Costly Self Control vs. Overwhelming Temptation

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Introduction

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More formally, if x is the set of options available, utility of current self is given by

$$\max_{\alpha \in B_v(x)} u(\alpha).$$

u is the utility function of the current self, v is the utility function of the future or deciding self, and $B_v(x)$ is set of v maximizers in x.

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3. Axiomatic foundations? Seems ad hoc.
4. Lack of a recursive structure for more periods.

Gul–Pesendorfer (GP) [2001], overcomes these problems.

Idea: Costly self–control. x is the set of options, and utility is:

$$\max_{\alpha \in x}[u(\alpha) - c(\alpha, x)]$$

where

$$c(\alpha, x) = \left[\max_{\beta \in x} v(\beta)\right] - v(\alpha).$$

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- *Compromise:* choice will maximize $u + v$, not v.
- *Cost to resisting temptation:* c.
- GP provide *axioms* and a *recursive dynamic version*.

GP's model also has Strotz as a limiting case.

$$\lim_{\lambda\to\infty}\left\{\max_{\alpha\in x}[u(\alpha)+\lambda v(\alpha)]-\max_{\beta\in x}\lambda v(\beta)\right\}=\max_{\alpha\in B_v(x)}u(\alpha).$$

Intuitively, as λ increases, the magnitude of temptation increases till there is no room for compromise.

In line with this, GP refer to Strotz model as overwhelming temptation.

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Can incorporate this by adding uncertainty about v. Consider a *random Strotz* model:

$$\int_W \max_{\beta \in B_w(x)} u(\beta) d\mu(w).$$

If μ is suitably atomless, this representation is continuous.

But still seems restrictive in allowing only extreme temptations; no compromise, no cost to resisting temptation.

Random Strotz model also handles intuitive examples GP can't.

Example: Broccoli, chocolate, and potato chips.
Plausible ordering:

$$\{b\} \succ \{b, c\}, \{b, p\} \succ \{b, c, p\}.$$

GP evaluate menu x by

$$\max_{\alpha \in x}[u(\alpha) + v(\alpha)] - \max_{\beta \in x} v(\beta).$$

Since adding p to $\{b, c\}$ worsens menu, $v(p) > \max\{v(b), v(c)\}$.

But then how can adding c to $\{b, p\}$ make it worse?

Surprisingly this representation has GP as a special case.

That is, for any GP representation, we can find μ such that for every x,

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From this, we conclude that we've been too quick to drop Strotz's overwhelming temptation approach in favor of GP's costly self–control approach.

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- Comparatives: If one random Strotz preference is "more prone to temptation" than another, what is relationship of representations?
- Recursive infinite horizon versions of the model.

Other related papers:

- Benabou and Pycia, 2002
- Fudenberg and Levine, 2006
- Chatterjee and Krishna, 2008
- Stovall, 2008

Model

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X = menus of lotteries — nonempty, closed subsets of $\Delta(B)$.
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Preference is over *menus*. Approach due to Kreps [1979], extended to lotteries by Dekel, Lipman, and Rustichini [2001].

Idea. Use agent's view of options he leaves himself to say what possibilities he foresees.

Kreps: $\{\text{chicken}, \text{fish}\} \succ \{\text{chicken}\}, \{\text{fish}\}$ reveals uncertainty about which will be more appealing.

GP: $\{\text{salad}\} \succ \{\text{salad}, \text{ice cream}\} \succ \{\text{ice cream}\}$ reveals concern about temptation.

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Important point: Assuming $\succeq$ is over menus means we assume modeler only sees *choice of menus*, not *choice from menus*.

Thus we do *not* directly see compromise (nor of course temptation costs) — we simply try to infer this from agent's view of menus.

Representations

A *random costly self control (or random GP) representation* is an EU function, u, and a measure over EU functions, ν, such that $\succeq$ is represented by

$$\int_V \left\{ \max_{\alpha \in x} [u(\alpha) + v(\alpha)] - \max_{\beta \in x} v(\beta) \right\} d\nu(v).$$

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A *random overwhelming temptation (or random Strotz) representation* is an EU function u and a measure over EU functions μ such that $\succeq$ is represented by

$$\int_W \max_{\beta \in B_w(x)} u(\beta)\, d\mu(w)$$

where

$$B_w(x) = \{\alpha \in x \mid w(\alpha) \geq w(\beta),\ \forall \beta \in x\}.$$

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Surprisingly, proof uses basic auction theory.

Proof: For $A \in [0,1]$, let

$$\mathcal{U}^*(A,x) = \max_{\beta \in x}[v(\beta) + Au(\beta)]$$

and let $\beta^*(A,x)$ be a maximizer.

So

$$\mathcal{U}^*(A,x) = \max_{\bar{A} \in [0,1]}[v(\beta^*(\bar{A},x)) + Au(\beta^*(\bar{A},x))].$$

Analogy:

- A is type, which is valuation of object
- $v(\beta^*(\bar{A},x))$ is transfer if reported type is $\bar{A}$
- $u(\beta^*(\bar{A},x))$ is probability of getting good if reported type is $\bar{A}$

Standard incentive compatibility argument gives

$$\mathcal{U}^*(s,x) = \mathcal{U}^*(0,x) + \int_0^s u(\beta^*(A,x))\, dA.$$

So for $s = 1$,

$$\mathcal{U}^*(1,x) - \mathcal{U}^*(0,x) = \int_0^1 u(\beta^*(A,x))\, dA.$$

LHS is $\max[u+v] - \max v$, RHS is random Strotz. (Latter uses fact that ties won't matter.)

Intuition. $\{\text{salad}\} \succ \{\text{salad, ice cream}\} \succ \{\text{ice cream}\}$

GP interpretation: $\{s, i\} \succ \{i\}$ means agent chooses s from $\{s, i\}$.
$\{s\} \succ \{s, i\}$ means this is costly, so utility of $\{s, i\}$ is

$$u(s) - c(s, \{s, i\}).$$

Random Strotz: Agent isn't sure about choice from $\{s, i\}$, so utility is

$$pu(s) + (1 - p)u(i).$$

Equivalence says that

$$c(s, \{s, i\}) = (1 - p)[u(s) - u(i)].$$

Need to see choices *from menus* to separate these models.

Example.

	u	v	$u+v$
s	2	0	2
i	0	1	1

So GP compute value of $\{s, i\}$ as $2 - 1 = 1$.

Random Strotz: Find A type indifferent between s and i. That is,

$$v(s) + A^* u(s) = v(i) + A^* u(i)$$

Get $A^* = 1/2$, so i is chosen for $A \in [0, 1/2)$, s otherwise. Hence value of menu is

$$\frac{1}{2}(0) + \frac{1}{2}(2) = 1.$$

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We conjecture that it is but have only a more restrictive result.

Theorem

The set of preferences with a Lipschitz continuous random GP representation equals the set of preferences with a Lipschitz continuous random Strotz representation.

Axiomatic Characterization

Proof of one direction of theorem based on axiomatic characterization of the two representations.

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Axioms: Dekel, Lipman, Rustichini, and Sarver (DLRS) axioms for additive EU representations (weak order, two continuity axioms, independence) plus one axiom to specialize to temptation, namely:

Definition

$\succeq$ satisfies *weak set betweenness* if $\{\alpha\} \succeq \{\beta\}$ for all $\alpha \in x$ and $\beta \in y$ implies

$$x \succeq x \cup y \succeq y.$$

Uniqueness

Suppose $\succeq$ has a random Strotz representation (u, μ).

$$\int_{w} \max_{\beta \in B_w(x)} u(\beta)\, d\mu(w)$$

Easy to show that u is unique up to positive affine transformations.

Clearly, for μ to be unique, must normalize space of ws. Fix a normalization, $\mathcal{W}$. For rest of talk, μ is defined over $\mathcal{W}$.

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Theorem

If $\succeq$ is nontrivial and has a random Strotz representation, then μ is unique.

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What is the relationship between the measures in the representations?

Intuition: Think of $\mathcal{W}$ as points on the globe with u as the "North Pole" and $-u$ as the "South Pole."

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Essentially, pick $v \in \mathcal{V}$ at random and then pick A at random (where distribution of A can depend on v) to construct $w = v + Au$.

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1. They have the same marginal distribution over equator points (v's).
2. For any equator point, μ_1 has a conditional distribution over movement toward the North Pole (A's) which dominates that of μ_2 in the usual FOSD sense.

Infinite Horizon Models

Gul and Pesendorfer [2004] give a recursive infinite horizon extension.

Set up: Menu now consists of lotteries over current consumption c and continuation menus z.

Representation: Utility to a menu x is

$$W(x) = \max_{(c,z)\in x} [u(c) + v(c) + \delta W(z)] - \max_{(c,z)\in x} v(c).$$

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Using our translation of GP into random Strotz:

$$W(x) = \int_0^1 \max_{(c,z)\in B_{v(c)+Au(c)+A\delta W(z)}} [u(c)+\delta W(z)]\, dA.$$

Example. Initial endowment of K units of divisible, costlessly storable commodity.

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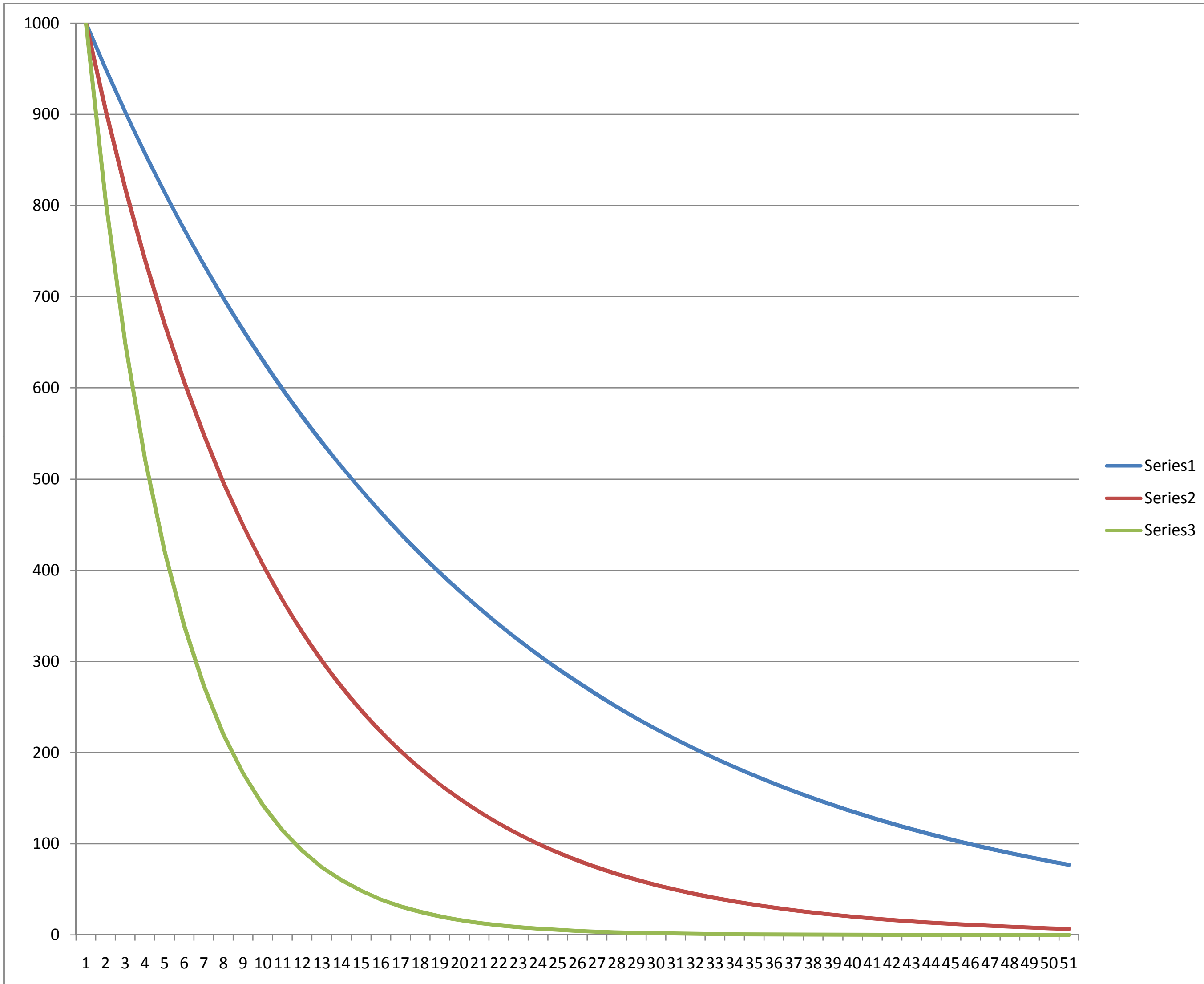
Under either GP or RS, get same value function $W(k)$.

Can also calculate the consumption/savings path in GP and random Strotz. Of course, it's random in the latter.


1000
900
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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51
Series1
Series2
Series3

Formal result: For any $\varepsilon \in (0, 1)$, let $p_T(\varepsilon)$ be the probability that RS savings at t are less than ε times GP savings at t for all $t \geq T$.

Claim: For every $\varepsilon > 0$,

$$lim_{T \to \infty} p_T(\varepsilon) = 1.$$

That is, savings under random Strotz are eventually trapped below ε of the GP savings with probability 1.

Intuition: In random Strotz, difference between $W(k)$ and what it would be without temptation is expected overconsumption; in GP, it is self–control costs.

Conclusion

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