Beauty Contests and Asset Prices under Asymmetric Information

Ryuichiro Ishikawa

and

Noritaka Kudoh

Issues

- We are interested in bubbles and crashes in financial markets.
- We are also interested in how traders learn from asset prices to form their expectations.

Finite Horizon and Bubbles

- Tirole (1982) showed that there cannot be a bubble under common knowledge, <u>because of backward induction</u>.
- Tirole (1985) showed that with <u>an infinite number of traders</u>, bubbles are possible.
  - Samuelson's (1958) OLG model of money.

Finite Horizon and Bubbles

- We focus on an environment in which there is a final trading date, so that backward induction is possible.
- Instead we consider an environment where traders' knowledge matters.
- In particular, we focus on higher-order expectations (HOE) of traders.

Higher-Order Expectations (1)

- Keynes wrote, "… professional investment may be likened to those newspaper competitions in which the competitors have to pick the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that <u>each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors</u>, all of whom are looking at the problem from the same point of view." (p. 156)

Higher-Order Expectations (2)

- Keynes continued, “We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the forth, fifth, and higher degrees.” (p. 156)

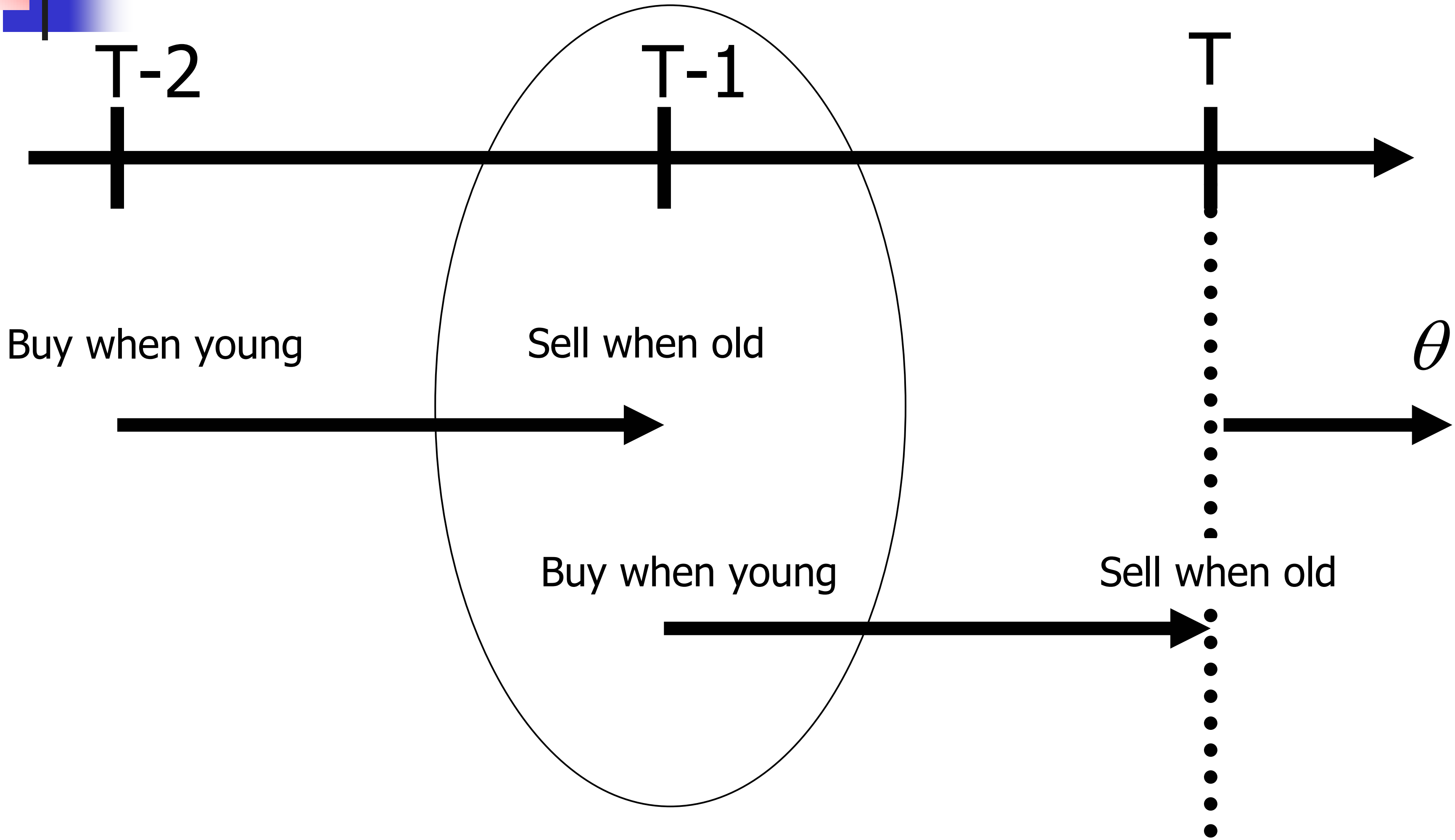
CARA-Gaussian Models

- Grossman (1976); Grossman and Stiglitz (1980):
  - CARA utility function.
  - Normal distribution for the fundamental.
  - Informed traders receive private signals.
  - Without noise traders, the price is fully revealing.
  - No one has an incentive to collect costly information (Grossman-Stiglitz paradox).

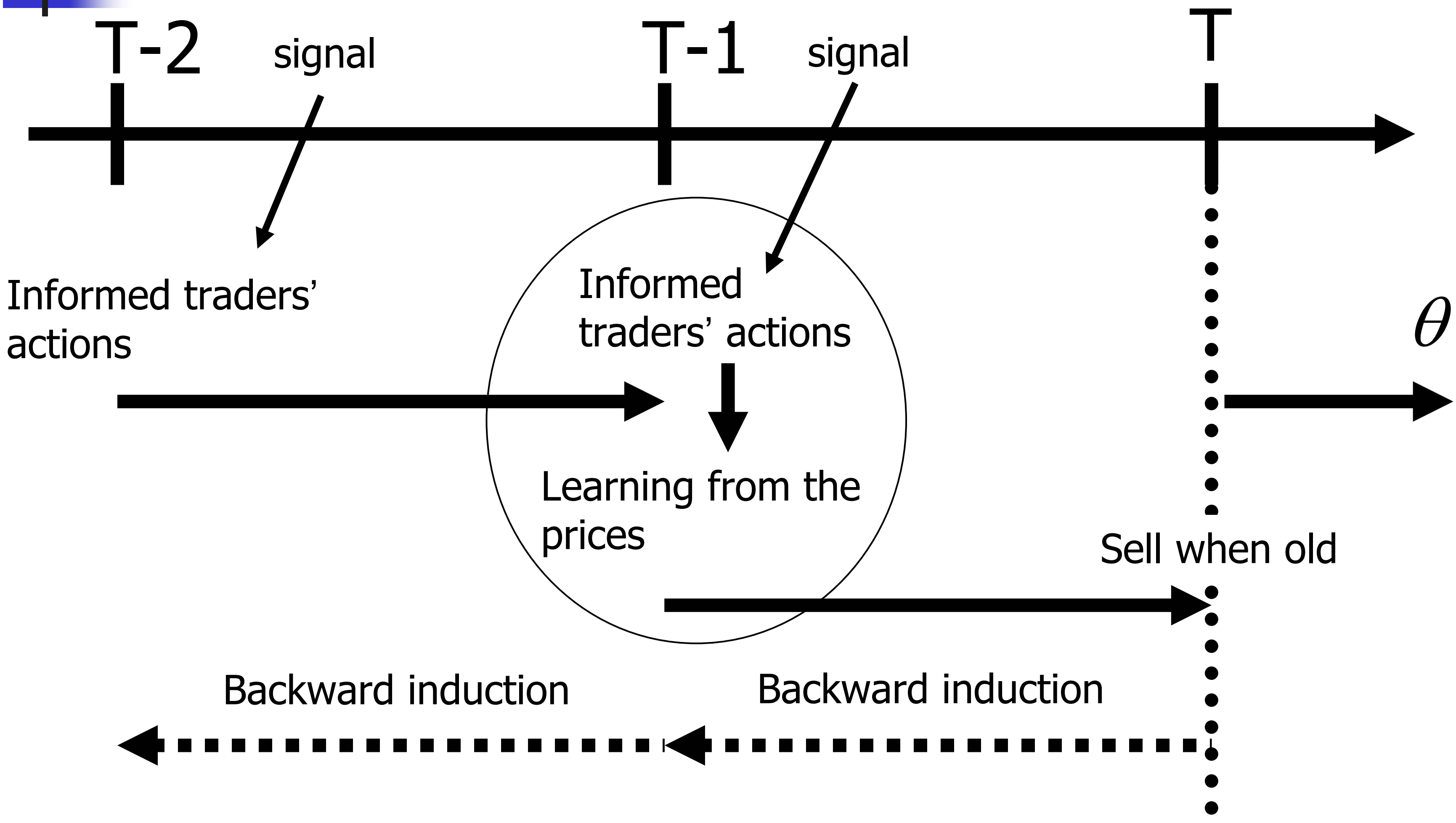
The AMS Model

- Allen-Morris-Shin (Review of Financial Studies, 2006).
- Finite horizon: T is the final round.
- Single risky asset with liquidation value $\theta \sim N(y, 1/\alpha)$.
- Traders live for 2 periods (OLG).
- Traders of generation T get θ in T+1.

The AMS Model



The AMS Model



The AMS Model

- The asset price at date t is

$$p_t = \overline{E}_t \overline{E}_{t+1} \cdots \overline{E}_T(\theta) + random$$

- In particular,

$$\underbrace{\overline{E}_t(\overline{E}_{t+1}(\theta))} \neq \overline{E}_t(\theta)$$

Average expectations about average expectations about the fundamentals = HOE = beauty contests.

In this paper…

- We introduce rational uninformed traders into the AMS framework to investigate the impact of asymmetric information on asset prices in the presence of beauty contests.

Asymmetric Information

- Gennotte and Leland (1990); Barlevy and Veronesi (2003):
  - Static model with uninformed traders.
  - A decrease in the price might be interpreted as a weak fundamental.
  - Multiple equilibria and crashes.
- Nonlinearity is introduced via noise traders.

Multiple linear equilibria

- Cespa (2002); Cespa and Vives (2007):
  - Two-period NREE model.
  - Noise trading has a serial correlation.
  - There is no informative signal in date 2.
  - Multiple linear equilibria (=crash?).

Question

- Does equilibrium multiplicity (=crash) arise as a result of beauty contests under asymmetric information?

Answer

- “Unlikely” under limit orders:
  - Traders submit their demand schedules.
- “Maybe” under market orders:
  - Traders submit the quantities demanded.

The Model (1)

- Finite horizon: T is the final round.
- Single risky asset with liquidation value $\theta \sim N(y, 1/\alpha)$.
- Traders live for 2 periods (OLG).
- Traders of generation T get θ in T+1.

The Model (2)

- Traders (OLG):
  - Informed traders: ϕ
  - Uninformed traders: $1-\phi$
  - Noise traders
- Noisy supply of asset: $s_t \sim N(0,1/\gamma)$

The Model (3)

- Each informed trader i receives a noisy signal about the fundamentals:

$$x_{it} = \theta + \varepsilon_{it}, \varepsilon_{it} \sim N(0,1/\beta)$$

- Informed trader i's information set:

$$\Omega_{it} = \{y, p_1, p_2, \cdots, p_t, x_{it}\}$$

The Model (4)

- Informed trader's decision problem:

$$\max_{X_{it}, M_{it}} E\left[U(w_i) \mid \Omega_{it}\right]$$

$$s.t.$$

$$p_t X_{it} + M_{it} = w_0$$

$$w_i = M_{it} + p_{t+1} X_{it}$$

The Model (5)

- W/O loss of generality, $w_0 = 0$:

$$\max_{X_{it}} E\left[U\left((p_{t+1} - p_t)X_{it}\right) \mid \Omega_{it}\right]$$

- Consumption function is of CARA class:

$$U(w) = -\exp\left\{-w/\tau\right\}$$

- τ: risk-tolerance parameter.

The Model (6)

- Gaussian property:

$$\max EU \Leftrightarrow \max E[w \mid \Omega] - \frac{1}{2\tau} \mathrm{Var}[w \mid \Omega]$$

- Asset demand function:

$$X_{it} = \frac{\tau\left[E\left[p_{t+1} \mid \Omega_{it}\right] - p_t\right]}{\mathrm{Var}\left[p_{t+1} \mid \Omega_{it}\right]}$$

Noise Traders (1)

- In general,

$$\int_0^1 X_{it}\,di = s_t + \underbrace{\int_0^1 X_{it-1}\,di}\ - \underbrace{s_{t-1}}$$

Old traders unwind their positions

- But, in date 1,

$$\int_0^1 X_{i1}\,di = s_1 \quad \Rightarrow \quad \int_0^1 X_{it}\,di = s_t$$

Noise Traders (2)

- Cespa (2002):

$$\int_0^1 X_{it}\,di = \int_0^1 X_{it-1}\,di + s_t$$

  - Noise traders are not OLG.
  - As a result,

$$\int_0^1 X_{it}\,di = s_{t-1} + s_t$$

Noise Traders (3)

- Cespa and Vives (2007):

$$\int_0^1 X_{it} di = \rho s_{t-1} + u_t$$

- Noise trading follows AR(1) process:
  - $\rho=0$: AMS
  - $\rho=1$: Cespa (2002)

Stock Market Equilibrium (1)

- Each trader submits his <u>demand schedule</u> to the market.
- Date T:

$$\underbrace{\int_0^{\phi} \frac{\tau\left[E\left[\theta \mid \Omega_{iT}\right]-p_T\right]}{\operatorname{Var}\left[\theta \mid \Omega_{iT}\right]} di}_{\text{Informed trading}} + \underbrace{(1-\phi) \frac{\tau\left[E\left[\theta \mid \Omega_{T}\right]-p_T\right]}{\operatorname{Var}\left[\theta \mid \Omega_{T}\right]}}_{\text{Uninformed trading}} = s_T$$

Noisy supply (s_T)

Stock Market Equilibrium (2)

- In the static equilibrium,

$$p_T = \xi_T^1 \overline{E}_T(\theta) + \left(1 - \xi_T^1\right) E\left[\theta \mid \Omega_T\right] - \xi_T^3 s_T,$$

$$\xi_T^1 \equiv \frac{1}{1 + \frac{1-\phi}{\phi} \frac{\mathrm{Var}\left[\theta \mid \Omega_{iT}\right]}{\mathrm{Var}\left[\theta \mid \Omega_T\right]}} \in (0,1)$$

Stock Market Equilibrium (3)

- When there are two trading dates,

$$p_T = \xi_{T-1}^1 \xi_T^1 \overline{E}_{T-1} \overline{E}_T(\theta) + \xi_{T-1}^1 \xi_T^2 \overline{E}_{T-1} E[\theta \mid \Omega_T] + \xi_{T-1}^2 \xi_T^1 E[\overline{E}_T \mid \Omega_{T-1}] + \xi_{T-1}^2 \xi_T^2 E[E[\theta \mid \Omega_T] \mid \Omega_{T-1}] - \xi_{T-1}^3 s_{T-1}$$

Stock Market Equilibrium (4)

- When there are two trading dates,

$$p_T = \xi^1_{T-1}\xi^1_T \underbrace{\overline{E}_{T-1}\overline{E}_T(\theta)}_{} + \left(1 - \xi^1_{T-1}\xi^1_T\right)\underbrace{y}_{} - \xi^3_{T-1} s_{T-1}$$

Average expectations about the average expectations about the fundamentals

Prior expectations about the fundamentals

Stock Market Equilibrium (5)

- In general,

$$p_t = K_t^1 \overline{E}_t \overline{E}_{t+1} \cdots \overline{E}_{T-1} \overline{E}_T(\theta) + \left(1 - K_t^1\right) y - \xi_t^3 s_t,$$

$$K_t^1 \equiv \xi_t^1 \xi_{t+1}^1 \cdots \xi_{T-1}^1 \xi_T^1 \in (0,1)$$

- In AMS,

$$p_t = \overline{E}_t \overline{E}_{t+1} \cdots \overline{E}_{T-1} \overline{E}_T(\theta) + random$$

Solving the Model (1)

- Two trading rounds.
- Method of undetermined coefficients:
  - Form price conjectures about p1 & p2
  - Learning from p1
  - Learning from p2
  - Solve for p2 and match coefficients
  - Solve for p1 and match coefficients

Solving the Model (2)

- An equilibrium is characterized by a solution to the system of equations regarding the coefficients.
- Key equation:

$$\gamma\mu_1^3 - \lambda A\mu_1^2 + (\beta_1 + B)\mu_1 - AB - A\phi_2\beta_2 = 0,$$

$$A \equiv \frac{\tau(\phi_1\tau\beta_1)(\phi_2\tau\beta_2)\gamma}{1+\tau^2\phi_2\beta_2\gamma}, B \equiv \alpha + (\phi_2\tau\beta_2)^2\gamma.$$

- There are at most three linear NREE.

Solving the Model (3)

- Proposition: If $\phi_1=\phi_2$ and $\beta_1=\beta_2$, then there is a unique linear NREE.
- Implication: with OLG noise traders, equilibrium multiplicity (=crash?) does not arise even under asymmetric information.
- Information aggregation is efficient (?).

Market Orders

- Informed traders submit market orders.
  - Their information set in date 1 is $\{y, x_1\}$.
  - In date 2, $\{y, p_1, x_2\}$.
- Uninformed traders submit limit orders.
  - Their information set in date 1 is $\{y, p_1\}$.
  - In date 2, $\{y, p_1, p_2\}$.
- There are at most eight linear NREE.
  - I am still working on it.

Conclusion

- Things to do:
  - (Hopefully) full characterization of all NREE.
  - Numerical examples.
- Future work:
  - News media
  - Market participation
  - Monetary policy