ON THE FISCAL TREATMENT OF LIFE EXPECTANCY RELATED CHOICES

JULIO DÁVILA[1,2] AND MARIE-LOUISE LEROUX[1]

[1]Université c. de Louvain, CORE
and
[2] Paris School of Economics - CNRS

January 2010

ABSTRACT. In an overlapping generations economy setup we show that, if individuals can improve their life expectancy at some cost —either in terms of resources or in terms of utility— then the laissez-faire competitive equilibrium steady state differs from the first-best steady state. This is due to the fact that under perfect competition individuals fail to anticipate the impact of their longevity-enhancing efforts on the returns to their annuitized savings. More specifically, at the competitive equilibrium steady state the individuals exert too much effort to increase their life expectancy and they consume too little, compared to the first-best steady state. We identify policies implementing the first-best steady state as a competitive equilibrium and show that it is not always necessary to resort to the taxation of health expenditures (if any), the announcement of a (possibly zero) lump-sum tax contingent to survival rates (and hence, indirectly, to individuals efforts) suffices to achieve the first-best. Interestingly enough the tax takes the value zero at the steady state.

The authors want to thank particularly Jacques Drèze from very insguhtful comments on a previous version of the paper, as well as David de la Croix. Julio Dávila gratefully thanks funding from the Belgian FNRS as "Promoteur d'un M.I.S. - Mobilité Ulysse F.R.S.-FNRS".

Typeset by $\mathcal{AMS}$-TEX

1. Introduction

In the last century, an unprecedented rise in life expectancy has been a pervasive phenomenon in both developed and developing countries. This has surely been due mostly to a host of causes affecting whole societies at large like, for instance, progress in medicine, improvements in agriculture, and better sanitary conditions, among others. Although there may also be among these causes a component that is related to individual behaviors or choices, its contribution to this dramatic increase in life expectancy is likely to have been small compared to those mentioned above. Nevertheless, an immediate consequence, among many others, of the increase in life expectancy is the pressure it puts on, for instance, the provision of health care, on pay-as-you-go pensions systems, on housing, etc.[1] Thus, as the constraints on these and other resources become tighter, the relative importance of the individual-specific causes of the increase in life expectancy may increase as well, and the question then arises about whether the decentralized choices made by the individuals about their efforts to have an ever increasing life expectancy are the right ones from an efficiency viewpoint. This is the question we address here.

Individuals can privately influence their life expectancy in various ways choosing to undertake actions and behaviors that tend to increase it, or to avoid those that may decrease it. Nevertheless, these choices typically imply a cost for them, either in terms of a disutility incurred or in terms of additional spending in, say, healthcare, and hence of lost consumption. In effect, while the most obvious way to increase life expectancy is to increase medical treatment and prevention —which requires the actual spending of income— individuals can also make behavioral choices to that end (e.g. exercising, abstaining from smoking, eating a healthy diet, driving safely) that do not necessarily require an additional spending, but may inflict nonetheless some disutility on the individual.[2]

Despite the undisputable positive aspect of having a higher longevity, this overall increase has had also some detrimental external effects on, for example, pension systems, publicly provided healthcare, urban development, and the environment.

[1]Of course, a longer life, specifically a healthier one, increases also the labor force available for production at any time, which works in the opposite direction, but for the sake of simplicity we are going to make abstraction of this fact.

[2]On the impact of health expenditures on life expectancy, see Poikolainen (1986). Several studies have also shown the impact of factors such as physical activity (Kaplan et al.,1987 and Okamoto, 2006), overweight (see Solomon and Manson, 1997 and Bender et al. 1998) and smoking (Doll and Hill, 1950).

The specific point this paper addresses is that, besides these well-known detrimental external effects, there exists another negative externality due to a higher life expectancy simply related to the impact that the individual's choice of *quantity* of life has on his *quality* of life, through the private resources he is left with for his extended life, if savings are annuitized. Becker and Philipson (1998) emphasized already how a rise in the quantity of life can affect its quality by showing that individuals investing in their longevity do not take into account that, by doing so, they influence the return of their annuitized savings. The result is too much investment in longevity compared to what would be optimal. Becker and Philipson (1998) thus suggests that one way to ensure a high return of savings should be to tax health expenditures (and thus, implicitly longevity). Some papers give recommendations in this direction. For example, Leroux (2008) showed that in the case of non-contractible effort to increase longevity, the social planner should tax second-period consumptions in order to reduce incentives for the individual to invest in longevity. Leroux et al. (2008a,b) studied the taxation of longevity-enhancing health expenditures and showed that three factors play a role in the choice of the adequate tax rate: (i) the possible misperception by the agents of their true survival probability; (ii) the Becker-Philipson effect, as described above; and, in case of asymmetric information, (iii) incentive constraints. Nevertheless, in Leroux (2008) and Leroux et al. (2008a,b) the framework was essentially static, with a 2-period-lived agent that solves a one-shot problem at the begining of the first period.

In this paper, on the contrary, we study the problem in a truly dynamic general equilibrium framework. Adressing the issue in a dynamic setup is the natural next step to undertake, since similar instances of inefficiencies due to an overlooked (by competitive agents) impact of individual saving decisions on the saving returns arise naturally in overlapping generations models as well (see Dávila (2008)). Thus we consider an overlapping generations economy in which individuals are identical except for the date they are born in. The representative agent is sure to live at least one period and at most two, conditional on a survival probability. He supplies inelastically labor when young and consumes from his labor income when young, and from his annuitized capital and monetary savings when old (if alive). We assume that the representative agent can influence his survival probability exerting some effort. We will distinguish between the case in which this effort entails a direct disutility but no additional spending (the disutility-effort case), and the case in which it requires some additional spending but has no direct impact on the agent's utility (the expenditure-effort case).[3] Thus, an expenditure-effort can be thought of

[3]We could as well assume that the individual exerts the two different types of efforts at the same time, but for the sake of simplicity, we consider them separately.

simply as health expenditures that enters the individual budget constraint and as resources unavailable for consumption or saving. A disutility-effort implies instead a cost in terms of utility only, entering negatively the utility function but not the budget constraint. It can be thought of generally as leading a "healthy" way of life (exercising, eating healthily, abstaining from smoking and other instantly gratificating pleasures, etc.), that might be unappealing to the individual at the time he exerts the effort, but that improves also his or her life expectancy and hence the prospects of enjoying utility from consumption in the second period of life.

Under the setup defined above, we show that, both in the disutility-effort and the expenditure-effort cases, the laissez-faire competitive equilibrium steady state level of individual effort is higher than the first-best steady state, and hence inefficient. For instance, in the expenditure-effort case the individuals do not take into account, as in Becker and Philipson (1998), that by investing in their longevity they also decrease the return of their annuitized savings —very much as in they do in Dávila (2008) by saving too much capital— and in that way they reduce their consumption possibilities in the second period. A similar effect is observed in the disutility-effort case. As a consequence, there is, as in the static case, room for a public intervention aiming at making the competitive equilibrium steady state with an annuity market for savings coincide with the first-best steady state. However, in the dynamic setup the policy instruments needed are different from those needed in the static case, and differ as well depending on whether the effort takes the form of a disutility or of an expenditure. In the disutility-effort case, we show to be optimal to announce a second-period lump-sum tax that depends on the second-period consumption of the previous generation and on the rate of growth of the population (net of the mortality rate between periods). Interestingly enough, at the competitive equilibrium steady state, the amount actually raised by the tax is zero in every period, so that the implementation of the first-best steady state allocation is achieved by the mere announcement of the policy. If, on the contrary, the effort is *only* an actual expenditure (e.g. health expenditure), the first-best can indeed be implemented by a tax on that expenditure at the young age and to make a lump-sum transfer of the same amount to the contemporary old —at the steady state, redistribution actually takes place, whenever there is demographic growth. However, resorting to a tax on health expenditures is actually not needed, in this case as well (and even when both types of efforts are simultaneously available to the agents) the first policy based on announcements of a lump-sum tax suffices to implement the first-best.

Our paper can be related to the growing literature dealing with endogenous longevity in overlapping generations setups. Some papers have already emphasized the role of endogenous longevity in shaping growth and savings patterns (see, for example, Chakraborty, 2004) as well as the environment (Jouvet et al., 2007). Other papers have studied how the golden rule is modified by the introduction of endogenous longevity, inducing the under-accumulation of capital when longevity depends on public health expenditures (De la Croix and Ponthière, 2008). These papers differ however from ours in several respects. First, all of them consider health expenditures as a publicly-provided good, so that individuals have no direct control over their life expectancy. Second, they consider, *for a given public policy*, either the competitive equilibrium steady state when the consumption-saving choice has longevity consequences, as in Chakraborty (2004) or the first-best steady state (as in De la Croix and Ponthière, 2008), but none of them shows that the laissez-faire competitive equilibrium steady state with annuitized savings typically differs from the first-best steady state. In particular, to the best of our knowledge, no paper has yet established that the combination of private health expenditures and of an annuity market requires an active fiscal policy if the first-best steady state is to be implemented as a competitive equilibrium. Moreover, we identify the different policies required for the implementation of the first-best depending on the specific form that the life expectancy-increasing effort can take.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows for the disutility-effort case that the competitive equilibrium steady state typically differs from the first-best steady state, and shows how to restore the first-best. Section 4, does the same but for the expenditure-effort case. Section 5 discusses the case when the two kinds of effort are available to the agents, and Section 6 concludes.

2. The model

Time is discrete, and at every date t, a generation G_t of identical agents is born. The size of the generations increases in time at a rate n.[4] The typical agents born at period t lives at least one period and at most two, conditional to a survival with probability $\pi(e^t)$ that he can influence exerting an effort e^t.[5] A period-t agent

[4]The size of generation t=0 is normalized to be a continuum of mass 1.

[5]Although agents are identical, we will allow (when relevant) for different agents within the same generation making different choices. In those cases the choice of, say, effort (but also consumptions and savings) will be indexed by the agent's identity $i \in G_t$ as in e^t_i. Nonetheless, throughout the paper we will focus on symmetric allocations (most of the time implicitly to avoid tiresome

supplies inelastically when young his labor (normalized to 1) for a real wage rate w_t that he can split as he wishes between first period consumption c_0^t and saving, which he can hold in either capital or intrinsically worthless money. His capital savings k^t earn a return r_{t+1} at $t+1$ from a fund in which they are placed and that lends them in the capital market to firms, while monetary holdings M^t bought at a real price $\frac{1}{p_t}$ at t are worth $\frac{1}{p_{t+1}}M^t$ at $t+1$. Savings (augmented of their return) are then used for second period consumption c_1^t. Note that the probability of survival $\pi(e^t)$ represents also the expected proportion of individuals born at t and choosing e^t that survives into the next period and, most importantly, that for large populations of agents choosing such an effort level e^t the actual survival rate will be arbitrarily close to $\pi(e^t)$.[6] Finally, effort can be costly to agents either in terms of utility (Section 3) or in terms of an income that could have otherwise been used for consumption (Section 4). The first case tries to capture the influence on life expectancy of individual behavioral choices that are unrelated to income but undesirable per se, while the second case can be simply thought of as standard health expenditures. Of course, the two types of efforts could be made simultaneously. We address first the two cases separately mostly for expositional reasons. The general case will be discussed in Section ?.

Consider first the utility-effort case. The probability of survival $\pi(e^t)$ depends on an effort level e^t that creates a linear disutility[7] γe^t (where γ represents thus the intensity of the effort disutility, assumed to be identical across individuals).[8] The probability π is assumed to satisfy

$$\begin{aligned} \pi'(e^t) &> 0 \\ \pi''(e^t) &< 0 \\ \lim_{e^t \to 0^+} \pi'(e^t) &= +\infty. \end{aligned} \tag{1}$$

repetitions) so that the index will typically be dropped, as above.

[6]As a simplification, we will identify expected and actual survival rates throughout the paper. As stated above, this implicitly requires assuming a sufficiently large population at each period, since the variance $\pi(e)(1-\pi(e))/N$ of the actual survival rate around the expected one $\pi(e)$ is arbitrarily close to zero for big enough populations of size N of agents choosing a level of effort e, and would not be acceptable in the case of small cohorts. A more precise model fully incorporating the exact random nature of actual survival rates would necessarily be more convoluted and we think that, as a first approach to the issue, the gain in transparency of working under this assumption more than justifies the simplification. At any rate, the results from a fully-fledged model are not expected to depart from those presented here, but to be fair this remains to be checked.

[7]Note that we obtain the same results by assuming convex disutility of effort. For simplicity of exposure, we stick to the linear case.

[8]For the case where, in a static setup, the effort disutility differs across individuals see Leroux (2008).

The utility from consumption when young c_0 and old c_1 is given by twice continuously differentiable, increasing, strictly concave functions u and v respectively satisfying the Inada condition, i.e.

$$
\begin{aligned}
u'(c_0) > 0 &< v'(c_1) \\
u''(c_0) < 0 &> v''(c_1) \\
\lim_{c_0 \to 0^+} u'(c_0) = +\infty &= \lim_{c_1 \to 0^+} v'(c_1)
\end{aligned} \tag{2}
$$

We also assume throughout the paper that the inequality

$$
\pi(e)\pi''(e)v(c_1)v''(c_1) \geq (\pi'(e)v'(c_1))^2 \tag{3}
$$

holds everywhere, which is enough to guarantee the quasi-concavity of the objective functions below, as well as that the first-order conditions are not only necessary but sufficient to characterize the optimal choices.[9] The expected lifetime utility of an agent born at time t and choosing c_0^t, c_1^t, and e^t is then

$$
U(c_0^t, c_1^t, e^t) = u(c_0^t) + \pi(e^t)v(c_1^t) - \gamma e^t. \tag{4}
$$

Since in this case effort has no impact on the agent's income, his budget constraints at periods t and $t+1$ are respectively

$$
\begin{aligned}
c_0^t + k^t + \frac{1}{p_t}M^t &\leq w_t \\
c_1^t \leq r_{t+1}k^t + \frac{1}{p_{t+1}}M^t &
\end{aligned} \tag{5}
$$

where w_t and r_{t+1} are the wage and returns to capital savings

Consider now the income-effort case. Here we assume that e^t is instead an amount of income that the individual spends in health care, which influences his survival

[9] This condition can be rearranged to become

$$
\frac{\pi''(e)}{\pi'(e)}e \cdot \frac{v''(c_1)}{v'(c_1)}c_1 \geq \frac{\pi'(e)}{\pi(e)}e \cdot \frac{v'(c_1)}{v(c_1)}c_1
$$

i.e. that the product of the elasticities of π' and v' is bigger than the product of the elasticities of π and v. In other words, the (proportional) marginal increases in survival probability and second period utility have to be *jointly* more sensitive to effort and consumption respectively than the probability and utility themselves, a sort of strong enough concavity of second period expected utility.

probability $\pi(e^t)$ (with the same properties as before). In this case, in the utility function above $\gamma = 0$ so that the agent's expected utility from choosing c_0^t, c_1^t, and e^t is now

$$U(c_0^t, c_1^t, e^t) = u(c_0^t) + \pi(e^t)v(c_1^t) \tag{6}$$

but the agent bears a cost in terms of real income e^t not available for consumption or savings, which reduces the first-period income available for consumption and saving:

$$\begin{aligned} c_0^t + k^t + \frac{1}{p_t}M^t + e^t &\leq w_t \\ c_1^t &\leq r_{t+1}k^t + \frac{1}{p_{t+1}}M^t. \end{aligned} \tag{7}$$

Note that, as opposed to other endogenous longevity models (e.g. Chakraborty (2004) and De la Croix and Ponthiere (2008)), in the two cases above the level of effort e^t is chosen by the individual himself.

Production is standard: at every period, firms produce, out of capital and labor, a single good that can be either consumed, saved to be used as capital for production the next period, or in the second case above devoted to a health expenditure. The production function $F(K, L)$ exhibits constant returns to scale and good and factors markets are perfectly competitive, so that the wage rate equals the marginal productivity of labor and the annuitized marginal productivity of capital remunerates the latter. Hence, at a competitive equilibrium (in which all the identical agents of any given generation face the same factor prices, and hence make the same choices, in particular the same level of capital savings) the factor prices faced by generation t must satisfy

$$\begin{aligned} w_t &= F_L(\frac{k^{t-1}}{1+n}, 1) \\ r_{t+1} &= F_K(\frac{k^t}{1+n}, 1)\frac{1}{\pi(e^t)} \end{aligned} \tag{8}$$

given that, at every period t, aggregate capital K_t equals at equilibrium the previous period aggregate savings in terms of capital $(1+n)^{t-1}k^{t-1}$ (for the sake of simplicity capital is assumed to depreciate completely in one period), aggregate labour L_t equals $(1 + n)^t$, and marginal productivities are homogeneous of degree 0. Note that, according to the equations above, capital savings are assumed to be invested into a fund that lends to firms and the return of which is therefore the marginal productivity of capital. Since the return to capital savings of any given generation t is annuitized, it depends on the actual survival rate of that generation as well, which for an economy with large enough cohorts can be identified to the probability of survival $\pi(e^t)$, and hence depends on the commonly chosen effort e^t. Indeed, the return to the aggregate savings invested in the fund is augmented in such an equilibrium by the fact that a proportion $1 - \pi(e^t)$ individuals of each generation does not survive into the next period and therefore profits are to be distributed among the proportion $\pi(e^t)$ of survivors only. This is a crucial feature of our model.

3. Case in which increasing life expectancy is costly in terms of utility

In this section, we assume that the longevity-enhancing effort has a cost in terms of utility only.[10] We characterize first the laissez-faire competitive equilibrium steady state, then the first-best steady state, we show that they typically differ, and we finally find the public intervention that makes them coincide, implementing thus the latter by means of the former.

3.1. Laissez-faire competitive equilibrium steady state.

We characterize here the competitive equilibrium steady state allocation under laissez-faire. Any agent's problem amounts to (i) choose how much to save and how to allocate his savings between capital and money, and (ii) to choose how much effort to make in order to increase his chances of surviving into the second period, that is to say, for an agent born in period t,

$$\begin{aligned} \max_{0 \le c_0^t, c_1^t, k^t, e^t, M^t} \quad & u(c_0^t) + \pi(e^t)v(c_1^t) - \gamma e^t \\ & c_0^t + k^t + \frac{1}{p_t}M^t \le w_t \\ & c_1^t \le r_{t+1}k^t + \frac{1}{p_{t+1}}M^t. \end{aligned} \tag{9}$$

This optimization problem is convex, since the objective function is quasi-concave[11] and the constrained set is convex. The first-order conditions characterizing therefore

[10]Think of it as, for instance, the exciting prospect of eating always a healthy diet, never smoking, non-stop exercising, etc.

[11]In effect, from

$$u(c_0) + \pi(e)v(c_1) - \gamma e \le u(c_0') + \pi(e')v(c_1') - \gamma e'$$

the solution to this problem[12] are

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t) - \gamma \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} \tag{10}$$

for some $\lambda^t > 0$ and $\mu^t > 0$, given the monotonicity of u and v, along with the budget constraints of the optimization problem above, or equivalently

$$\begin{aligned} \frac{u'(c_0^t)}{v'(c_1^t)} = \pi(e^t)\frac{p_t}{p_{t+1}} &= \pi(e^t)r_{t+1} \\ c_0^t + k^t + \frac{1}{p_t}M^t &= w_t \\ c_1^t = r_{t+1}k^t + \frac{1}{p_{t+1}}M^t & \\ \pi'(e^t)v(c_1^t) = \gamma. & \end{aligned} \tag{11}$$

The first equation in the first line equates the marginal rate of substitution (actually $\frac{u'(c_0^t)}{\pi(e^t)v'(c_1^t)}$) between first and second period consumptions to the rate at which income can be transferred from the first to the second period of life (namely $\frac{p_t}{p_{t+1}}$); it determines thus the agent's optimal level of savings. The second equation in the first line, on the other hand, requires the absence of arbitrage between the two saving instruments (money and capital) for the agent to be willing to hold them both. The second and tird lines are the agent's budget constraints, and the last line states that the agent's optimal level of effort equates the direct marginal cost of

---

it follows that

$$\begin{aligned} & u(c_0) + \pi(e)v(c_1) - \gamma e \\ & \quad \leq \lambda[u(c_0) + \pi(e)v(c_1) - \gamma e] + (1-\lambda)[u(c_0') + \pi(e')v(c_1') - \gamma e'] \\ & \quad = [\lambda u(c_0) + (1-\lambda)u(c_0')] + [\lambda\pi(e)v(c_1) + (1-\lambda)\pi(e')v(c_1')] - [\lambda\gamma e + (1-\lambda)\gamma e'] \\ & \quad \leq u(\lambda c_0 + (1-\lambda)c_0') + [\lambda\pi(e)v(c_1) + (1-\lambda)\pi(e')v(c_1')] - \gamma[\lambda e + (1-\lambda)e'] \\ & \quad \leq u(\lambda c_0 + (1-\lambda)c_0') + \pi(\lambda e + (1-\lambda)e')v(\lambda c_1 + (1-\lambda)c_1') - \gamma[\lambda e + (1-\lambda)e'] \end{aligned}$$

where the last two inequalities follow from the fact that $u(c_0)$ and $\pi(e)v(c_1)$ are concave (the latter under the assumption made in (3) that $\pi(e)\pi''(e)v(c_1)v''(c_1) \geq (\pi'(e)v'(c_1))^2$ holds everywhere).

[12]The second-order conditions guaranteeing that the first-order conditions are not just necessary but sufficient as well for a (local, and from the convexity of the problem also) global maximum have been checked to be satisfied. Details to be added or available upon request.

increasing effort (the right-hand side) and its marginal benefit (the left-hand side), i.e. the marginal increase of the survival probability times the utility of second period consumption.

At a competitive equilibrium of an economy with large enough generations, the two conditions in (8) equating at every period, the wage rate to the marginal productivity of labor and the rental rate of capital to its annuitized marginal productivity, must be satisfied as well. Therefore, adding up the budget constraints of the young and old alive at any time t, it must hold

$$\begin{aligned} c_0^t + \frac{\pi(e^{t-1})}{1+n} c_1^{t-1} + k^t + \frac{1}{p_t} M^t \\ = F_L(\frac{k^{t-1}}{1+n}, 1) + F_K(\frac{k^{t-1}}{1+n}, 1) \frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{p_t} \frac{M^{t-1}}{1+n} \end{aligned} \tag{12}$$

where (because of the feasibility of the allocation of resources and the constant returns to scale of the technology) the first three terms of the left-hand side cancel out with the first two of the right-hand side at equilibrium, so that at any t it must hold

$$\frac{M^t}{M^{t+1}} = \frac{1+n}{\pi(e^t)}. \tag{13}$$

Thus, at equilibrium, the individual monetary holdings must always decrease at a slower pace than in the standard 2-period lifetime case with certainty (where they decrease every period by a constant factor $\frac{1}{1+n}$). This follows from the fact that some individuals die at the end of the first period and the claims on resources they could have made using their monetary savings disappear with them (no one inherits them).

At a competitive equilibrium steady state the monetary savings held by the agents must be constant in real terms, i.e. $\frac{M^t}{p^t} = \frac{M^{t+1}}{p^{t+1}}$ always, and therefore prices must decrease at the same rate, so that it holds

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)} \tag{14}$$

where e is the steady state level of effort chosen by each individual. Therefore, the competitive equilibrium steady state under laissez-faire consists of a profile

$\tilde{c}_0, \tilde{c}_1, \tilde{e}, \tilde{k}, \tilde{m}$ satisfying

$$\begin{aligned}\frac{u'(c_0)}{v'(c_1)} &= 1 + n = F_K(\frac{k}{1+n}, 1)\\ c_0 + k + m &= F_L(\frac{k}{1+n}, 1)\\ \frac{\pi(e)}{1+n}c_1 &= F_K(\frac{k}{1+n}, 1)\frac{k}{1+n} + m\\ \pi'(e)v(c_1) &= \gamma.\end{aligned} \tag{15}$$

The next proposition establishes the uniqueness of the competitive equilibrium steady state of this economy.

Proposition 1. *In the standard Diamond* (1965) *overlapping generations economy with production and money —augmented to allow for the choice of a higher life expectancy at a cost in terms of utility— the competitive equilibrium steady state is unique.*[13]

Proof. Assume both c_0, c_1, k, m, e and c_0', c_1', k', m', e satisfy (15). Then from

$$F_k(\frac{k}{1+n}, 1) = 1 + n = F_k(\frac{k'}{1+n}, 1)$$

it follows that $k = k'$, and from

$$\frac{u'(c_0)}{v'(c_1)} = 1 + n = \frac{u'(c_0')}{v'(c_1')}$$

if, without loss of generality, $c_0 < c_0'$, follows that $c_1 < c_1'$, that $m > m'$ (from the first budget constraint) and $e > e'$ (from the second budget constraint). But then

$$\pi'(e)v(c_1) < \pi'(e')v(c_1')$$

which cannot be, so that $c_0 = c_0'$. Similarly, $c_1 = c_1'$. It follows then straightforwardly from the equations in (15) that $m = m'$ and $e = e'$ as well. Q.E.D.

[13]Moreover, it is regular solution to the system (15) and hence a continuously differentiable function of the growth rate n and the disutility rate from effort γ. Just check the adequate Jacobian at a solution to the system (we did it): it is regular.

3.2 The first-best steady state.

The first-best steady state maximizes instead the utility of the representative agent under the feasibility constraint, so that for an economy with large enough generations[14] it is characterized by the solution to the problem

$$\begin{aligned} \max_{0\le c_0,c_1,k,e} \; & u(c_0)+\pi(e)v(c_1)-\gamma e \\ \text{s.t.} \quad & c_0+\frac{\pi(e)}{1+n}c_1+k\le F(\frac{k}{1+n},1) \end{aligned} \tag{16}$$

The resource constraint in the optimization problem above requires that the output per worker (net of capital replacement) allows at any time to satisfy the consumption of the young and old agents alive that period, the latter being only a proportion $\frac{1}{1+n}$ of the former because of the population growth of which, moreover, only a fraction $\pi(e)$ would have survived, for large enough cohorts. It should be noted that the optimization problem in (16) is not convex since, although the objective function of this problem is indeed quasi-concave under (3), the constrained set is not an upper contour set of a quasi-concave function, but rather of a *difference* of two quasi-concave functions —since both the left-hand side and the right-hand side functions in the constraint in (16) are quasi-concave— which needs not be convex. Fortunately, this is not a problem since, under the assumptions made, the problem (16) has a unique solution (see Proposition 2 below) that moreover is interior to the positive orthant, so that it can be characterized as the only solution to the first-order conditions satisfying the second-order conditions for a local maximum.[15]

In effect, the boundary behavior of u, v, and π in (1) and (2) implies that any solution to the problem (16) above will be interior, and the first-order conditions characterizing an interior solution to the problem are

$$\begin{pmatrix} u'(c_0) \\ \pi(e)v'(c_1) \\ 0 \\ \pi'(e)v(c_1)-\gamma \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{\pi(e)}{1+n} \\ 1-F_K(\frac{k}{1+n},1)\frac{1}{1+n} \\ \frac{\pi'(e)}{1+n}c_1 \end{pmatrix} \tag{17}$$

for some $\lambda>0$, given the monotonicity of u and v, along with the resource constraint in the optimization problem above. Equivalently, the first-best steady state is the

[14]For the identification of actual and expected survival rates made in the feasibility constraint below to be acceptable.

[15]From the absence of non-interior solutions and the uniqueness of the interior one, the local maximum is necessarily a global maximum.

only profile (see Proposition 1 below) c_0^*, c_1^*, e^*, k^* satisfying the equations:

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} = 1+n =& F_K(\frac{k}{1+n},1) \\ c_0 + \frac{\pi(e)}{1+n}c_1 + k =& F(\frac{k}{1+n},1) \\ \pi'(e)v(c_1) = \gamma +& \pi'(e)v'(c_1)c_1. \end{aligned} \tag{18}$$

The last condition states in this case that the first-best level of effort should be such that the marginal cost of effort (the right-hand side) should equate its marginal benefit (the left-hand side). While the marginal benefit is still simply the marginal increase of the survival probability times the utility of second period consumption, the marginal cost of increasing survival consists now of the sum of the direct marginal utility cost of increasing effort (namely γ) and an indirect cost (not internalized by competitive agents) in terms of the additional pressure on resources (i.e. $\lambda\frac{\pi'(e)}{1+n}c_1$, or equivalently $\pi'(e)v'(c_1)c_1$ from the second first-order condition in (17)). This latter effect follows from the fact that an increase in everyone's survival chances creates an additional demand for the existing resources. This additional cost of an increased life expectancy is not taken into account by the individuals when choosing their effort level in a competitive equilibrium under laissez-faire, which accounts for the departure from the first-best of the competitive equilibrium steady state.

Under the assumptions made (in particular the concavity of second period utility), the solution to the first-order conditions (18) is unique, as the next proposition shows.

Proposition 2. *In the standard Diamond* (1965) *overlapping generations economy with production —augmented to allow for the choice of a higher life expectancy at a cost in terms of utility— the first-best steady state is unique.*[16]

Proof. Assume both c_0, c_1, k, e and c_0', c_1', k', e' satisfy the equations (18) characterizing any solution to the problem (6), so that

$$F(\frac{k}{1+n},1) = 1+n = F(\frac{k'}{1+n},1)$$

[16]Moreover, it is regular solution to the system (18) and hence a continuously differentiable function of the growth rate n and the disutility rate from effort γ. Just check the adequate Jacobian at a solution to the system (we did it): it is regular.

from which $k = k^* = k'$ for some k^*, and

$$\frac{u'(c_0)}{v'(c_1)} = 1 + n = \frac{u'(c_0')}{v'(c_1')}$$

$$c_0 + \frac{\pi(e)}{1+n} c_1 = F(\frac{k^*}{1+n}, 1) - k^* = c_0' + \frac{\pi(e')}{1+n} c_1'$$
$$\pi'(e)[v(c_1) - v'(c_1)c_1] = \gamma = \pi'(e')[v(c_1') - v'(c_1')c_1']$$

Assume without loss of generality that $c_0 < c_0'$. Then, since $u'', v'' < 0$, from the first line above $c_1 < c_1'$ as well. Hence from the second line $e > e'$, since $\pi' > 0$. But $v(c) - v'(c)c$ is strictly increasing in c, from $v'' < 0$, and $\pi(e)$ decreasing in e, from $\pi'' < 0$, so that

$$\pi'(e)[v(c_1) - v'(c_1)c_1] < \pi'(e')[v(c_1') - v'(c_1')c_1']$$

contradicting the third line above. Hence $c_0 = c_0'$. Similarly, $c_1 = c_1'$, and hence from the feasibility condition in (18) also $e = e'$. Q.E.D.

Moreover, it is straightforward (although tiresome, we did it) to check that the unique solution to the system (18) satisfies the second-order conditions guaranteeing that the solution to the first-order conditions is a strict local maximum. This combined with the uniqueness established in Proposition 1 above is enough to establish that the local maximum is indeed global (as already mentioned, uniqueness is essential, since the constrained set is not necessarily convex).

Note that the equations (15) characterizing the laissez-faire steady state would be equivalent to those characterizing the first-best steady state in (18)[17] if it were not for the term $\pi'(e)v'(c_1)c_1$ appearing in the last equation on the first-best conditions (18), but missing from the competitive equilibrium steady state conditions (15). As a consequence, the laissez-faire competitive equilibrium steady state is not the first-best steady state, as the next proposition establishes.

Proposition 3. *In the standard Diamond* (1965) *overlapping generations economy with production and money —augmented to allow for the choice of a higher*

[17]To be more precise they would rather imply the first-best conditions, but under the conditions guaranteeing the uniqueness of the first-best steady state that amounts to the same thing (up to a residual determination of m as $F_L(\frac{k}{1+n}, 1) - c_0 - k$ if one starts from the first-best).

life expectancy at a cost in terms of utility— the first-best steady state is not a competitive equilibrium outcome under laissez-faire.

Proof. Let (c_0^*, c_1^*, k^*, e^*) be the first-best steady state solution to (18), and $(\tilde{c}_0, \tilde{c}_1, \tilde{k}, \tilde{m}, \tilde{e})$ be the laissez-faire competitive equilibrium steady state solution to (15). It follows trivially from the last equation in each of the systems (18) and (15) that, should the two steady states coincide, then since

$$\pi'(\tilde{e})v(\tilde{c}_1) = \gamma = \pi'(e^*)[v(c_1^*) - v'(c_1^*)c_1^*] \tag{16}$$

it would hold also

$$\pi'(e^*)v'(c_1^*)c_1^* = 0 \tag{17}$$

which cannot hold for an interior steady state guaranteed by the good behavior at the boundary of the representative agent's utility. Q.E.D.

As noted above, the term $\pi'(e)v'(c_1)c_1$ —which from the first-best first order conditions (18) is equivalent to $\lambda\frac{\pi'(e)}{1+n}c_1$ — measures the indirect cost of an increase in life expectancy implied by the additional pressure put on resources by a bigger fraction of survivors. This cost is not taken into account by the individuals in a symmetric competitive equilibrium. In effect, price-taking individuals disregard the impact of their joint efforts —through a higher life expectancy— on the return to their own savings. More specifically, they take as given the return to capital r_{t+1} while it happens to be at equilibrium a function $F_K(\frac{k^t}{1+n}, l)/\pi(e^t)$ of their own common level of effort e^t. The same remark holds for the return to monetary savings which, with perfect foresight, they take as given to be p_t/p_{t+1}, while it turns out to depend at equilibrium on the common level of effort e^t according to $(1+n)/\pi(e^t)$. As a consequence, the agents overinvest in their life expectancy with respect to the efficient level, living in expectation longer lives while saving in terms of capital the same amount, which leads them to enjoy lower levels of consumption in both periods, as the following proposition shows.

Proposition 4. *In the standard Diamond* (1965) *overlapping generations economy with production and money —augmented to allow for the choice of a higher life expectancy at a cost in terms of utility— the agents consume too little (in both periods) and devote too much effort to increase their life expectancy at the laissez-faire competitive equilibrium steady state* $(\tilde{c}_0, \tilde{c}_1, \tilde{k}, \tilde{m}, \tilde{e})$, *compared to the first-best* *steady state* (c_0^*, c_1^*, k^*, e^*), *i.e.*

$$\begin{aligned} c_1^* &> \tilde{c}_1 \\ c_0^* &> \tilde{c}_0 \\ k^* &= \tilde{k} \\ e^* &< \tilde{e}. \end{aligned} \tag{18}$$

Proof. Firstly, $\tilde{k} = k^*$ follows trivially from the equalization of the marginal productivity of capital to the rate of growth of the population in both the laissez-faire competitive equilibrium steady state and the first-best steady state.

As for the level of effort e, let us see first that necessarily $e^* \leq \tilde{e}$.

(1) Assume $e^* > \tilde{e}$, and assume also that $c_1^* \geq \tilde{c}_1$. Then

$$\frac{\pi(e^*)}{1+n} c_1^* > \frac{\pi(\tilde{e})}{1+n} \tilde{c}_1 \tag{19}$$

and hence $c_0^* < \tilde{c}_0$ from the equation

$$c_0^* + \frac{\pi(e^*)}{1+n} c_1^* = F(\frac{k^*}{1+n}, 1) - k^* = F(\frac{\tilde{k}}{1+n}, 1) - \tilde{k} = \tilde{c}_0 + \frac{\pi(\tilde{e})}{1+n} \tilde{c}_1 \tag{20}$$

so that

$$u'(c_0^*) > u'(\tilde{c}_0). \tag{21}$$

Moreover, since $c_1^* \geq \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} \geq \frac{1}{v'(\tilde{c}_1)}. \tag{22}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \geq \frac{u'(c_0^*)}{v'(\tilde{c}_1)} > \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{23}$$

which cannot be since both at the competitive equilibrium steady state and the first-best steady state these marginal rates of substitution are equal to the rate of growth of the population $1 + n$.

(2) Assume otherwise that $e^* > \tilde{e}$ and $c_1^* < \tilde{c}_1$. Then $\pi'(e^*) < \pi'(\tilde{e})$ since π is concave, and $v(c_1^*) < v(\tilde{c}_1)$, so that

$$\pi'(e^*) v(c_1^*) < \pi'(\tilde{e}) v(\tilde{c}_1) \tag{24}$$

but then for the last equations in conditions (8) and (15) to hold that would require

$$\pi'(e^*)v'(c_1^*)c_1^* < 0 \tag{25}$$

which cannot be either.

Therefore, necessarily $e^* \leq \tilde{e}$.

Let us see now that $e^* < \tilde{e}$ indeed.

(1) Assume that $e^* = \tilde{e}$ and that $c_1^* > (<)\tilde{c}_1$. Then

$$\frac{\pi(e^*)}{1+n}c_1^* > (<)\frac{\pi(\tilde{e})}{1+n}\tilde{c}_1 \tag{26}$$

and hence $c_0^* < (>)\tilde{c}_0$ by (20), from which

$$u'(c_0^*) > (<)u'(\tilde{c}_0). \tag{27}$$

Moreover, since $c_1^* > (<)\tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} > (<)\frac{1}{v'(\tilde{c}_1)}. \tag{28}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} > (<)\frac{u'(c_0^*)}{v'(\tilde{c}_1)} > (<)\frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{29}$$

which again cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1+n$.[18]

(2) Assume that $e^* = \tilde{e}$ and assume moreover that $c_1^* = \tilde{c}_1$. Then

$$\frac{\pi(e^*)}{1+n}c_1^* = \frac{\pi(\tilde{e})}{1+n}\tilde{c}_1 \tag{30}$$

and hence $c_0^* = \tilde{c}_0$, i.e. $(c_0^*, c_1^*, e^*) = (\tilde{c}_0, \tilde{c}_1, \tilde{e})$ which cannot be by Proposition 1.

Therefore, necessarily $e^* < \tilde{e}$.

Finally, assume $c_1^* \leq \tilde{c}_1$. Then, as previously,

$$\frac{\pi(e^*)}{1+n}c_1^* < \frac{\pi(\tilde{e})}{1+n}\tilde{c}_1 \tag{31}$$

[18]Note that although admittedly repetitive, the argument cannot be collapsed into a single step.

and hence $c_0^* > \tilde{c}_0$ by (20), from which

$$u'(c_0^*) < u'(\tilde{c}_0). \tag{32}$$

Moreover, since $c_1^* \leq \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} \leq \frac{1}{v'(\tilde{c}_1)}. \tag{33}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \leq \frac{u'(c_0^*)}{v'(\tilde{c}_1)} < \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{34}$$

which cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1 + n$.[19]

Therefore, necessarily $c_1^* > \tilde{c}_1$.

As a consquence, since both at the first-best steady state and the laissez-faire competitive steady state, it holds

$$\frac{u'(c_0^*)}{v'(c_1^*)} = 1 + n = \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{35}$$

$c_1^* > \tilde{c}_1$ implies $c_0^* > \tilde{c}_0$ as well.

Q.E.D.

In the following section, we show how to decentralize the first-best steady state as a competitive equilibrium.

3.3 Implementation of the first-best steady state as a competitive equilibrium steady state.

Note that many instances of unhealthy behaviors with a direct link with life expectancy that do not have an impact on the agent's budget constraints (like not

[19]The same remark as in footnote ? applies here.

exercising or taking prolonged sunbaths) go, for that same reason, untaxed.[20] Moreover, in many cases, it is not possible to tax them indirectly either, by taxing, for example, saving returns (held in terms of either capital or money). In effect, on the one hand, taxing savings may disincentive the prospect of a high life expectancy and, thus, it could discourage a healthy behavior. But, on the other hand, taxing savings distorts the consumption-saving decision, modifying the condition equating the intertemporal marginal rate of substitution of consumption to the return to savings in (15), which would make it impossible to coincide with the first-best steady state.[21] Therefore, an alternative type of intervention is needed.

Consider instead the following policy. Announce at each period t to the newborn generation that in the second period a lump-sum tax or subsidy of an amount $\tilde{c}_1^{t-1} \ln \frac{\tilde{\pi}^t}{\tilde{\pi}^{t-1}}$ will be raised from them or transferred to them respectively, according to its sign, with $\tilde{c}_1^{t-1}$ being the observed (average of possibly different) second period consumptions of members of generation $t-1$, and $\tilde{\pi}^t$, $\tilde{\pi}^{t-1}$ being the observed survival rates of generations t and $t-1$. Note that although $\tilde{\pi}^t$, the survival rate of generation t, is not known at the time t of the announcement (everything else is), it will crucially be nonetheless known at the time the policy will have to be implemented in $t+1$.

In the problem that any given agent of period t generation faces now (with the second period lump-sum tax/subsidy) he clearly takes as given the factor prices w_t, r_{t+1}, the observed past consumptions $\tilde{c}_1^{t-1}$ and the survival rate of the previous generation $\tilde{\pi}^{t-1}$. As for the survival rate of his own generation $\tilde{\pi}^t$, whether the agent thinks of it as being independent of his choice of effort e^t or not —and in this last case by how much he thinks he can he influence it— turns out to be a subtler point than one would have though at first sight. In effect, since the cohorts are supposed to be large —in particular large enough to justify the identification of actual survival rates $\tilde{\pi}^t$ with expected survival rates $\pi(e^t)$— one would think that each agent would think of his influence over the actual survival rate as negligible, so that he would maximize utility assuming $\tilde{\pi}^t$ to be constant with respect to e^t. Nevertheless, proceeding this way amounts to implicitly assume the following inconsistency from the agent, under common knowledge of rationality: knowing that all the agents of his generation are

[20]Others (like smoking and drinking alcohol) do. And others still that could be taxed (like eating junk food) are not, yet. Nevertheless, harmful behaviors, to one-self or to others, are taxed indeed, through fines (e.g. for speeding and other instances of dangerous driving).

[21]For instance, in Leroux (2008), it is shown that, in a static partial equilibrium framework, the first-best allocation can be restored through a tax on savings or, equivalently, on second period consumption. In this case, the individual has less incentives to invest in a higher life expectancy as his second period consumption is distorted downward.

identical to him, should he think (because of each agent being negligible in his cohort) that $\tilde{\pi}^t$ is constant with respect to his and everybody else's individual effort, i.e. that $\frac{\partial \tilde{\pi}^t}{\partial e^t}(e^t) = 0$ for all e^t, then he would know that all the agents in his generation *would face* the same problem and hence would make the same choice e^t, but then he would know also that $\tilde{\pi}^t$ would be (arbitrarily close to) $\pi(e^t)$, i.e. he would know that the actual survival rate would depend indeed nontrivially on his (and everybody else's common) choice of effort, so that $\frac{\partial \tilde{\pi}^t}{\partial e^t}(e^t) \neq 0$ for some e^t !! Is this enough to justify assuming that the representative agent identifies $\tilde{\pi}^t$ to $\pi(e^t)$? Well, not yet, but it is nonetheless enough to first discard that rational agents assume $\frac{\partial \tilde{\pi}^t}{\partial e^t}(e^t) = 0$ for all e^t. In other words, it is enough to justify assuming that any rational agent concludes $\tilde{\pi}^t$ to be a non-constant function ϕ of his own effort. If everybody else, being identical to him, makes exactly the same conjecture about the dependence of the actual survival rate on *his own effort* through ϕ as well,[22] then all the agents of any given generation *actually face* the same problem

$$\begin{aligned} &\max_{0 \le c_0^t, c_1^t, k_1^t, e^t, M^t} u(c_0^t) + \pi(e^t) v(c_1^t) - \gamma e^t \\ &\qquad c_0^t + k^t + \frac{1}{p_t} M^t \le w_t \\ &\qquad c_1^t \le r_{t+1} k^t + \frac{1}{p_{t+1}} M^t - \tilde{c}_1^{t-1} \ln \frac{\phi(e^t)}{\tilde{\pi}^{t-1}} \end{aligned} \tag{36}$$

(where the agent substitutes $\phi(e^t)$ to $\tilde{\pi}^t$ in the tax), and therefore they make the same choices, in particular of e^t so that, for large enough cohorts, $\tilde{\pi}^t$ is at every period indeed $\pi(e^t)$. As a consequence, rational agents aware of the fact of all the agents being identical will conjecture ϕ to be π, under common knowledge of rationality. The solution to the problem above is therefore characterized by the

[22]This is by no means obvious and can be arguably contested. On the one hand, all of them being identical makes of assuming they making the same conjecture under the same circumstances only natural but, on the other hand, no condition on any given agent's ϕ other than not being constant can be concluded from rationality and common knowledge of rationality. This is a true assumption. The problem is that other than this opens the door to arbitrarily distinct individual conjectures, with the accompanying problem of possible indeterminacy of equilibrium at best, or of its nonexistence at worst.

first-order conditions[23]

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t)-\gamma \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ \tilde{c}_1^{t-1}\frac{\pi'(e^t)}{\pi(e^t)} \end{pmatrix} \tag{37}$$

for some $\lambda^t, \mu^t > 0$, along with the budget constraints of the optimization problem above or, equivalently, by the system of equations

$$\begin{aligned} \frac{u'(c_0^t)}{v'(c_1^t)} = \pi(e^t)\frac{p_t}{p_{t+1}} &= \pi(e^t)r_{t+1} \\ c_0^t + k^t + \frac{1}{p_t}M^t &= w_t \\ c_1^t = r_{t+1}k^t + \frac{1}{p_{t+1}}M^t &- \tilde{c}_1^{t-1}\ln\frac{\pi(e^t)}{\pi(e^{t-1})} \\ \pi'(e^t)v(c_1^t) &= \gamma + \pi'(e^t)v'(c_1^t)\tilde{c}_1^{t-1} \end{aligned} \tag{38}$$

while, as before, at equilibrium the two conditions (5) determining the wage and rental rates need to be satisfied as well. Moreover, adding up the budget constraints of the agents living at any given period t one gets

$$\begin{aligned} c_0^t + \frac{\pi(e^{t-1})}{1+n}c_1^{t-1} + k^t + \frac{1}{p_t}M^t = \\ F_L(\frac{k^{t-1}}{1+n},1) + F_K(\frac{k^{t-1}}{1+n},1)\frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{1+n}\frac{1}{p_t}M^{t-1} \\ - \frac{\pi(e^{t-1})}{1+n}(1+n)^t c_1^{t-2}\ln\frac{\pi(e^{t-1})}{\pi(e^{t-2})}. \end{aligned} \tag{39}$$

and because of the feasibility condition and the constant returns to scale the first three terms of the left-hand side cancel out with the first two of the right-handside, so that (39) it is equivalent to

$$\frac{1}{p_t}M^t = \frac{\pi(e^{t-1})}{1+n}\frac{1}{p_t}M^{t-1} - \frac{\pi(e^{t-1})}{1+n}(1+n)^t c_1^{t-2}\ln\frac{\pi(e^{t-1})}{\pi(e^{t-2})} \tag{40}$$

[23] As a matter of fact, the problem (36) is not convex if ϕ is π, since the second constraint determines a non-convex upper contour set (the left-hand side not being quasi-concave). Nonetheless, the maximum exists, since the constrained set is still compact, and given the boundary behavior of u, v, and π it must be in the interior of the positive orthant and satisfy the first-order conditions. The uniqueness of the competitive equilibrium steady state under this policy (which follows from its coincidence with the unique first-best steady state, see below Proposition 3) guarantees that at the steady state the agents are maximizing indeed their utility under their given constraints.

which, at the steady state, implies again

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)} \tag{41}$$

as the last term in (40) vanishes. Therefore, a competitive equilibrium steady state under this policy would consist of a profile $\tilde{c}_0, \tilde{c}_1, \tilde{e}, \tilde{k}, \tilde{m}$ satisfying

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K(\frac{k}{1+n}, 1) \\ c_0 + k + m &= F_L(\frac{k}{1+n}, 1) \\ c_1 &= F_K(\frac{k}{1+n}, 1)\frac{1}{\pi(e)}k + \frac{1+n}{\pi(e)}m \\ \pi'(e)v(c_1) &= \gamma + \pi'(e)v'(c_1)c_1. \end{aligned} \tag{42}$$

which is exactly the first-best steady state system of equations (18). Therefore, the existence and uniqueness of the first-best steady state guarantees the existence and uniqueness of the competitive equilibrium steady state implementing it under this policy.

Note that the value of the tax/susbidy $\tilde{c}_1^{t-1} \ln \frac{\tilde{\pi}^t}{\tilde{\pi}^{t-1}}$ is zero at the steady state, so that no tax or subsidy is actually raised or handed out in that case, keeping the government budget trivially balanced. As a matter of fact, the mere announcement of the policy makes the agents modify their choices in such a way that the first-best steady state is attained in a decentralized way when this was not possible under laissez-faire. This result is summarized in the next proposition.

Proposition 5. *In the standard Diamond* (1965) *overlapping generations economy with production and money —augmented to allow for the choice of a higher life expectancy at a cost in terms of utility— the first-best steady state* (c_0^*, c_1^*, k^*, e^*) *is a competitive equilibrium steady state under a second period a lump-sum tax* (*or subsidy if negative*) *of an amount* $\tilde{c}_1^{t-1} \ln \frac{\tilde{\pi}^t}{\tilde{\pi}^{t-1}}$ *at period* t*, where* $\tilde{c}_1^{t-1}$ *and* $\tilde{\pi}^t$ *are the observed average second period consumptions at* $t-1$ *and survival rate at* t *respectively.*[24]

This policy restores the first-best steady state for two reasons. First, adjusting their effort, the individuals directly reduce (collectively) the tax they face (or increase the

[24] The actual amount raised or transferred is zero at the steady state.

subsidy they receive) when old and, second, they adjust their probability of survival according to the prospect of facing a tax which reduces their future consumption or a subsidy that increases it. By imposing a lump-sum subsidy or tax on consumption when old, the planner makes the agents internalize the true consequences for their own life-expectancy choices and thus provides the incentives to choose the right level of effort.

4. Case in which increasing life expectancy is costly in terms of resources

Assume now that the individual can increase his life expectancy at some cost in terms of resources, so that the individual can divert part of his first period income away from consumption and saving, in order to increase his chances of survival. Thus, this effort appears directly in the individual's first period budget constraint instead of directly in the individual's utility. As in the previous case, we will characterize the competitive equilibrium steady state under laissez-faire, the first-best steady state, and finally the policy that implements the first-best steady state as a competitive equilibrium outcome.

4.1 Competitive equilibrium steady state under laissez-faire.

The representative agent's problem under perfect competition is in this case

$$\begin{aligned} &\max_{0\leq c_0^t, c_1^t, k^t, e^t, M^t} u(c_0^t) + \pi(e^t)v(c_1^t) \\ &\qquad c_0^t + k^t + \frac{1}{p_t}M^t + e^t \leq w_t \\ &\qquad c_1^t \leq r_{t+1}k^t + \frac{1}{p_{t+1}}M^t \end{aligned} \tag{46}$$

As in the utility-effort case, the individual has to decide how much to save as well as the composition of his savings portfolio in terms of capital and money. The difference now comes from the fact that, he must decide as well how much of his income to devote to health expenditures e^t in order to pin down the optimal (from his viewpoint) life expectancy. The solution to the agent's problem is characterized by the first-order conditions[25]

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t) \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} \tag{47}$$

for some $\lambda^t, \mu^t > 0$, along with the budget constraints in the problem above. Equivalently, agent t's choice is the solution to

$$\begin{aligned} \frac{u'(c_0^t)}{v'(c_1^t)} &= \pi(e^t)\frac{p_t}{p_{t+1}} = \pi(e^t)r_{t+1} \\ c_0^t + k^t + \frac{1}{p_t}M^t + e^t &= w_t \\ c_1^t &= r_{t+1}k^t + \frac{1}{p_{t+1}}M^t \\ \pi'(e^t)v(c_1^t) &= \pi(e^t)v'(c_1^t)r_{t+1}. \end{aligned} \tag{48}$$

At the competitive equilibrium, the wage and rental rate are still determined by the conditions (5) determining the wage and rental rate of capital, so that the return to savings invested in capital by a generation depends on the survival rate of that same generation. Under competitive conditions, the individuals take these variables as given. Again, from the addition of the budget constraints of the agents alive at any given period t

$$\begin{aligned} c_0^t + \frac{\pi(e^{t-1})}{1+n}c_1^{t-1} + k^t + \frac{1}{p_t}M^t + e^t = \\ F_L(\frac{k^{t-1}}{1+n}, 1) + F_K(\frac{k^{t-1}}{1+n}, 1)\frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{p_t}\frac{M^{t-1}}{1+n} \end{aligned} \tag{49}$$

it follows that the feasibility of the allocation is equivalent to

$$\frac{M_t}{M_{t+1}} = \frac{1+n}{\pi(e^t)} \tag{50}$$

[25] Here the problem is, finally, convex and well behaved. The second-order conditions guaranteeing the first-order conditions to be sufficient have been checked to be satisfied. Details to be added or upon request.

which at a steady state implies also

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)}. \tag{51}$$

Therefore, a competitive equilibrium steady state under laissez-faire consists of a profile $\tilde{c}_0, \tilde{c}_1, \tilde{e}, \tilde{k}, \tilde{m}$ such that

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K(\frac{k}{1+n}, 1) \\ c_0 + k + m + e &= F_L(\frac{k}{1+n}, 1) \\ c_1 &= \frac{1}{\pi(e)} F_K(\frac{k}{1+n}, 1)k + \frac{1+n}{\pi(e)} m \\ \pi'(e)v(c_1) &= (1+n)v'(c_1). \end{aligned} \tag{52}$$

4.2 First-best steady state.

The first-best steady state results in this case from solving the problem[26]

$$\begin{aligned} &\max_{0 \le c_0, c_1, k, e} u(c_0) + \pi(e)v(c_1) \\ &c_0 + \frac{\pi(e)}{1+n} c_1 + k + e \le F(\frac{k}{1+n}, 1) \end{aligned} \tag{43}$$

where e denotes the resources devoted to increase the individuals' life expectancy (through their probability of survival) as, say, health expenditures, and that enters directly the feasibility constraint. The solution to the optimization problem above is characterized by the first-order conditions[27]

$$\begin{pmatrix} u'(c_0) \\ \pi(e)v'(c_1) \\ 0 \\ \pi'(e)v(c_1) \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{\pi(e)}{1+n} \\ 1 - F_K(\frac{k}{1+n}, 1)\frac{1}{1+n} \\ 1 + \frac{\pi'(e)}{1+n} c_1 \end{pmatrix} \tag{44}$$

[26] Here also the objective function is quasi-concave but the constrained set is not necessarily convex since it is the upper contour set of a function that is not quasi-concave. Nonetheless, the same trick as in the previous case (namely, absence of non-interior solutions + uniqueness + SOC's guarateeing the solution to the FOC's to be strict local max = global max) saves the day.

[27] Here again the SOC's for the FOC's to be sufficient were checked to be satisfied. Details to be added or upon request.

for some $\lambda > 0$, along with the constraint of the problem above. Equivalently, a first-best steady state consists of a profile c_0^*, c_1^*, e^*, k^* satisfying

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K(\frac{k}{1+n}, 1) \\ c_0 + \frac{\pi(e)}{1+n}c_1 + k + e &= F(\frac{k}{1+n}, 1) \\ \pi'(e)v(c_1) &= (1+n)v'(c_1) + \pi'(e)v'(c_1)c_1. \end{aligned} \tag{45}$$

Note that the first line is the same condition as the one obtained in the case where increasing life expectancy is costly in terms of utility in (18): first, the equality between the inter-temporal marginal rate of substitution and the rate at which consumption can be transferred between the two periods, and second, the maximization of output net of capital replacement. The feasibility condition in the second line includes now as an expenditure the resources e devoted to pin down the life expectancy of the individual, i.e. health expenditures. Thus output net of replacement of used up capital must be at any period equal to the consumption of young individuals, plus the consumption of the survivors of the preceding generation, *and the health expenditures*.

Finally, the last condition differs from the one obtained in the utility-effort case in (18). Indeed, the term $(1+n)v'(c_1)$ is now substituted to the term γ in the right-hand side. As before, this condition still requires that, at the first-best steady state, the marginal benefit of increasing the life expectancy, $\pi'(e)v(c_1)$, exactly matches its marginal cost which, in this case, consists of (i) the direct impact that an increase in health expenditures has on second period consumption —reducing it at a rate $\frac{1+n}{\pi(e)}$ and hence reducing second period utility at a rate $(1+n)v'(c_1)$ (first term on the right-hand side)— and of (ii) the indirect cost (common to both the utility-effort and the resources-effort cases) in terms of the additional pressure on resources following from bigger cohorts of survivors (the second term $\lambda\frac{\pi'(e)}{1+n}c_1 = \pi'(e)v'(c_1)c_1$ in the right-hand side).

As in the previous case, under the assumptions made (in particular the concavity of second period utility), the solution to the first- order conditions (45) is unique, as the next proposition shows.

Proposition 6. *In the standard Diamond* (1965) *overlapping generations economy with production —augmented to allow for the choice of a higher life expectancy at* *a cost in terms of income— the first-best steady state is unique.*[28]

Proof. Assume both c_0, c_1, k, e and c_0', c_1', k', e' satisfy the equations (45) characterizing any solution to the problem (43), so that

$$F(\frac{k}{1+n}, 1) = 1 + n = F(\frac{k'}{1+n}, 1)$$

from which $k = k^* = k'$ for some k^*, and

$$\frac{u'(c_0)}{v'(c_1)} = 1 + n = \frac{u'(c_0')}{v'(c_1')}$$

$$c_0 + \frac{\pi(e)}{1+n}c_1 + e = F(\frac{k^*}{1+n}, 1) - k^* = c_0' + \frac{\pi(e')}{1+n}c_1' + e'$$

Assume without loss of generality that $c_0 < c_0'$. Then, since $u'', v'' < 0$, from the first line above $c_1 < c_1'$ as well. Hence from the second line $e > e'$, since $\pi' > 0$ and the identity function is increasing as well. But (45) requires

$$\pi'(e)[v(c_1) - v'(c_1)c_1] = (1+n)v'(c_1')$$

to be satisfies by both e, c_1 and e', c_1', which cannot be since the left-hand side increases from e, c_1 to e', c_1' ($v(c) - v'(c)c$ is strictly increasing in c, from $v'' < 0$, and $\pi'(e)$ decreasing in e, from $\pi'' < 0$), while the right-hand side idecreases from c_1 to c_1'. Hence $c_0 = c_0'$. Similarly, $c_1 = c_1'$, and hence from the feasibility condition in (45) also $e = e'$, since the left-hand side is monotone in e. Q.E.D.

Only the last equation in the system above differs from the one in the first-best system of equations in (45). Indeed, compared to the first-best system (45), the term $\pi'(e)v'(c_1)c_1$ is missing in (52), which is simply due to the fact that the return to savings invested in capital, $r_{t+1} = \frac{1}{\pi(e^t)}F_K(\frac{k^t}{1+n}, l)$ and in money, $p_t/p_{t+1} = (1+n)/\pi(e^t)$, are taken as given by the individual under perfect competition. He does not take into account that, by investing in his longevity, he is also going to modify the overall return of his savings and thus, his consumption possibilities when old. As a consequence, the suboptimality of the competitive equilibrium steady state follows, as the following proposition establishes.

[28]Moreover, it is a regular solution to the system (8) and hence a continuously differentiable function of the growth rate n and the disutility rate from effort γ. Just check the adequate Jacobian at a solution to the system: it is regular.

Proposition 4. *In the standard Diamond* (1965) *overlapping generations economy with production and money —augmented to allow for the choice of a higher life expectancy at a cost in terms of resources— the first-best steady state is not a competitive equilibrium outcome under laissez-faire.*

Proof. Letting (c_0^*, c_1^*, k^*, e^*) be the first-best steady state solution to (45), and $(\tilde{c}_0, \tilde{c}_1, \tilde{k}, \tilde{m}, \tilde{e})$ be the laissez-faire competitive equilibrium steady state solution to (52), it follows trivially from the last equation in each of the systems (45) and (52) that should the two coincide, then since

$$\pi'(\tilde{e})v(\tilde{c}_1) = (1+n)v'(\tilde{c}_1) = (1+n)v'(c_1^*) = \pi'(e^*)[v(c_1^*) - v'(c_1^*)c_1^*] \tag{53}$$

it would hold also

$$\pi'(e^*)v'(c_1^*)c_1^* = 0 \tag{54}$$

which cannot hold for an interior steady state guaranteed by the good behavior at the boundary of the agent's utility. Q.E.D.

As in the previous utility-effort case, the fact that the individuals do not take into account the stress that a higher life expectancy puts on the available resources leads them to invest too much resources into it compared to what would be the optimal amount, i.e. $\tilde{e} > e^*$. The next proposition establishes this.

Proposition 5. *In the standard Diamond* (1965) *overlapping generations economy with production and money —augmented to allow for the choice of a higher life expectancy at a cost in terms of resources— the agents devote too much effort when young to increase their life expectancy and consume too little when old at the laissez-faire competitive equilibrium steady state $(\tilde{c}_0, \tilde{c}_1, \tilde{k}, \tilde{m}, \tilde{e})$, compared to the first-best steady state (c_0^*, c_1^*, k^*, e^*), i.e.*

$$\begin{aligned} c_1^* &\geq \tilde{c}_1 \\ k^* &= \tilde{k} \\ e^* &\leq \tilde{e} \end{aligned} \tag{55}$$

Proof. [29] Firstly, $\tilde{k} = k^*$ follows trivially from the equalization of the marginal

[29] The proof parallels that of the utility-effort case, but maybe surprisingly has a few twists that make it significantly different. Notably, a consequence of them is that no relation can be established between the first period consumptions $\tilde{c}_0$ and c_0^*, as well as that neither $c_1^* > \tilde{c}_1$ nor $e^* < \tilde{e}$ are guaranteed anymore.

productivity of capital to the rate of growth of the population in both the laissez-faire competitive equilibrium steady state and the first-best steady state.

As for the level of effort e and the second -period consumption c_1, let us see first that necessarily $e^* \leq \tilde{e}$ and $c_1^* \geq \tilde{c}_1$.

(1) Assume $e^* > \tilde{e}$ and $c_1^* \geq \tilde{c}_1$. Then

$$\frac{\pi(e^*)}{1+n}c_1^* + e^* > \frac{\pi(\tilde{e})}{1+n}\tilde{c}_1 + \tilde{e} \tag{56}$$

and hence $c_0^* < \tilde{c}_0$ from the equation

$$c_0^* + \frac{\pi(e)}{1+n}c_1^* + e^* = F(\frac{k^*}{1+n}, 1) - k^* = F(\frac{\tilde{k}}{1+n}, 1) - \tilde{k} = \tilde{c}_0 + \frac{\pi(\tilde{e})}{1+n}\tilde{c}_1 + \tilde{e} \tag{57}$$

so that

$$u'(c_0^*) > u'(\tilde{c}_0). \tag{58}$$

Moreover, since $c_1^* \geq \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} \geq \frac{1}{v'(\tilde{c}_1)}. \tag{59}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \geq \frac{u'(c_0^*)}{v'(\tilde{c}_1)} > \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{60}$$

which cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1+n$.

As a consequence, either $e^* \leq \tilde{e}$, or $c_1^* < \tilde{c}_1$, or both hold.

(2) Assume that both $e^* \leq \tilde{e}$ and $c_1^* < \tilde{c}_1$ hold. Then, as previously,

$$\frac{\pi(e^*)}{1+n}c_1^* + e^* < \frac{\pi(\tilde{e})}{1+n}\tilde{c}_1 + \tilde{e} \tag{61}$$

and hence $c_0^* > \tilde{c}_0$ by (57), from which

$$u'(c_0^*) < u'(\tilde{c}_0). \tag{62}$$

Moreover, since $c_1^* \leq \tilde{c}_1$, then

$$\frac{1}{v'(c_1^*)} \leq \frac{1}{v'(\tilde{c}_1)}. \tag{63}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \leq \frac{u'(c_0^*)}{v'(\tilde{c}_1)} < \frac{u'(\tilde{c}_0)}{v'(\tilde{c}_1)} \tag{64}$$

which cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1 + n$.

Therefore, either $e^* \leq \tilde{e}$ and $c_1^* \geq \tilde{c}_1$, or $e^* > \tilde{e}$ and $c_1^* < \tilde{c}_1$.

(3) Assume $e^* > \tilde{e}$ and $c_1^* < \tilde{c}_1$. Then $v'(c_1^*) > v'(\tilde{c}_1)$ holds, as well as $\pi'(e^*) < \pi'(\tilde{e})$ and $v(c_1^*) < v(\tilde{c}_1)$, and hence

$$\pi'(e^*)v(c_1^*) < \pi'(\tilde{e})v(\tilde{c}_1) \tag{65}$$

But since,

$$\begin{aligned} \pi'(e^*)v(c_1^*) &= (1+n)v'(c_1^*) + \pi(e^*)v'(c_1^*)c_1^* \\ \pi'(\tilde{e})v(\tilde{c}_1) &= (1+n)v'(\tilde{c}_1) \end{aligned} \tag{66}$$

then necesarily $\pi(e^*)v'(c_1^*)c_1^* < 0$, which cannot be.

Therefore $e^* \leq \tilde{e}$ and $c_1^* \geq \tilde{c}_1$.

Q.E.D.

It is worth noting that, as opposed to what happened in the disutility-effort case, nothing can be said now about how do the first-period consumptions c_0^* and $\tilde{c}_0$ compare. This is simply due to the fact that when e enters the budget constraint, it gives one additional degree of freedom to the problem, which leaves undetermined how c_0^* and $\tilde{c}_0$ compare.

4.3 Implementation of the first-best steady state as a competitive equilibrium steady state.

Contrarily to what happened in the utility-effort case, health expenditures can now be taxed or subsidized directly. This simplifies considerably the implementation of the first-best steady state. For instance, assume that the government taxes health expenditures at a rate σ^t and hands at $t+1$ a lump-sum transfer T^t to agents born at time t. In this case, the representative agent's problem becomes

$$\max_{c_0^t, c_1^t, k^t, e^t, M^t} u(c_0^t) + \pi(e^t)v(c_1^t)$$

$$\begin{aligned} c_0^t + k^t + \frac{1}{p_t}M^t + (1+\sigma^t)e^t &\leq w_t \\ c_1^t &\leq r_{t+1}k^t + \frac{1}{p_{t+1}}M^t + T^t \end{aligned} \tag{67}$$

The solution to this problem is characterized by the first-order conditions[30]

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t) \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1+\sigma^t \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} \tag{68}$$

for some $\lambda^t, \mu^t > 0$, and the budget constraints in the problem above, or, equivalently, by

$$\begin{aligned} \frac{u'(c_0^t)}{v'(c_1^t)} = \pi(e^t)\frac{p_t}{p_{t+1}} &= \pi(e^t)r_{t+1} \\ c_0^t + k^t + \frac{1}{p_t}M^t + (1+\sigma^t)e^t &= w_t \\ c_1^t &= r_{t+1}k^t + \frac{1}{p_{t+1}}M^t + T^t \\ \pi'(e^t)v(c_1^t) &= \pi(e^t)v'(c_1^t)r_{t+1}(1+\sigma^t) \end{aligned} \tag{69}$$

At a competitive equilibrium, the conditions (5) determining the wage and rental rate of capital still hold. We require also that the government runs a balanced budget at every period, so that in every period t it must hold

$$e^t\sigma^t = T^{t-1}\frac{\pi(e^{t-1})}{(1+n)} \tag{70}$$

where the amount raised by taxes on health expenditures on the left-hand side matches at every period the amount handed out to the survivors of the previous

[30]The problem is this time a nicely behaved convex one. No hassles for once.

generation, on the right-hand side. Finally, adding up the budget constraints of the agents alive at any given period

$$\begin{aligned} c_0^t + \frac{\pi(e^{t-1})}{1+n} c_1^{t-1} + k^t + \frac{1}{p_t} M^t + (1+\sigma^t)e^t = \\ F_L(\frac{k^{t-1}}{1+n}, 1) + F_K(\frac{k^{t-1}}{1+n}, 1)\frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{p_t}\frac{M^{t-1}}{1+n} + T^{t-1}\frac{\pi(e^{t-1})}{(1+n)} \end{aligned} \tag{71}$$

it follows that the feasibility condition is again equivalent to

$$\frac{M_t}{M_{t+1}} = \frac{1+n}{\pi(e^t)} \tag{72}$$

which at the steady state requires

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)}. \tag{73}$$

Therefore, the competitive equilibrium steady state is characterized now by a profile $\tilde{c}_0, \tilde{c}_1, \tilde{e}, \tilde{k}, \tilde{m}$ satisfying

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} = 1+n = F_K(\frac{k}{1+n}, 1) \\ c_0 + k + m + (1+\sigma)e = F_L(\frac{k}{1+n}, 1) \\ c_1 = \frac{1}{\pi(e)} F_K(\frac{k}{1+n}, 1)k + \frac{1+n}{\pi(e)} m + T \\ \pi'(e)v(c_1) = v'(c_1)(1+n)(1+\sigma) \\ e\sigma = T\frac{\pi(e)}{1+n}. \end{aligned} \tag{74}$$

Comparing conditions (74) with those characterizing the first-best steady state in (45), it is straightforward to check that they share the same solution if the tax rate is[31]

$$\sigma = \frac{\pi'(e)}{1+n} c_1 \tag{75}$$

[31] It can be easily verified that an equivalent expression for the optimal tax rate at the first-best steady state is

$$\sigma = \frac{v'(c_1)c_1}{v(c_1) - v'(c_1)c_1}$$

Note, that if $v(.)$ has constant elasticity of substitution, $v(x) = x^\epsilon$, this tax takes the form $\varepsilon/(1-\varepsilon)$ and depends thus only on the parameter ϵ and not on the particular value of the steady state second period consumption c_1.

Therefore, in order to implement the first-best steady state, the government just needs to announce at the beginning of each period t that (i) health expenditures are going to be taxed then at a rate

$$\sigma^t = \frac{\pi'(e^{t-1})}{1+n} c_1^{t-1} \tag{76}$$

(which depends only on known variables and cannot be manipulated by individuals born in period t) and (ii) a lump-sum transfer will be made to period-t agents at $t+1$ of an amount equal to[32]

$$T^t = e^{t-1}\sigma^t \frac{1+n}{\pi(e^{t-1})} = \frac{\pi'(e^{t-1})e^{t-1}}{\pi(e^{t-1})} c_1^{t-1} \tag{77}$$

The lump-sum transfer depends thus on the elasticity of the survival probability with respect to health expenditures and on the consumption when old of the previous generation. Replacing these two expressions into conditions in (74) characterizing the competitive equilibrium steady state with taxes, it is straightforward to check that at the steady state the conditions coincide with those of the first-best steady state in (45),[33] so that such tax-and-transfers scheme implements the first-best steady state. This result is summarized in the next proposition.

Proposition 6. *In the standard Diamond* (1965) *overlapping generations economy with production and money —augmented to allow for the choice of a higher life expectancy at a cost in terms of resources— the first-best profile* (c_0^*, c_1^*, k^*, e^*) *satisfying* (45) *is a competitive equilibrium outcome if such expenditure is taxed at a rate*

$$\sigma^t = \frac{\pi'(e^{t-1})}{1+n} c_1^{t-1} \tag{78}$$

for each generation t, and a second period lump-sum transfer is made to each generation t of an amount

$$T^t = \frac{\pi'(e^{t-1})e^{t-1}}{\pi(e^{t-1})} c_1^{t-1}. \tag{79}$$

[32]Note that the formulation of the transfer T^t is defined such that it depends only on variables which cannot be manipulated by the individuals born in period t. The consequence of such an assumption is that the budget balance condition, although satisfied at the steady state, will not be satisfied ex post, outside the steady state.

[33]Under assumtions guaranteeing the uniqueness of the latter.

Finally, consider the *expected* per capita net taxes paid by any given generation t, i.e. $\tau^t = \sigma^t e^t - \pi(e^t)T^t$ (note that the transfer T^t is conditional on the individual's survival, while the contribution is paid in first period, with certainty). Replacing for the expressions of σ^t and T^t, it amounts to

$$\tau^t = \pi'(e^{t-1})c_1^{t-1}[\frac{e^t}{1+n} - \frac{\pi(e^t)e^{t-1}}{\pi(e^{t-1})}] \tag{80}$$

which, at the steady state, becomes

$$\tau = \pi'(e)c_1 e[\frac{1}{1+n} - 1] < 0. \tag{81}$$

These expected net taxes are negative simply because of our assumption of positive demographic growth as (if $n = 0$, we would also have $\tau = 0$). This is not incompatible with budget balance at each period, which is guaranteed by (70).

5. Discussion

6. Conclusion

In this paper, we address in a dynamic setup the externality created by expenses or individual behaviors that have an impact on the individual's life expectancy. Becker and Philipson (1998) first showed in a static setup how the individuals' attempts to increase the "quantity" of their life also affect the "quality" of it in a way that they do not perfectly anticipate, which typically leads to an inefficient outcome. More specifically, we show, in this paper, that in an overlapping generations economy with production à la Diamond (1965) the competitive equilibrium steady state still differs from the first-best steady state because of this external effect of longevity on the return to savings, both when individuals can affect their life expectancy by means of health expenditures, or when they can do it by just improving their habits in a way that is costly for them in terms of utility (but at no cost in terms of resources). The externality is created by the fact that individuals do not take into account that their life expectancy affects the return to their annuitized savings (held either in money or in capital) and, hence, their consumption possibilities when old. In this case, they are likely to invest too much in their longevity in comparison to what would be optimal. We show nonetheless that the first-best steady state can be decentralized as a competitive equilibrium in both cases if the government announces and implements the adequate policy of taxes and transfers, and we identify this policies.

Still our paper could be extended in several ways. First, we consider a type of effort which is costly in terms of utility and in terms of resources but we excluded the case where the effort requires time investment. This would certainly have implications on the labour supply. Moreover, we assume a perfect annuity market, which may be far from what is observed in reality; we would certainly relax this assumption in a extension of this paper.

References

1 Becker, G. and Philipson, T., "Old age longevity and mortality contingent claims", *Journal of Political Economy*, Vol.106 (1998), 551-573.

2 Bender, J., Trautner, C., Spraul, M. and Berger, M., "Assessment of excess mortality in obesity", *American Journal of Epidemiology*, Vol. 147, No 1 (1998), 42-47.

3 Chakraborty, S. , "Endogenous lifetime and economic growth", *Journal of Economic Theory*, Vol. 116 (2004), 119–137

4 Dávila, J., "The taxation of capital returns in overlapping generations economies without financial assets.", CORE DP 75 (2008)

5 De la Croix, D. and G. Ponthière, "On the gloden rule of capital accumulation under endogenous longevity", CORE DP 49 (2008)

6 Doll, R. and Hill, B., "Smoking and carcinoma of the lung", *British Medical Journal*, Vol. 2 (1950), 739-747.

7 Jouvet P-A, P. Pestieau, and G. Ponthière, "Longevity and environmental quality in an OLG model", Economix WP 19 (2007).

8 Kaplan, G.A, T.E. Seeman, R.D. Cohen, L.P. Knudsen and J. Guralnik, "Mortality among the elderly in the Alameda county study: behavioral and demographic risk factors", *American Journal of Public Health*, Vol. 77, No3 (1987), 307-312

9 Leroux, M-L, "Endogenous differential mortality, non monitored effort and optimal non linear taxation", CORE DP 29 (2008)

10 Leroux, M-L, P. Pestieau, and G. Ponthière, "Optimal linear taxation under endogenous longevity, CORE DP 51 (2008)

11 Leroux, M-L, P. Pestieau, and G. Ponthière, "Should we subsidize longevity?" , CORE DP 58 (2008)

12 Okamoto, K., "Life expectancy at the age of 65 years and environ- mental factors: an ecological study in Japan" *Archives of Gerontology and Geriatrics*, Vol. 43 (2006), 85-91.

13 Poikolainen, K. and Escola, J., "The effect of health services on mortality decline in death rates from amenable to non-amenable causes in Finland, 1969-1981", *The Lancet*, Vol.1, No 8474 (1986), 199-202.

14 Solomon, C. and Manson, J.E., "Obesity and mortality: a review of the epidemiological data", *American Journal of Clinical Nutrition*, Vol. 66, No 4 (1997), 1044-1050.