\mathbf{m}}$ is mathematically equivalent to the equilibrium of the

Grossman-Stiglitz model which does not have type G strategy with dispersed information. As they show, $\hat{\mathbf{m}}$ is uniquely determined because the difference $CE_B - CE_U$ is a strictly decreasing function of m_B when $m_G = 0$. Moreover, result (i) assures the unique measure of strategy B investors, $\hat{m}_B$, is necessarily positive. For the hypothetical equilibrium state $\hat{\mathbf{m}}$, we can find a unique positive number δ such that

(A21) $\delta \equiv \frac{1}{2a}[\ln \tau_{v|s_g,p}(\hat{\mathbf{m}}) - \ln \tau_{v|s_b}] > 0$.

Then, as long as $c_g < c_b + \delta$, we have

(A22) $CE_G(\hat{\mathbf{m}}) - CE_B(\hat{\mathbf{m}}) = \frac{1}{2a}[\ln \tau_{v|s_g,p}(\hat{\mathbf{m}}) - \ln \tau_{v|s_b}] - (c_g - c_b) > 0$.

This implies that $\hat{\mathbf{m}}$ is no longer equilibrium when strategy G is available for investors. In other words, the equilibrium state, if it exists, must include positive measure of strategy G investors as long as $c_g < c_b + \delta$.

In conclusion, the state without both types of informed investors cannot be equilibrium as long as $c_g \in (c_b, c_b + \delta)$. That is, the equilibrium, if it exists, must include both types of informed traders.

QED

Proof of PROPOSITION 4: (i) Let H(K) be hybrid strategy with type G signal of accuracy level K. Note the following relationship of the precisions of y:

(A23) $\tau_{y|s_g(K),s_b,p} = \tau_{y|s_b,p} + K\sigma_g^{-2}$

When an investor already has type B information, additional type G signal has independent error term; thus, the precision of y is just larger by the precision of the error term.

Let $\mathbf{m}_0$ be the equilibrium state when hybrid strategy is not available. Suppose $\mathbf{m}_0$ is not equilibrium when hybrid strategy is available. Then there must be some K such that $CE_{H(K)}(\mathbf{m}_0) - CE_B(\mathbf{m}_0) > 0$.

Let K* be the unique accuracy level chosen by strategy G investors in ($\mathbf{m}_0$). Then, the equilibrium condition implies the following:

(A24) $CE_{G(K^*)}(\mathbf{m}_0) = CE_U(\mathbf{m}_0)$,

(A25) $CE_{G(K^*)}(\mathbf{m}_0) \geq CE_{G(K)}(\mathbf{m}_0)$ for all *K*.

Thus, we have

(A26) $\frac{1}{2a}[\ln \tau_{v|p}(\mathbf{m}_0) - \ln \tau_{v|s_g(K^*),p}(\mathbf{m}_0)] + \bar{c} + K^* \cdot c = 0$, and

(A27) $\frac{1}{2a}[\ln \tau_{v|s_g(K^*),p}(\mathbf{m}_0) - \ln \tau_{v|s_g(K),p}(\mathbf{m}_0)] + (K - K^*) \cdot c \geq 0$ for all *K*.

Adding both sides of (A26) and (A27) yields

(A28) $\frac{1}{2a}[\ln \tau_{v|p}(\mathbf{m}_0) - \ln \tau_{v|s_g(K),p}(\mathbf{m}_0)] + \bar{c} + K \cdot c \geq 0$.

By (A28), we obtain for any *K*

(A29) $$CE_{H(K)}(\mathbf{m}_0) - CE_B(\mathbf{m}_0) = \frac{1}{2a}[\ln \tau_{v|s_g(K),s_b,p}(\mathbf{m}_0) - \ln \tau_{v|s_b,p}(\mathbf{m}_0)] - \bar{c} - K \cdot c$$

$$\leq \frac{1}{2a} \ln \frac{\tau_{v|s_g(K),s_b,p}(\mathbf{m}_0) \cdot \tau_{v|p}(\mathbf{m}_0)}{\tau_{v|s_b,p}(\mathbf{m}_0) \cdot \tau_{v|s_g(K),p}(\mathbf{m}_0)} < 0 .$$

The last inequality comes from the fact that the fraction in the logarithm is less than 1. We can verify this as follows: For any strategy distribution **m**, we have

(A30) $$\frac{\tau_{v|s_g(K),s_b,p} \cdot \tau_{v|p}}{\tau_{v|s_b,p} \cdot \tau_{v|s_g(K),p}} = \frac{(\mathrm{var}[y \mid s_b, p] + \sigma_z^2)(\mathrm{var}[y \mid s_g(K), p] + \sigma_z^2)}{(\mathrm{var}[y \mid s_g(K), s_b, p] + \sigma_z^2)(\mathrm{var}[y \mid p] + \sigma_z^2)}$$

$$= \frac{(\tau_{y|s_b,p} + \sigma_z^2 \tau_{y|s_b,p}(\tau_{y|s_b,p} + K\sigma_g^{-2}))(\tau_{y|p} + \sigma_z^2 \tau_{y|p}(\tau_{y|p} + K\sigma_g^{-2})) + K\sigma_g^{-2}(\tau_{y|p} + \sigma_z^2 \tau_{y|p}(\tau_{y|p} + K\sigma_g^{-2}))}{(\tau_{y|s_b,p} + \sigma_z^2 \tau_{y|s_b,p}(\tau_{y|s_b,p} + K\sigma_g^{-2}))(\tau_{y|p} + \sigma_z^2 \tau_{y|p}(\tau_{y|p} + K\sigma_g^{-2})) + K\sigma_g^{-2}(\tau_{y|s_b,p} + \sigma_z^2 \tau_{y|s_b,p}(\tau_{y|s_b,p} + K\sigma_g^{-2}))} .$$

This is strictly less than 1 because $\tau_{y|s_b,p} > \tau_{y|p} > 0$; thus

(A31) $\tau_{y|p} + \sigma_z^2 \tau_{y|p}(\tau_{y|p} + K\sigma_g^{-2}) < \tau_{y|s_b,p} + \sigma_z^2 \tau_{y|s_b,p}(\tau_{y|s_b,p} + K\sigma_g^{-2})$ for any **m**.

Inequality (A29) contradicts the hypothesis that there is a hybrid strategy that gives a higher payoff than any of the strategies in the equilibrium without hybrid strategy. Therefore, the equilibrium remains equilibrium even when hybrid strategy is available for investors.

(ii) Let $\mathbf{m}_1$ be an equilibrium state with uninformed investors and investors adopting H(K^*). Then we must have

(A32) $CE_{G(K^*)}(\mathbf{m}_1) \leq CE_U(\mathbf{m}_1)$.

Otherwise, investors are better off switching to type G strategy with accuracy level K^*. This implies

$$\text{(A33)} \quad C(K^*) \geq \frac{1}{2a}[\ln \tau_{v|s_g(K^*),p}(\mathbf{m}_1) - \ln \tau_{v|p}(\mathbf{m}_1)].$$

The benefit of switching from the hybrid strategy to strategy B is

$$\text{(A34)} \quad CE_B(\mathbf{m}_1) - CE_{H(K^*)}(\mathbf{m}_1)$$

$$= \frac{1}{2a}[\ln \tau_{v|s_b,p}(\mathbf{m}_1) - \ln \tau_{v|s_g(K^*),s_b,p}(\mathbf{m}_1)] + C(K^*)$$

$$\geq \frac{1}{2a}[\ln \tau_{v|s_b,p}(\mathbf{m}_1) - \ln \tau_{v|s_g(K^*),s_b,p}(\mathbf{m}_1)] + \frac{1}{2a}[\ln \tau_{v|s_g(K^*),p}(\mathbf{m}_1) - \ln \tau_{v|p}(\mathbf{m}_1)] \text{ by (A33)}$$

$$= \frac{1}{2a} \ln \frac{\tau_{v|s_b,p}(\mathbf{m}_1) \cdot \tau_{v|s_g(K^*),p}(\mathbf{m}_1)}{\tau_{v|s_g(K^*),s_b,p}(\mathbf{m}_1) \cdot \tau_{v|p}(\mathbf{m}_1)} > 0.$$

The final inequality is shown in the same way as in the proof of (i). This implies that investors are better off switching to strategy B; hence, $\mathbf{m}_1$ is not qualified as equilibrium. Contradiction.

QED

References

Admati, Anat. 1985. A noisy rational expectations equilibrium for multi-asset securities market. *Econometrica* 53: 629-657.

Bray, Margaret. 1982. Learning, estimation, and the stability of rational expectations. *Journal of Economic Theory* 26: 318-339.

Brunnermeier, Markus. 2001. *Asset Pricing under Asymmetric Information*. Oxford: Oxford University Press.

Dasgupta, Amil and Andrea Prat. 2007. Information Aggregation in Financial Markets with Career Concerns. Mimeo.

Easley, David and Maureen O'Hara. 2004. Information and the cost of capital. *Journal of Finance* LIX: 1553-1583.

Fudenberg, Drew and David Levine. 1998. *Theory of Learning in Games*. Cambridge: MIT Press.

Grossman, Sanford and Joseph Stiglitz. 1980. On the impossibility of informationally efficient markets. *American Economic Review* 70: 393-408.

Hellwig, Martin. 1980. On the aggregation of information in competitive markets. *Journal of Economic Theory* 22: 477-498.

Huberman, Gur and Tomer Regev. 2001. Contagious speculation and a cure for cancer: A nonevent that made stock prices soar. *Journal of Finance* LVI: 387-396.

Lee, In Ho. 1993. On the convergence of informational cascade. *Journal of Economic Theory* 61: 395-411.

Malkiel, Burton. 1999. *A Random Walk down Wall Street*. New York: W. W. Norton & Company.

Routledge, Bryan R. 1999. Adaptive learning in financial markets. *Review of Financial Studies* 12: 1165-1202.

Schwager, Jack. 1989. *Market Wizards*. New York: New York Institute of Finance.

Shiller, Robert. 2000. *Irrational Exuberance*. Princeton: Princeton University Press.

Shleifer, Andrei. 2000. *Inefficient Markets: An Introduction to Behavioral Finance*. Oxford: Oxford University Press.

Soros, George. 1998. *The Crisis of Global Capitalism*. New York: Public Affairs.

Footnotes

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[1] Shiller (2000) names this theory the "free-rider argument."

[2] Replicator dynamics and best response dynamics are originally continuous processes in continuous time. To make a numerical simulation, we approximate them by difference equations in discrete time. These equations have adjustment parameters which specify the size of jump intervals per unit time. The parameters should be small enough to give a good approximation. Extremely large adjustment parameters can cause divergence. Of course such results are ignored as irrelevant results.

[3] This point was suggested by Katsunari Yamaguchi in his comments to the previous version of the paper. The author gratefully appreciates it.

[4] Measurability needs to be assumed because set $\{i \text{ in } I| \varepsilon_i > c\}$ is not always Lebesgue measurable. Note that $\{\varepsilon_i\}$ is a set of random variables. Since the order of investors does not matter, if we can rearrange the index after signal realization so that G is an interval in I and ε_i is increasing in i, then ε_i is a measurable function.

[5] As stated in footnote 4, we need to rearrange the order of index so that the subsets B, G, and U are all Lebesgue measurable.

[6] See Brunnermeier (2001), p.14-16, for the definition of rational expectations equilibrium.

[7] See Brunnermeier (2001), p.12, for the projection theorem.

[8] Then, β_g represents the impact of all strategy G investors with heterogeneous accuracy levels.

[9] Routledge (1999) studied investors' learning of these two things in the model of Grossman and Stiglitz (1980).

[10] See section 3.6.2 of Fudenberg and Levine (1998) and its reference for some alternative justification for replicator dynamics as social learning.

Review copies of figures

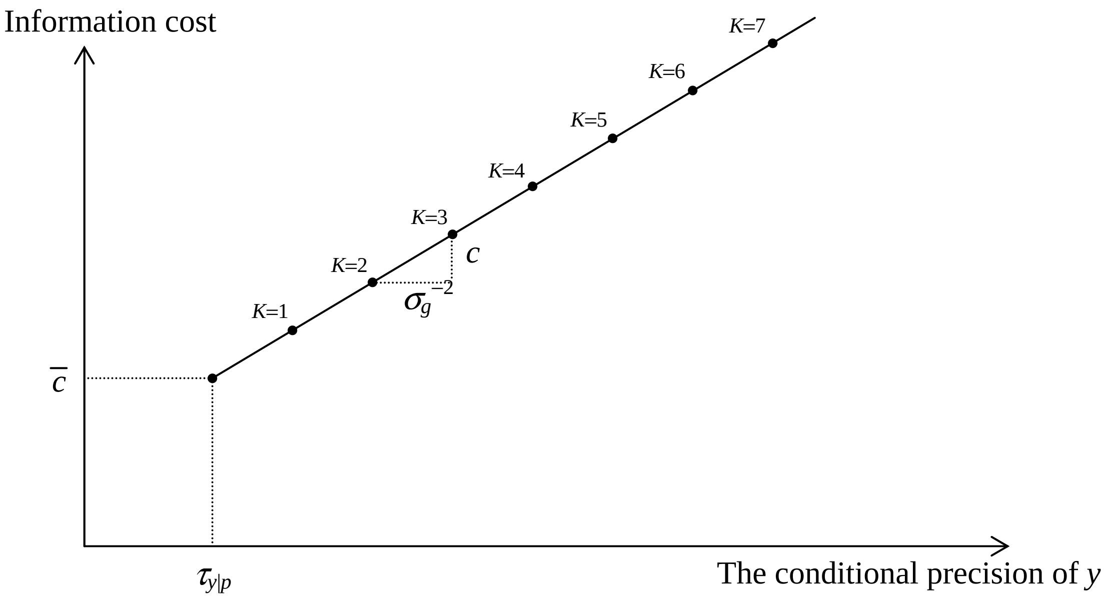

Figure 1. Linear technology of independent investigation.

Linear technology means that, when investors of type G information are allowed to choose the number of independent signals to get, an additional independent signal, which increases the precision of y by σ_g^{-2}, increase the cost by a fixed amount c.

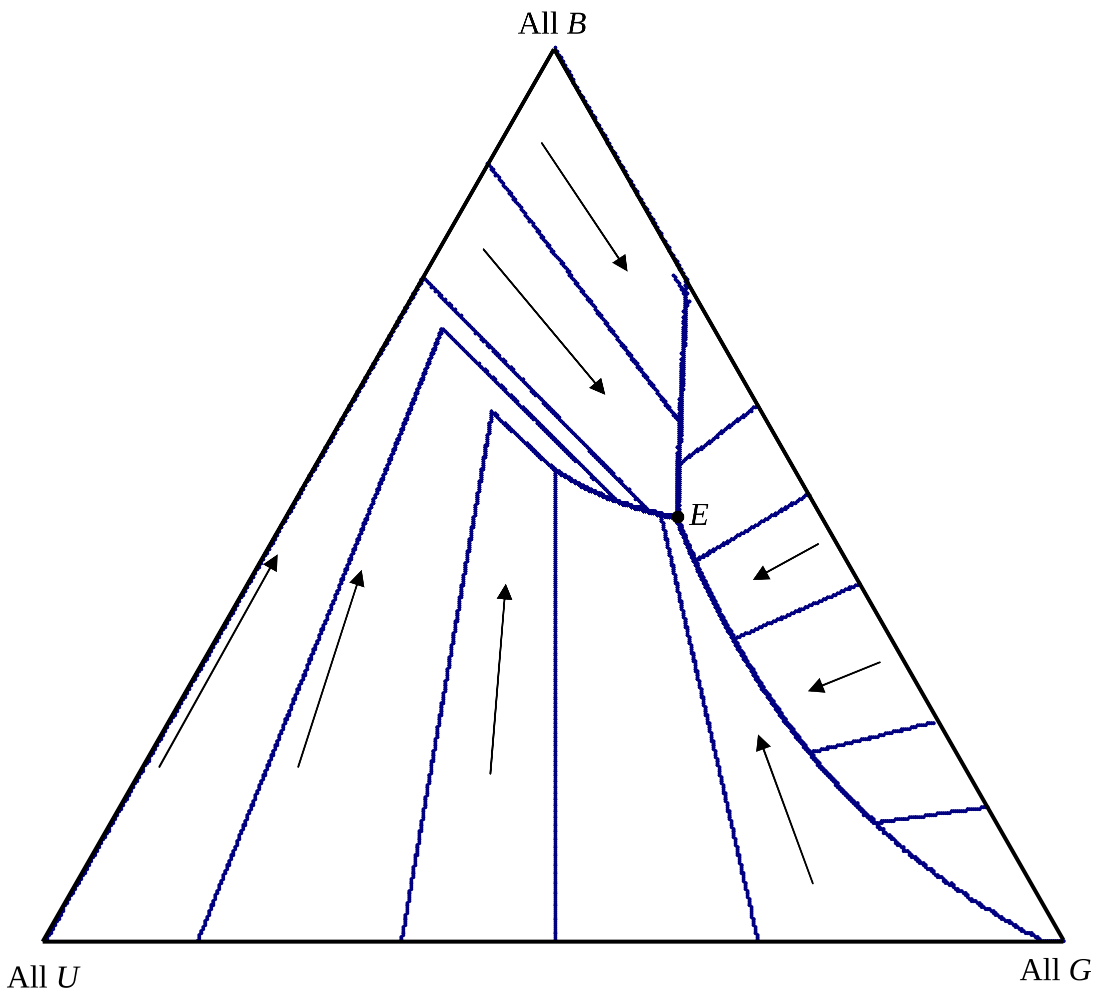

Figure 2.1a. Evolution of strategy distribution under best response dynamics

This graph shows how strategy distribution evolves under best response dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
1.4	1.1	2	2.2	0.7	0.32	0.53

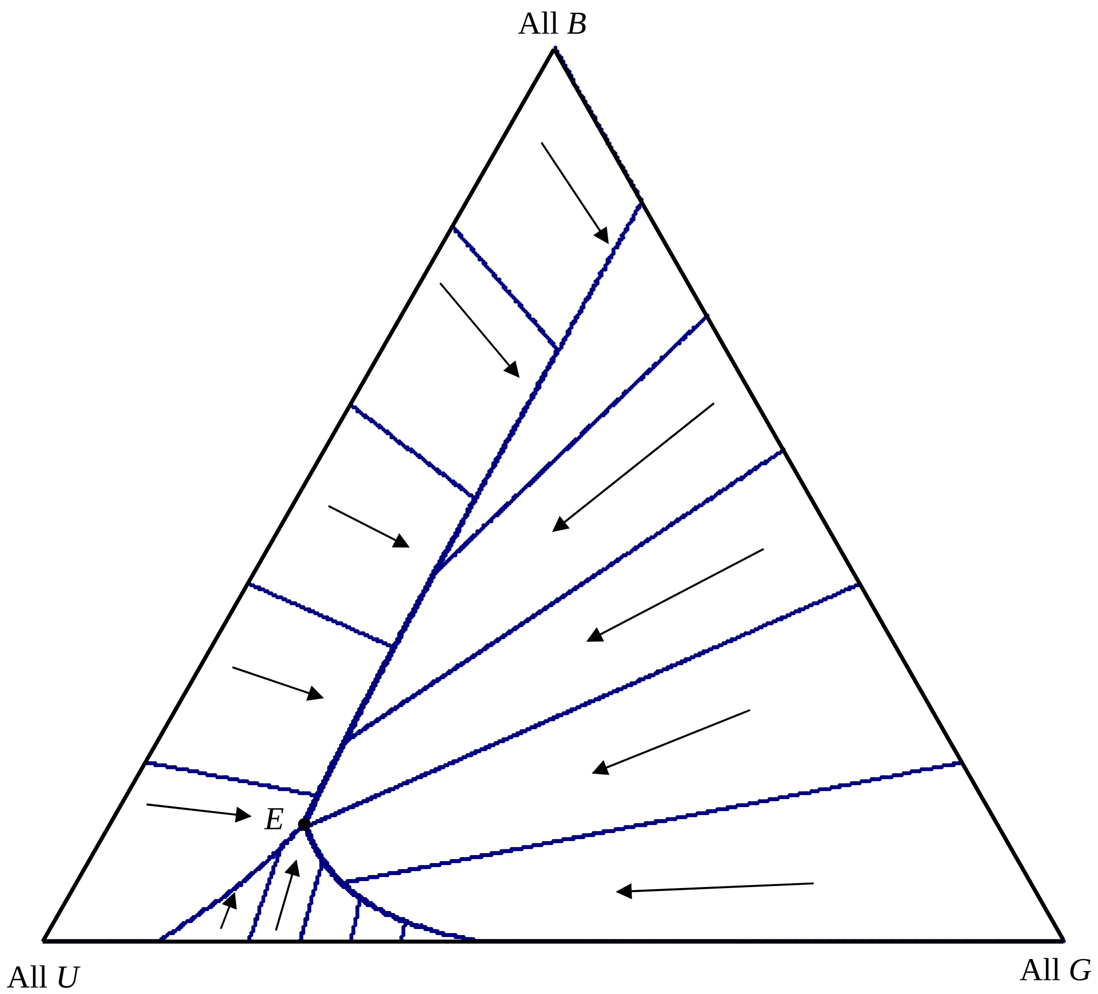

Figure 2.1b. Evolution of strategy distribution under best response dynamics

This graph shows how strategy distribution evolves under best response dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
3	1	0.1	1	0.7	0.05	0.3

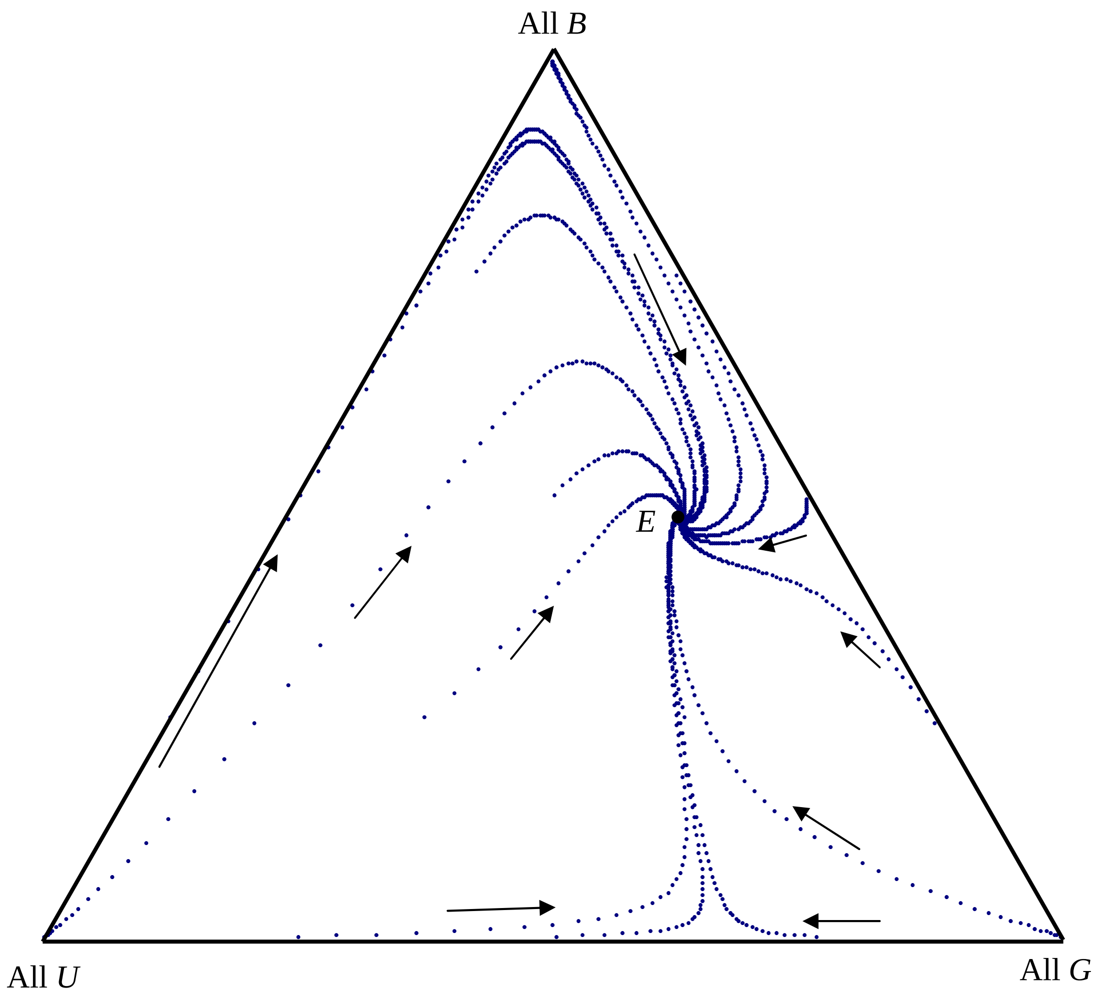

Figure 2.2a. Evolution of strategy distribution under replicator dynamics

This graph shows how strategy distribution evolves under replicator dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
1.4	1.1	2	2.2	0.7	0.32	0.53

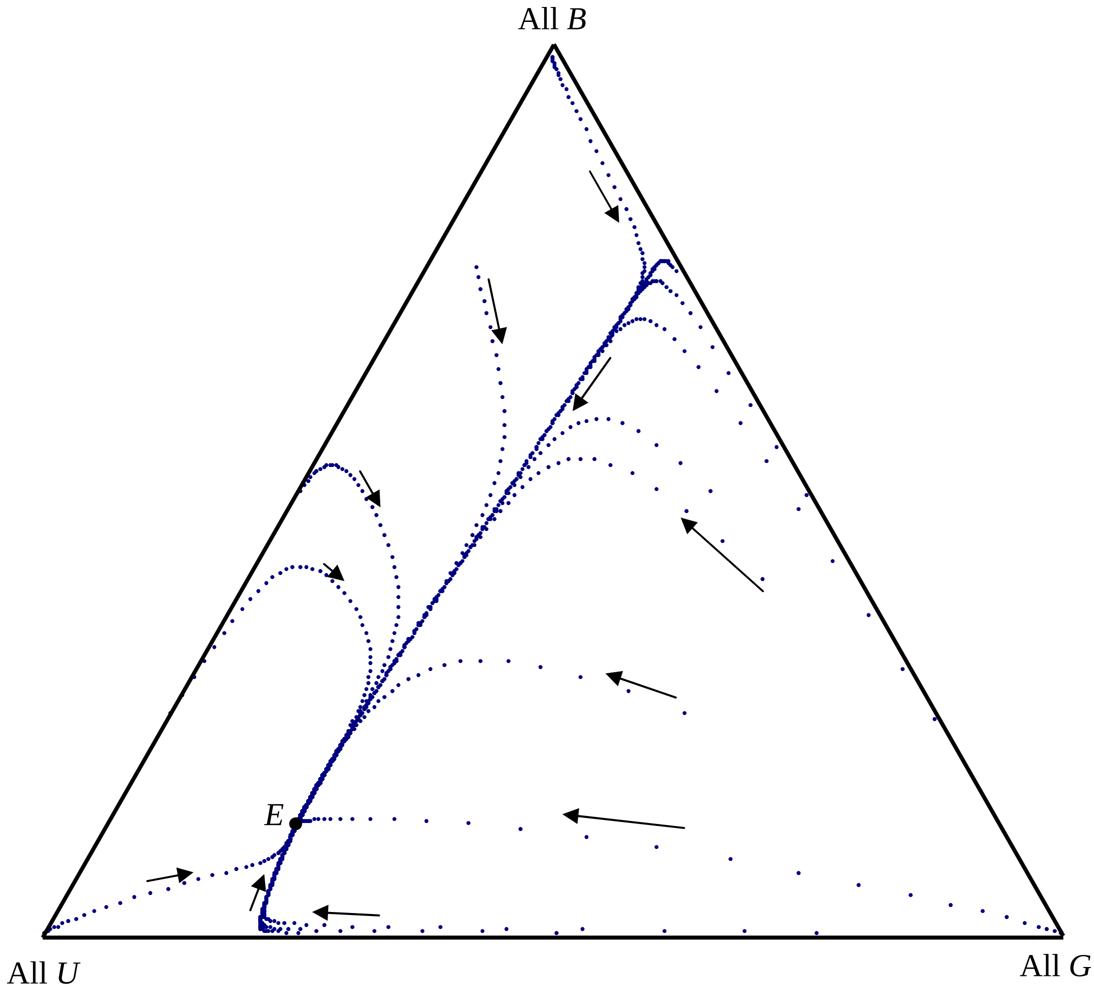

Figure 2.2b. Evolution of strategy distribution under replicator dynamics

This graph shows how strategy distribution evolves under replicator dynamics in the case specified by the following parameters.

σ_b^2	σ_g^2	σ_x^2	σ_y^2	a	c_b	c_g
3	1	0.1	1	0.7	0.05	0.3